Actual gradiometer measurement

Actual Accelerometer output

measured acceleration (x,y,z = sensitive axes of the accelero) ⟵ **real acceleration (X,Y,Z = orthonormal reference frame)**

$$?_{meas} \; ? \; K_0 + (I + dK + dO + dS)\, ?_{real} + ?_n + k_2 ?^2$$

Source of errors

bias **noise**

CALIBRATION

Scale factor **Misalignment** **Coupling** **Quadratic terms**

dK diagonal

$$dO \; ? \begin{pmatrix} 0 & ? & ? \\ ? & 0 & ? \\ ? & ? & 0 \end{pmatrix} \qquad dS \; ? \begin{pmatrix} 0 & ? & ? \\ ? & 0 & ? \\ ? & ? & 0 \end{pmatrix}$$

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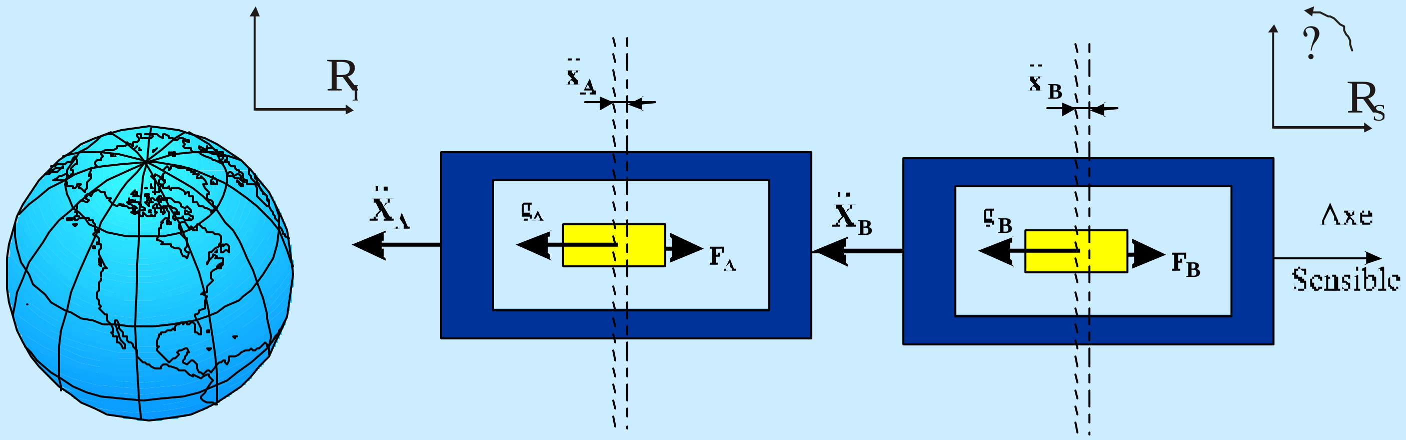
Actual gradiometer measurement

Proof-Mass motion:



$$m_{I_A} ?\ddot{\vec{X}}_A ? \ddot{\vec{x}}_A ? ? m_{g_A} g_A ? F_A ? F_{p_A}$$

=> **Accelerometer A Output**

$$\frac{\hat{F}_A}{m_{I_A}} ? ?1 ? K_A ?F_A ? E ?F_A ?? E_{n_A}$$

Difference of 2 accelerometer measurements:

$$\frac{\hat{F}_A}{m_{I_A}} ? \frac{\hat{F}_B}{m_{I_B}} ? ?\ddot{\vec{X}}_A ? \ddot{\vec{X}}_B ?? ?K_A ? K_B ?\frac{\ddot{\vec{X}}_A ? \ddot{\vec{X}}_B}{2}$$

$$? ?\ddot{x}_A ? \ddot{x}_B ?? 2? ?\dot{x}_A ? \dot{x}_B ?? ?? ? ? ? ? ?? ?x_A ? x_B ?$$

$$? ?\frac{m_{g_A}}{m_{I_A}} ? \frac{m_{g_B}}{m_{I_B}} ?? \frac{g_A ? g_B}{2} ?$$

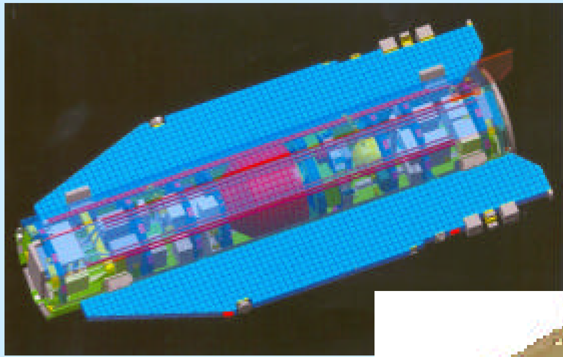
$$? \frac{F_{P_A}}{m_{I_A}} ? \frac{F_{P_B}}{m_{I_B}}$$

$$? E ?F_A ?? E ?F_B ?? E_{n_A} ? E_{n_B}$$

- (1) Common mode acceleration
- (2) Proof-masses relative motion
- (3) Difference of Gravity field effects
- (4) Non gravitational acceleration
- (5) Instrument

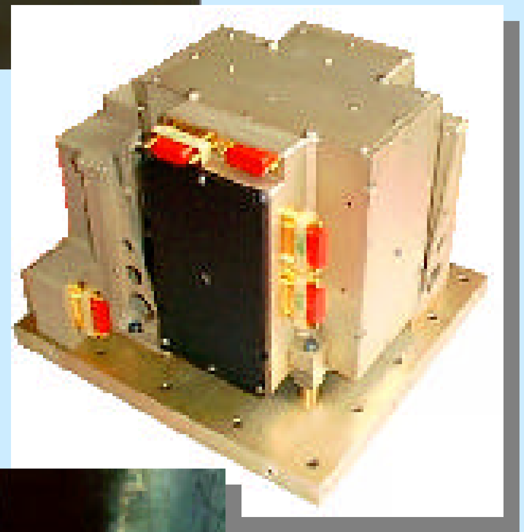
A drag-free satellite:
a request for space gradiometry

(1/2)



to reduce $\ddot{X}_i$, ? , $\mathring{?}$

✍ **Drag-Free system composed of :**

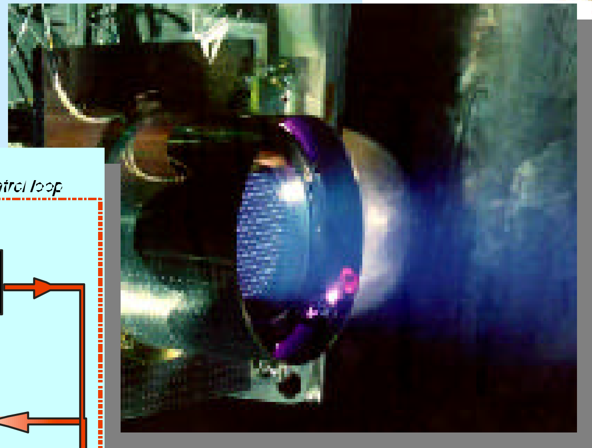


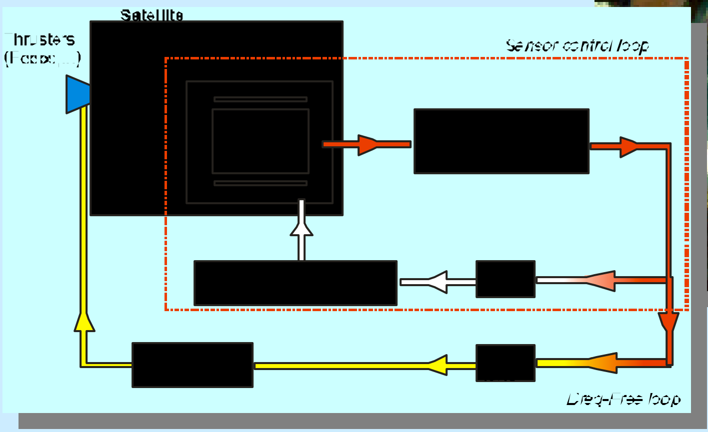
+ Accelerometers or Inertial mass and position sensing (sensors)

+ Propulsion system (actuators)

+ Control laws of servo-loop (attitude and orbit)

- ***Sensor : High sensitivity and stability***
- ***Electric or Cold gas propulsion :***
  - ? Low thrust
  - ? Fine continuous (or not) control for manoeuvres
  - ? Linearity and low thrust noise





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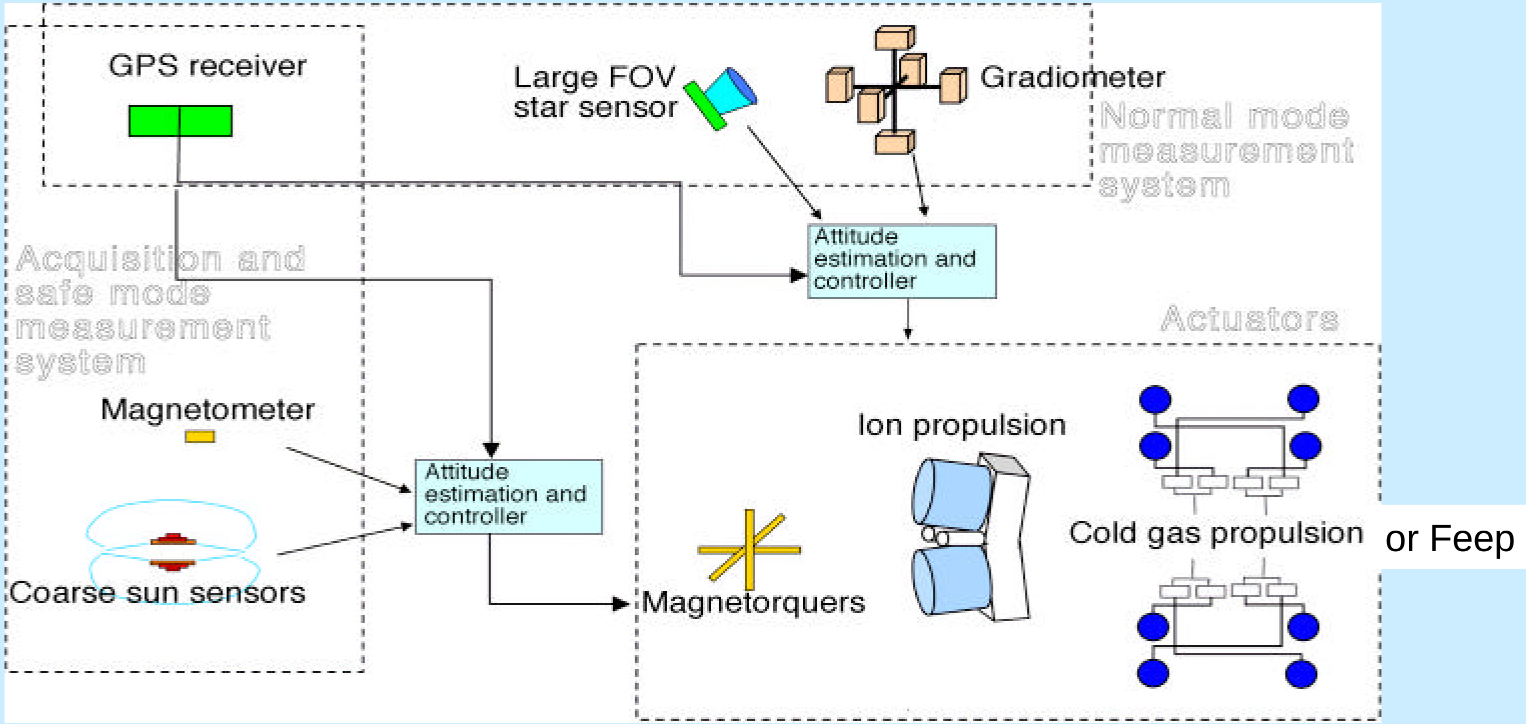
GOCE : Attitude and Drag Control architecture (2/2)



✍ **Compensation of S/C external forces and torques: atmospheric drag, radiation pressure, magnetic, gravity gradients... (associated with fine attitude pointing)**

to obtain a pure gravitational orbit @ low altitude

to preserve fine instrumentation from disturbing acceleration

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Attitude and Drag Control

GOCE

System Requirements depending on instrument calibration

GOCE specifications

- Linear acceleration : 5.10^{-7} m/s^2 (maxi), < $2.5 \cdot 10^{-8}$ ms^{-2}/? Hz (mbw)
- Angular acceleration: 10^{-6} rad/s^2 (maxi), < $1.5 \cdot 10^{-8}$ rads^{-2} /? Hz (mbw)
- Angular velocity : 10^{-5} rad/s (maxi)+orbital y < 10^{-6} rads^{-1} /? Hz (mbw) along x,z and < $5 \cdot 10^{-7}$ rads^{-1} /? Hz (mbw) y
- Earth Pointing : 0.5 mrad (maxi) < 2.10^{-5} rad/? Hz (mbw)
- Low flying altitude (250 km / 2.10^{-5} ms $^{-2}$ drag) drives:
  - redundant system for the ‘nominal’ modes (measurement and thrusters)
  - Two propulsion technologies : Ion thrusters and FEEP/Cold Gaz fine thrusters

*mbw : measurement frequency bandwidth = 5.10^{-3} Hz -10^{-1} Hz

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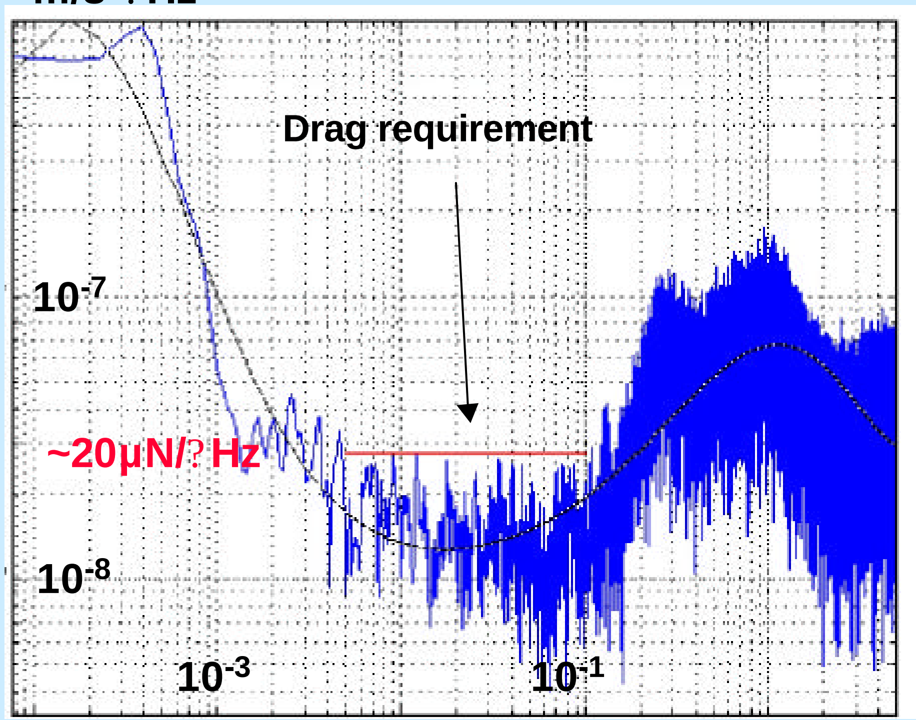
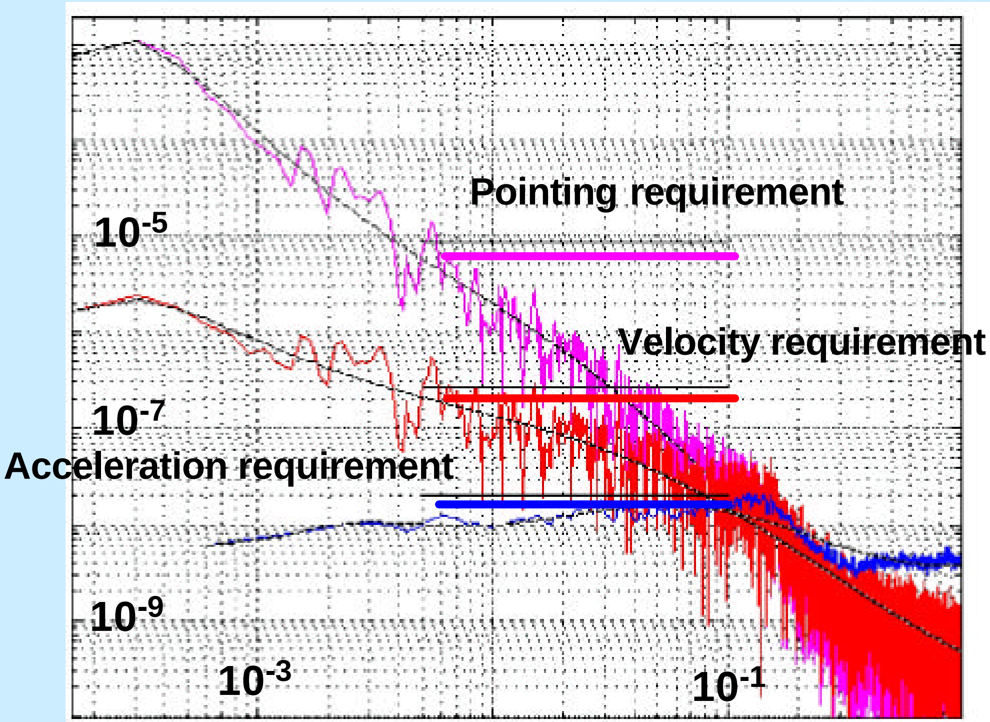
GOCE : Attitude and Drag Control Performances

m/s²?Hz



Frequency (Hz)

unit/?Hz



Pointing requirement: 8.6·10^{-6} rad/?Hz fulfilled ? **(2 mE?Hz)**

Drag control requirements: 2.5·10^{-8} m/s²?Hz fulfilled ? **(0.9 mE?Hz)**

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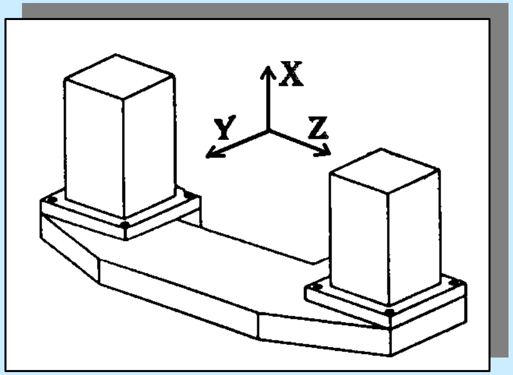
One-axis Gradiometer sensitivities



$$?_{meas1} = K_{01} + (I + dM_1)?_{real1} + ?_{n1} + k_{21}?_{?}^2$$

$$?_{meas2} = K_{02} + (I + dM_2)?_{real2} + ?_{n2} + k_{22}?_{?}^2$$

Common acceleration

Differential acceleration

$$?_{meas,c} ? \frac{?_{meas,1} ? ?_{meas,2}}{2} ? ?f\ (\text{drag} \ldots)$$

$$?_{meas,d} ? \frac{?_{meas,1} ? ?_{meas,2}}{2} ? ?f\ ' (? \ ?? \ ??????$$

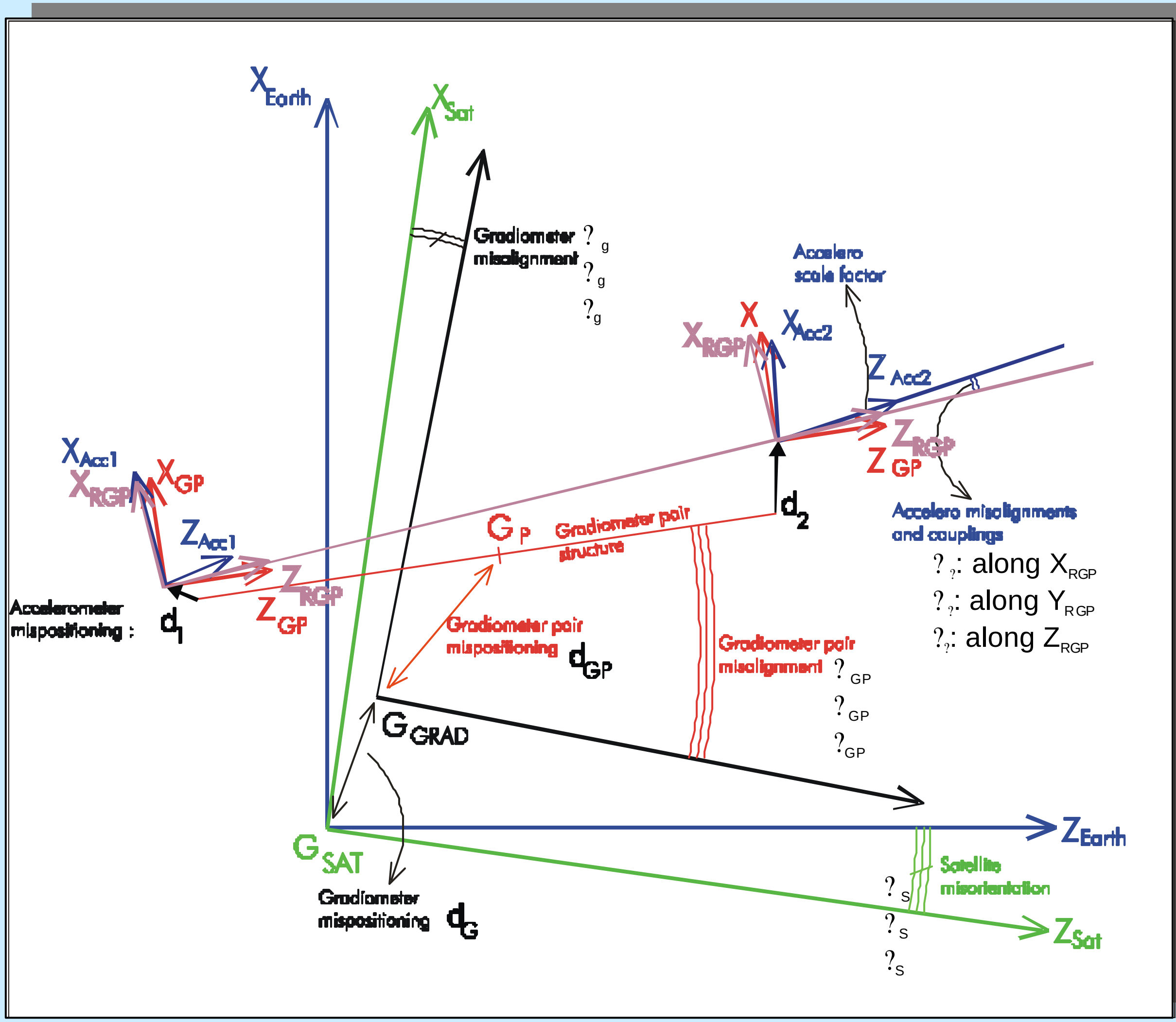
$$\begin{pmatrix} ??_{meascx} \\ ??_{meascy} \\ ??_{meascz} \\ ??_{measdx} \\ ??_{measdy} \\ ??_{measdz} \end{pmatrix} ? \frac{1}{2} \begin{pmatrix} K_{02x} ? K_{01x} \\ K_{02y} ? K_{01y} \\ K_{02z} ? K_{01z} \\ K_{02x} ? K_{01x} \\ K_{02y} ? K_{01y} \\ K_{02z} ? K_{01z} \end{pmatrix} ? \begin{pmatrix} K_{cx} & ?_c ? ?_c & ? ?_c ? ?_c & K_{dx} & ?_d ? ?_d & ? ?_d ? ?_d \\ ? ?_c ? ?_c & K_{cy} & ?_c ? ?_c & ? ?_d ? ?_d & K_{dy} & ?_d ? ?_d \\ ?_c ? ?_c & ? ?_c ? ?_c & K_{cz} & ?_d ? ?_d & ? ?_d ? ?_d & K_{dz} \\ K_{dx} & ?_d ? ?_d & ? ?_d ? ?_d & K_{cx} & ?_c ? ?_c & ? ?_c ? ?_c \\ ? ?_d ? ?_d & K_{dy} & ?_d ? ?_d & ? ?_c ? ?_c & K_{cy} & ?_c ? ?_c \\ ?_d ? ?_d & ? ?_d ? ?_d & K_{dz} & ?_c ? ?_c & ? ?_c ? ?_c & K_{cz} \end{pmatrix} \begin{pmatrix} ??_{cX} \\ ??_{cY} \\ ??_{cZ} \\ ??_{dX} \\ ??_{dY} \\ ??_{dZ} \end{pmatrix} ? \begin{pmatrix} ?_{ncx} \\ ?_{ncy} \\ ?_{ncz} \\ ?_{ndx} \\ ?_{ndy} \\ ?_{ndz} \end{pmatrix} ? \begin{pmatrix} 2k_{2dx}?_{cx}?_{dx} ? k_{2cx}(?_{cx}^2 ? ?_{dx}^2) \\ 2k_{2dy}?_{cy}?_{dy} ? k_{2cy}(?_{cy}^2 ? ?_{dy}^2) \\ 2k_{2dz}?_{cz}?_{dz} ? k_{2cz}(?_{cz}^2 ? ?_{dz}^2) \\ 2k_{2cx}?_{cx}?_{dx} ? k_{2dx}(?_{cx}^2 ? ?_{dx}^2) \\ 2k_{2cy}?_{cy}?_{dy} ? k_{2dy}(?_{cy}^2 ? ?_{dy}^2) \\ 2k_{2cz}?_{cz}?_{dz} ? k_{2dz}(?_{cz}^2 ? ?_{dz}^2) \end{pmatrix}$$

To be reduced or calibration

To be reduced or measured

Calibration Approach:

(1/3)



Many reference frames !
Positions !
Alignments !

RGP=Real Gradiometer Pair

(frame linked to the accelerometer centers)

GP=Gradiometer Pair

(frame linked to the structure)

Calibration principle:

✍ **to shake the instrument at well known frequency, phase and direction**

✍ **to observe common and differential outputs**

Calibration Approach: 2 possible technics (2/3)

ARISTOTELES

(ESA mission cancelled in 1993)

Independent calibration device to provide well known accelerations along/about absolute reference frames

✍ **Linear and angular shaking**

✍ **Quality of the shaker = quality of the calibration**

GOCE

Accelerometers are used to control the shaking of the satellite, with thrusters also used for DFC (servo-loop)

✍ **Linear and angular shaking along accelerometer reference frame with accelerometer performance**

✍ **no need of a specific device**

GOCE calibration is less demanding for the instrument and the satellite
example: perpendicularity of linear shaking ~ 10^{-2} rad in GOCE,
instead of 5.10^{-6} rad in ARISTOTELES !!!

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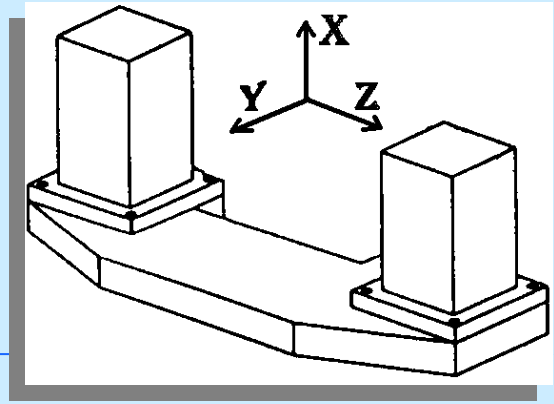
Calibration Approach: GOCE Performance (3/3)



Output expression: $?_{meas} ? M.?_{real} ? ...$

Measurement correction: $?_{real} ? M^{?1}.?_{meas} ? ... ? B.?_{meas} ? ...$ → **Estimation of M^{-1}**

$$?_{meas,dz} ? \frac{B_{d,zx}}{B_{c,zz}} ?_{meas,cx} ? \frac{B_{d,zy}}{B_{c,zz}} ?_{meas,cy} ? \boxed{\frac{B_{d,zz}}{B_{c,zz}}} ?_{meas,cz} ? \frac{B_{c,zx}}{B_{c,zz}} ?_{meas,dx} ? \frac{B_{c,zy}}{B_{c,zz}} ?_{meas,dy} ? \frac{1}{B_{c,zz}} ?_{dZ}$$
$$? noise ? bias ? non ? linear ? terms$$

$$? \frac{dK_{dz}}{K_{cz}} ? 2\frac{?_c ?_d}{K_{cz}K_{cy}} ? 2\frac{?_c ?_d}{K_{cz}K_{cx}} ? \frac{?_c^2 ? ?_d^2}{K_{cz}K_{cy}} ? \frac{dK_{dy}}{K_{cy}} ? \frac{dK_{dz}}{K_{cz}} ? ? \frac{?_c^2 ? ?_d^2}{K_{cz}K_{cx}} ? \frac{dK_{dx}}{K_{cx}} ? \frac{dK_{dz}}{K_{cz}} ? ? HOT$$

6 - axis excitation and observation (5.10^{-7} ms^{-2} @ a few 10^{-2} Hz)
Whole matrix evaluated:
- **scale factor matching 5.10^{-6}**
- **alignments 10^{-5} rd**

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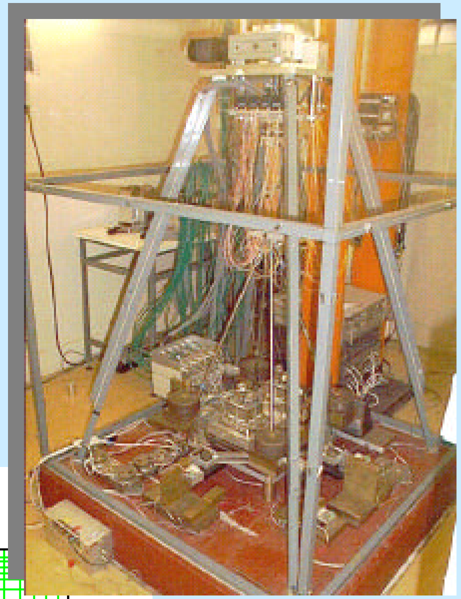
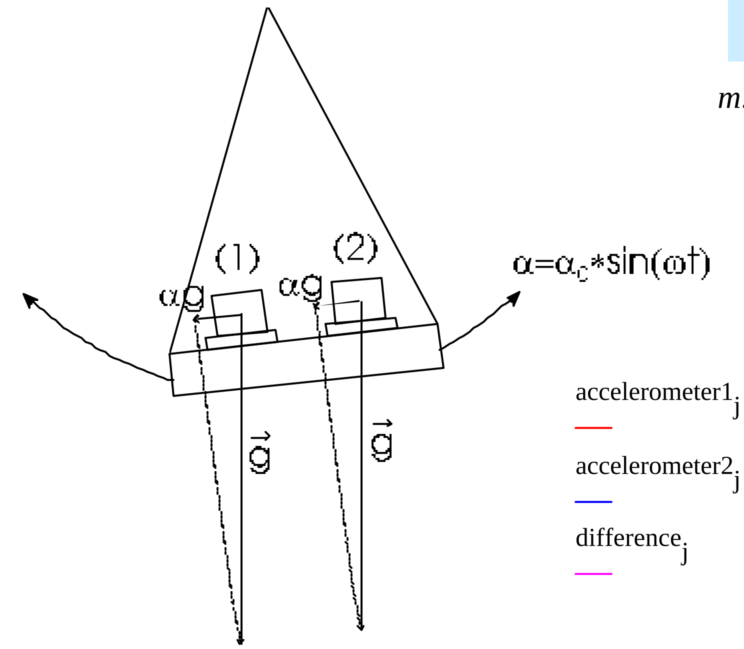
Ground verification:
Scale factor matching

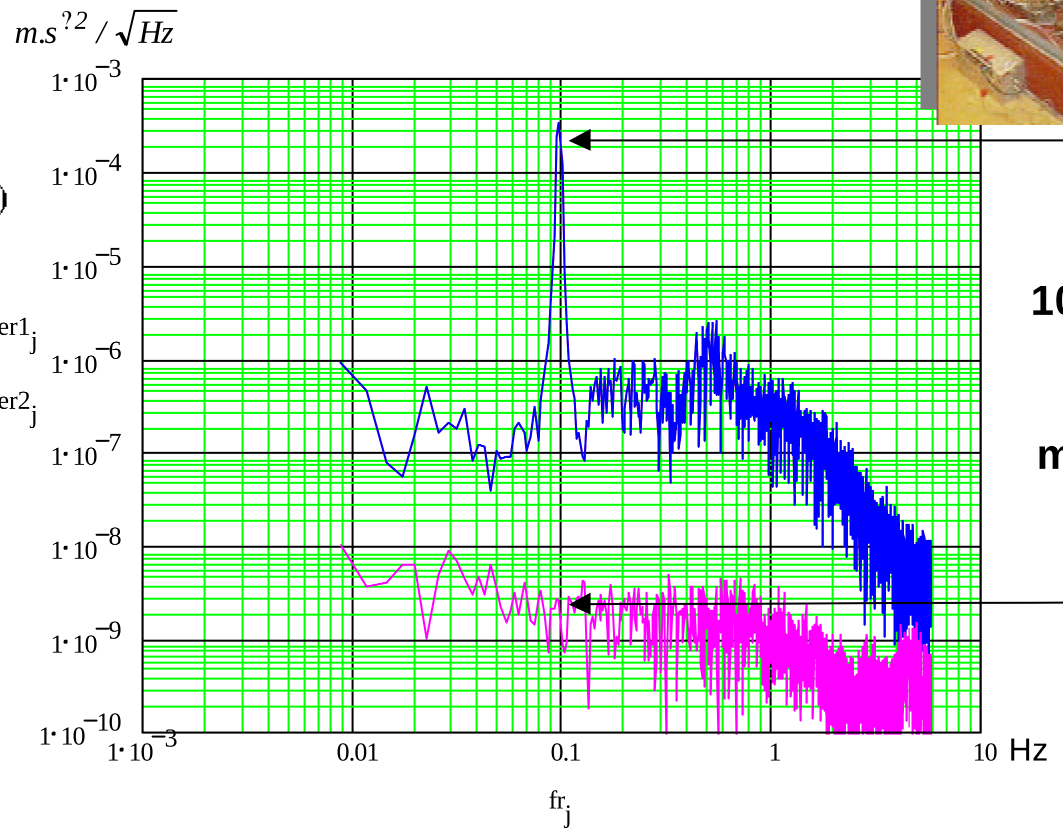


Test bench shaking at low freq.



the projection of ‘g’ is the useful signal.

After numerical correction



10^{-5} rejection of the common mode signal

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Scale factor matching versus frequency

(✍ higher gravity field spherical harmonics to be measured)

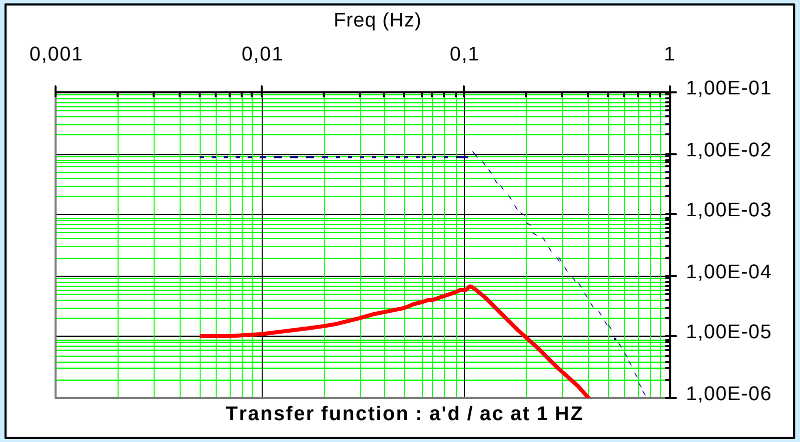
Scale factor difference versus frequency

a(t) continuous → Accelerometer loop → a'(t) continuous → Decimation at 10 Hz → a'*(t) 10 Hz → Decimation at 1 Hz → a'**(t) 1 Hz

Error on transfer function

Geometric differential scale factor

$$? \; a^{**'}_{d,14}(f) \; ? \; \left|H^{1Hz}\right|^2 ? \, k_c^{l\,2} \left|H_c^{10Hz}\right|^2 \left|H_d^V\right|^2 \; ? \; k_c^{l\,2} \left|H_d^{10Hz}\right|^2 \left|H_c^V\right|^2 \; ? \; k_d^{l\,2} \left|H_c^V\right|^2 \; ? \ldots ? ? \; a_c$$

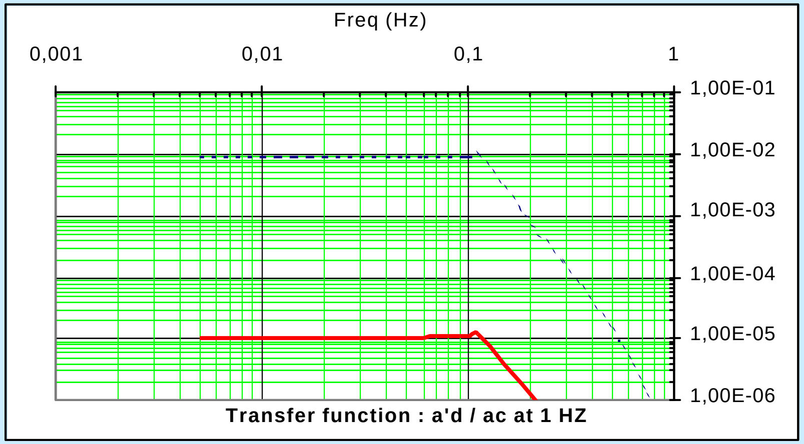


Before calibration

10^{-2}

10^{-5}

After calibration



10% of relative error on constant time

1% of relative error on constant time

- ✍ necessity of very plat frequency low
- ✍ calibration in measurement bend width

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Effects of accelerometer non linearities

Instrument non linearities due to : *- accelerometer electrostatic*

- operationelectronics non linearities

Induced high frequency signal aliasing (satellite/instrument structure behaviour, environment fluctuations, thrusters…)

Model with quadratic and cubic non linearities ? acc² and acc³

Quadratic non linearity sensitivity : calibrated in orbit and corrected

Cubic non linearities and decimation : limited by design and environment control

Aliasing

$$\left|H^{1Hz}\right|^2 ?\left|H_d^{10Hz}\right|^2 ?\ a_c^{'} ?\left|H_c^{10Hz}\right|^2 ?\ a_d^{'} ?\ H_d^{10Hz}(p) ?\ a_c^{'} a_d^{'} H_c^{10Hz}(?\,p) ?\ H_c^{10Hz}(p) ?\ a_d^{'} a_c^{'} H_d^{10Hz}(?\,p) ?$$

Differential cubic aliasing due to error between accelerometer (geometric and electronic)

Parameter	Symbol	Value
Relative error on mass of the proof-mass	$?m/m$	$7\ 10^{-4}$
Relative error on surface of electrode	$?S_c/S=?S_d/S$	10^{-3}
Relative error of gap	$?e/e$	$3\ 10^{-3}$
Relative common error on polarisation voltage	$?V_{pc}/V_p$	10^{-5}
Relative differential error on polarisation voltage	$?V_{pd}/V_p$	2.10^{-3}
Relative capacitive sensor bias	y_0/e	$4.5\ 10^{-5}$
Common error on amplifier gain	$?Gm_c$	10^{-3}
Differential error on amplifier gain	$?Gm_d$	10^{-4}
Error on DVA bias	$?\ V_c ?? ?V_d$	10^{-3} V

Cubic factor

Common: 5.30 10^{14} V.s⁴/m³

Differential: 6.64 10^{12} V.s⁴/m³

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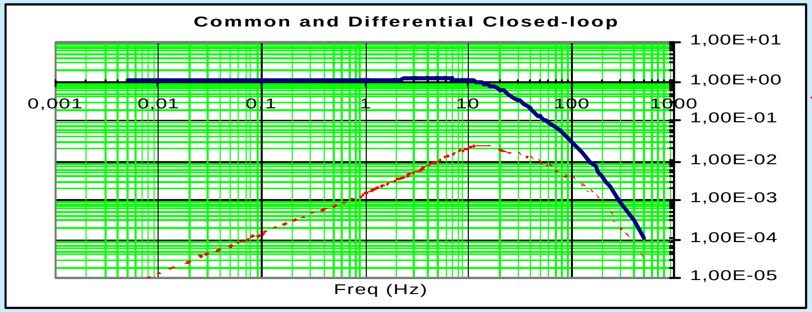
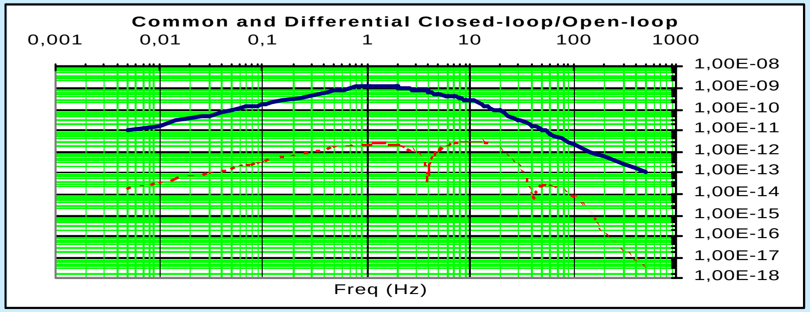
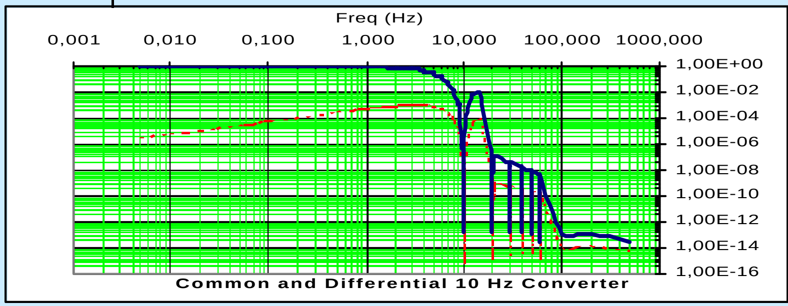
Rejection of high frequency: accelerometer loop

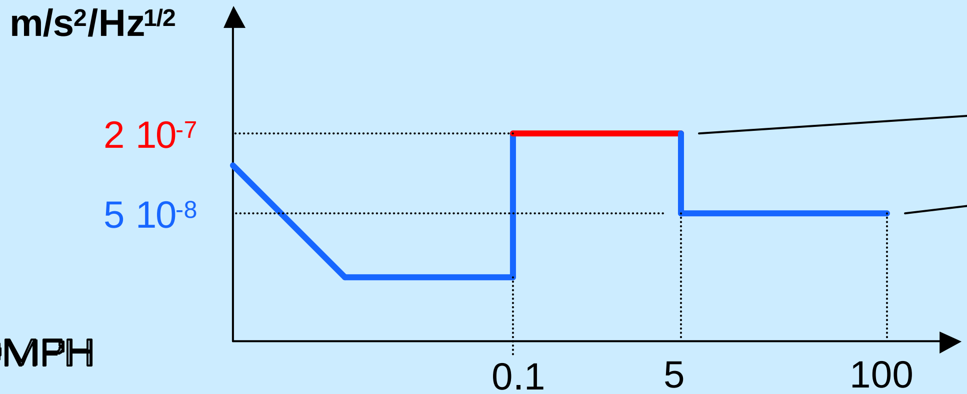
Maximal Modulus	Common	Differential
Closed-loop	1.22	$2.37\ 10^{2}$
H^{Y} (m/V)	$1.16\ 10^{-9}$	$3.17\ 10^{12}$
H^{10Hz}	1	$5.71\ 10^{-5}$







PSD of common acceleration



Maximal cubic aliasing

$0.017\ mE/Hz^{1/2}$

$0.005\ mE/Hz^{1/2}$

$0.018\ mE/Hz^{1/2}$

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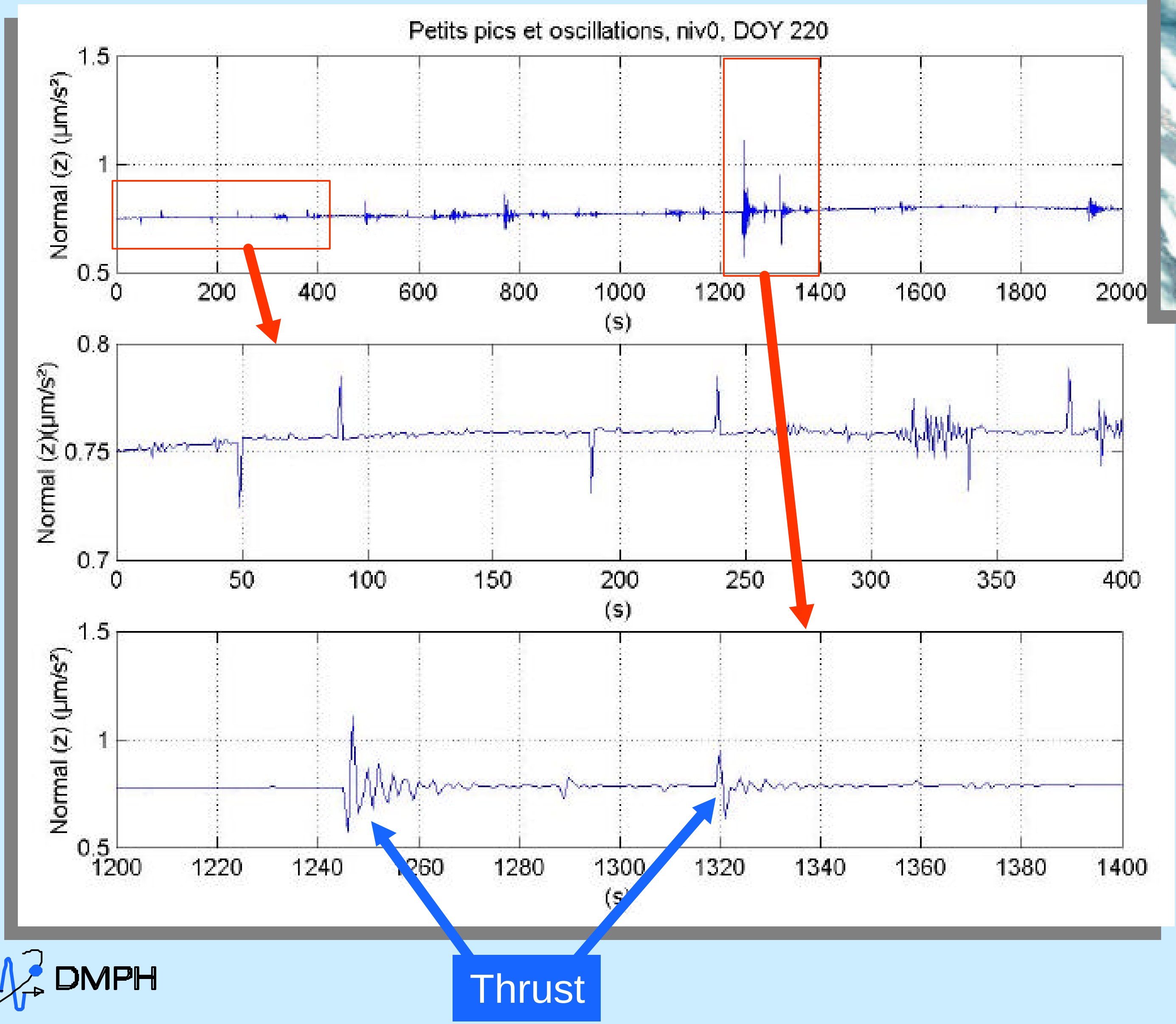
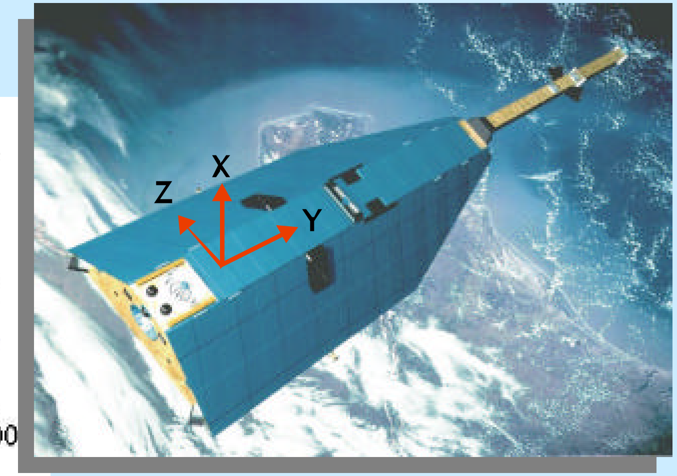
STAR measurements





Heater switch on / off

Magnetometer boon oscillation
(observed along 2 axes
not along X)

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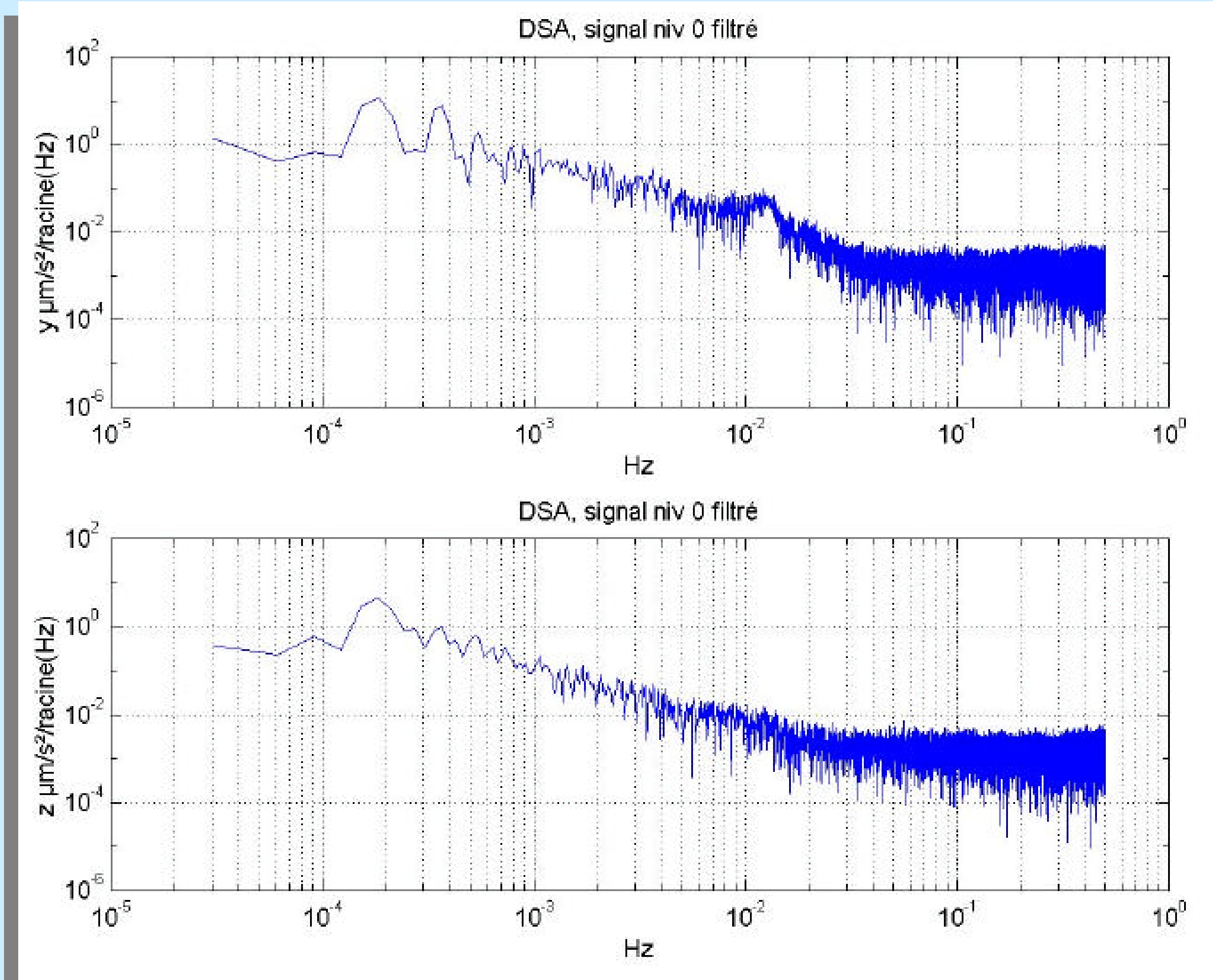
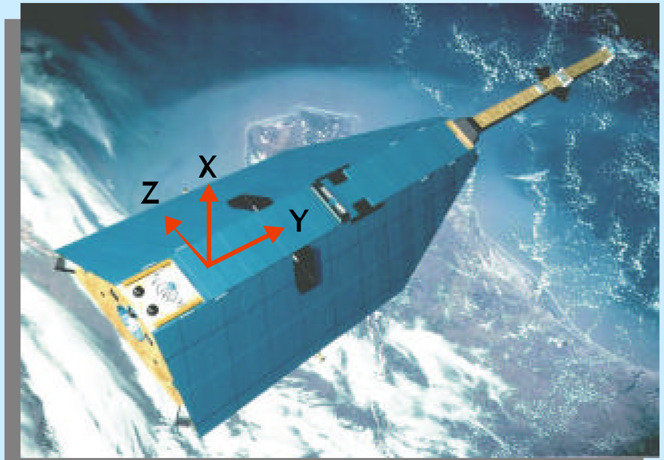
STAR measurements





Grégory Pradels 12/00

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Gradiometer accuracy

✍ **Soft and study environment: orbit + satellite + instrument case**

✍ **Differential measurement & rejection of non gravity sensor signal:**

- ✍ ***Concept + configuration***
- ✍ ***Structure + positionning + alignment***
- ✍ ***Calibration + linearity***

✍ **Gravity field measurement accuracy:**

- ***Accelerometer sensitivity***
- ***Data processing***

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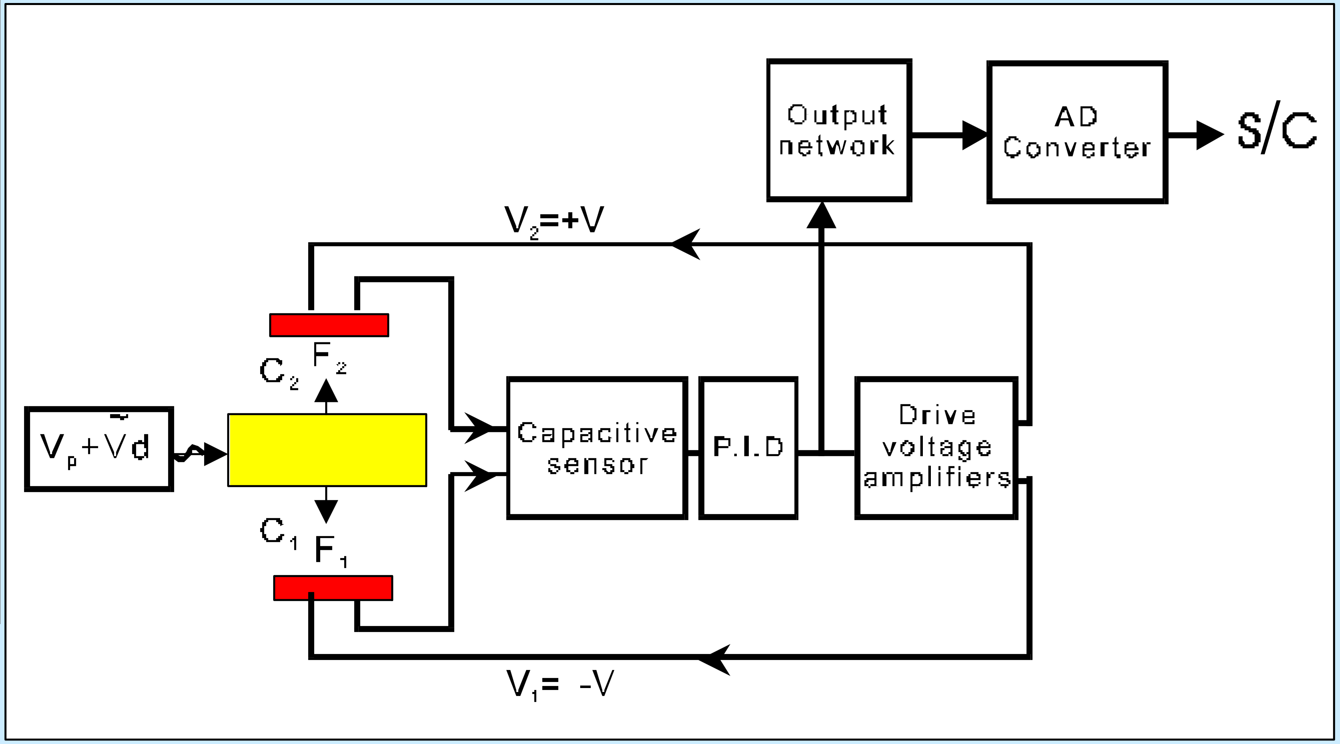
Accelerometer main features



- Proof mass : motion and attitude servo-controlled
- Capacitive position sensor
- Electrostatic actuators
- 6 degrees of freedom measurement

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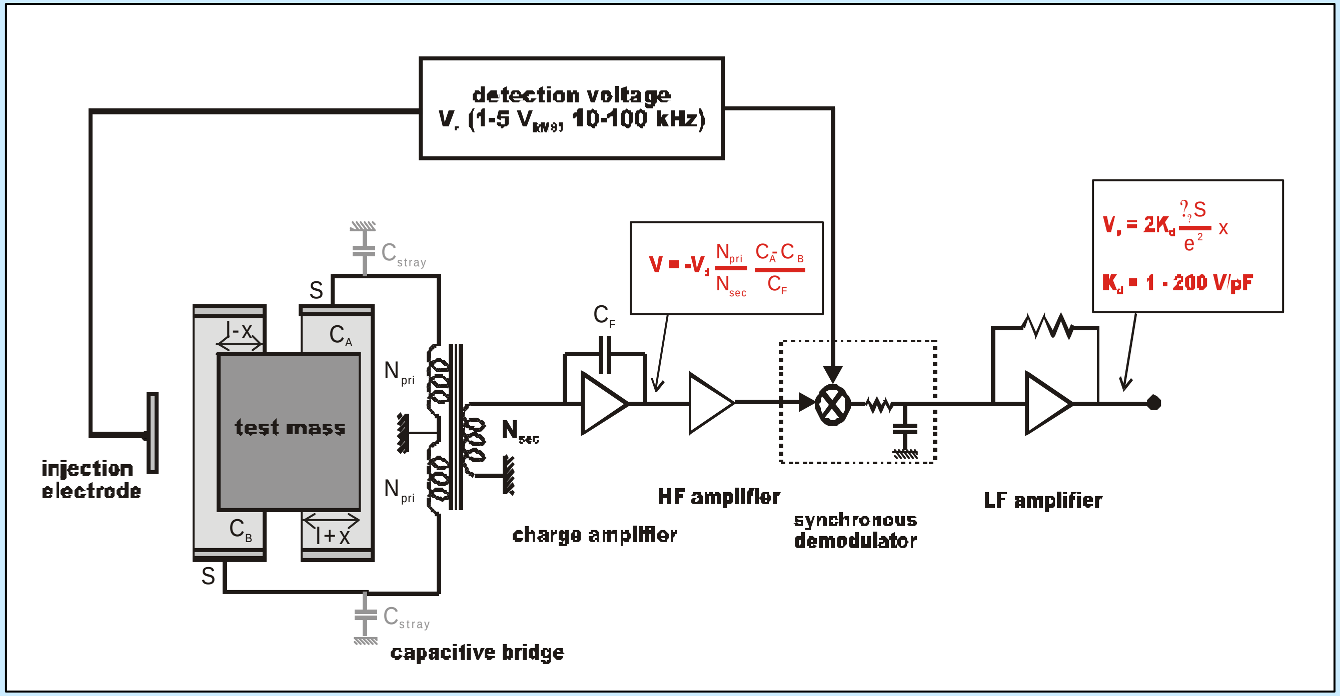
The capacitive sensor



To optimise :

- the resolution
- the bias stability
- the rejection of stray capacitance
- the force back action

GRADIO configuration

Scale factor : 50 V/pF (i.e. 2.4 10^4 V / m)

Resolution : 1.2 10^{-7} pF / $Hz^{1/2}$ within 5.10^{-3} Hz to 100 Hz

i.e. 5.10^{-12} m / $Hz^{1/2}$ for 300 µm gap

and compatible with 10^{-13} N / $Hz^{1/2}$

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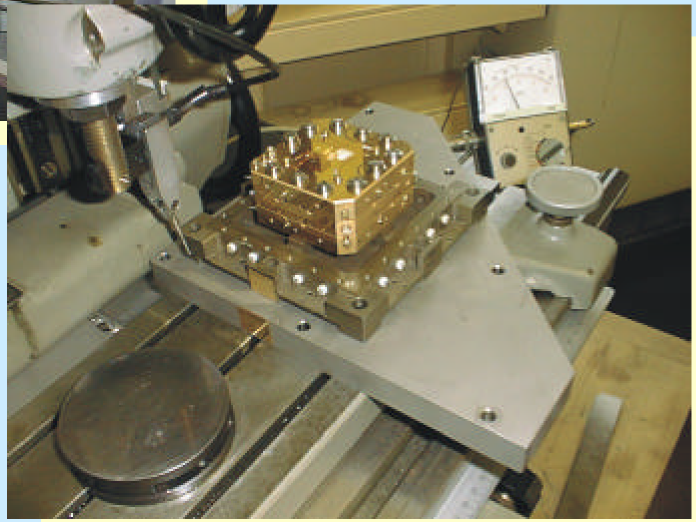
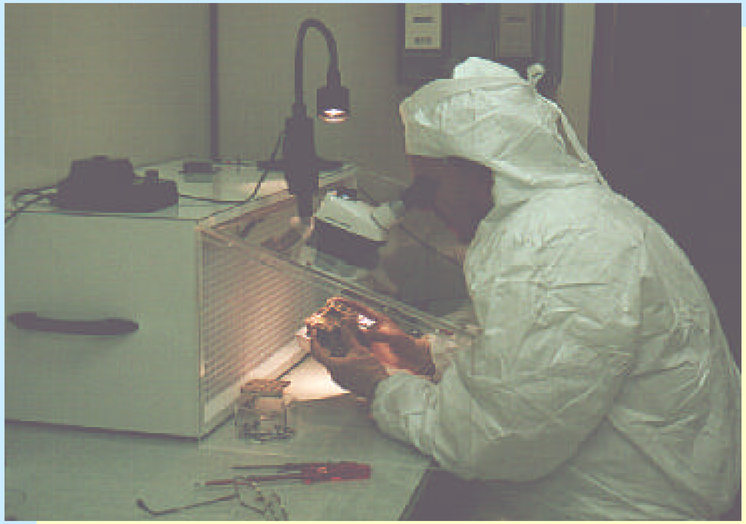
Sensor Core





- ULE material
- Optical grinding
- Ultrasonic machining
- Gold coating (sputtering)
- Clean room integration
- micromys, arcsecond

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Accelerometer Output model

$$?_{mes_i} \; ? \; K_{0_i}$$

$$? \; K_{1_i} ?_i$$

$$? \; K_{2_i} ?_i^2$$

$$? \; \underset{j ? i}{?} \; K'_{1ij} ?_j$$

$$? \; noise$$

Bias

- Disymetries of restoring electrostatic forces
- Parasitic forces on proof-mass
- Measurement electronics

Scale factor

- Mechanical geometry
- Reference voltage

Non-linearities (Quadratic term)

- Disymetries of restoring electrostatic forces

Cross-coupling

- Orthogonality of surfaces

Noise

- Fluctuations of bias sources
- Random desturbing forces on proof-mass
- Thermal sensitivity & random temperature fluctuations

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Bias Fluctuations

Disturbing force on proof-mass

Radiometer effect

$$n? \;\; ? \;\; \frac{1}{2}\frac{PS}{mT} ? n? T ?$$

Radiation presure

$$n? \;\; ? \;\; \frac{32}{3}\frac{? ST^{3}}{mc} ? n? T ?$$

Earth's magnetic field fluctuations

$$n? \;\; ? \;\; 3\frac{?_m V}{?_0 m} ? n \frac{B^2_{Earth}}{R_{orbit}} ?$$ (is attenuated by 50)

Spacecraft's magnetic field fluctuations

$$n? \;\; ? \;\; 3\frac{?_m V}{?_0 m} ? n \frac{B^2_{sc}}{R_{sc}} ?$$ (is attenuated by 50)

Lorentz force fluctuations

$$n? \;\; ? \;\; \frac{?_{orbit} R_{orbit}}{m} ? n Q_c B_{Earth} ?$$ (is attenuated by 50)

Gas damping

$$n? \;\; ? \;\; \frac{1}{m}\sqrt{4k_b T ?}$$

Gold wire stiffness

$$n? \;\; ? \;\; \frac{k_w}{m} x_n$$

Satellite gravity gradient

$$n? \;\; ? \;\; \frac{2Gm ? n}{?^{3}}$$

Gold wire damping

$$n? \;\; ? \;\; \frac{1}{m}\sqrt{4k_b T H_w}$$

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Electronics noise

$$n? \;\; ? \;\; \frac{?S}{me^2} \, ? \, \frac{?S}{S} \;\; ? \;\; \frac{V_{\mathrm{det}bias}}{eG(V\,/\,F)} \; ? \begin{matrix} ?nV_p \;\; ? \\ ?nV_d \;\; ? \\ ?nV_{cpd} \; ? \\ ?nV_{dva} \; ? \end{matrix}$$

Electronics Noise sources

Geometrical dissymetries

Detector bias

Position sensor noise

$$n? \;\; ? \;\; ?? \, {}^{2}_{p} \; ? \; ? \, {}^{2} ?x_n$$

Drive Amplifier noise

$$n? \;\; ? \;\; \frac{?S}{me^2} V_p ?nV_{dva} ?$$

(can be rejected by control loop gain...)

Contact Potential Difference noise

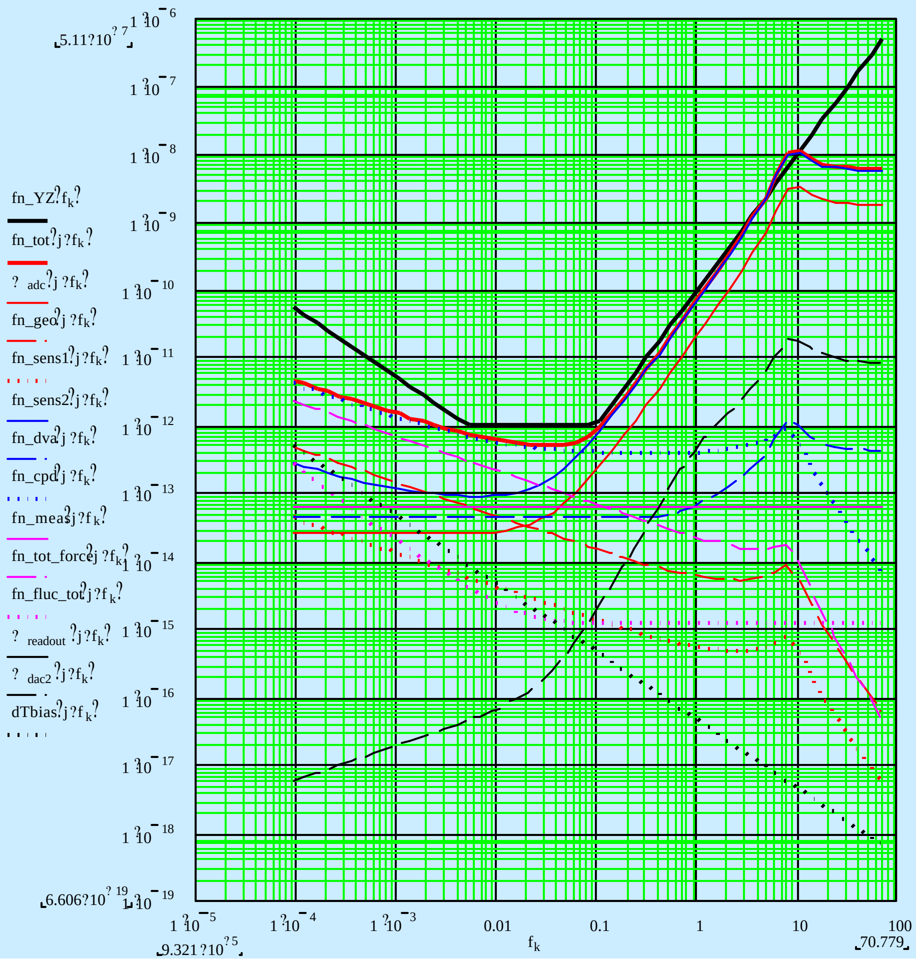
$$n? \;\; ? \;\; \frac{?S}{me^2} ?V_p nV_{cpd} \; ? \; V_{cpd} nV_p ?$$

Measurement amplifiers

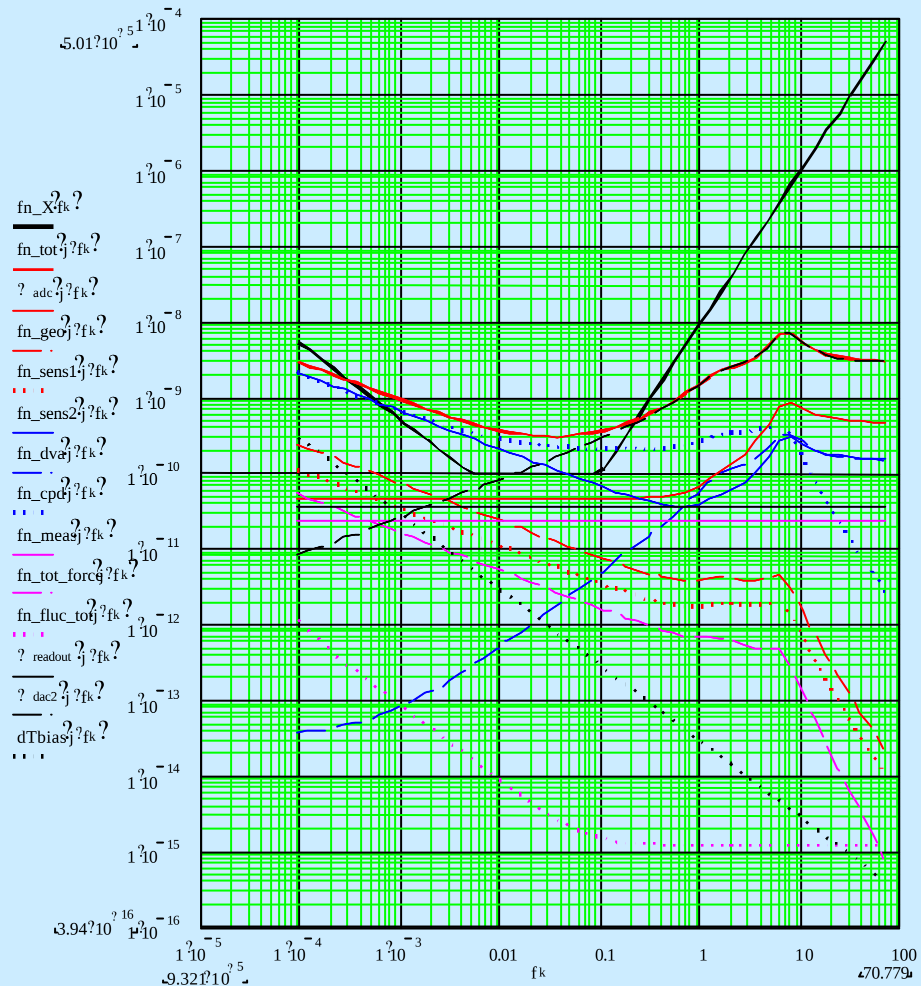
$$n? \;\; ? \;\; \frac{?S}{me^2} V_p ?nV_{meas} ?$$

(Measurement noise re-expressed at electrode level throught DVA and Meas. ampli. gains)

Most updated Budget concerning Accelerometer noise
GOCE



Science: Ultra-sensitive



Science: Less-sensitive

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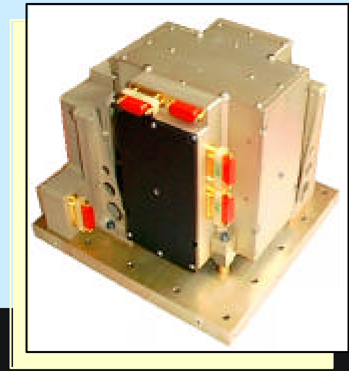
STAR Accelerometer for CHAMP Mission

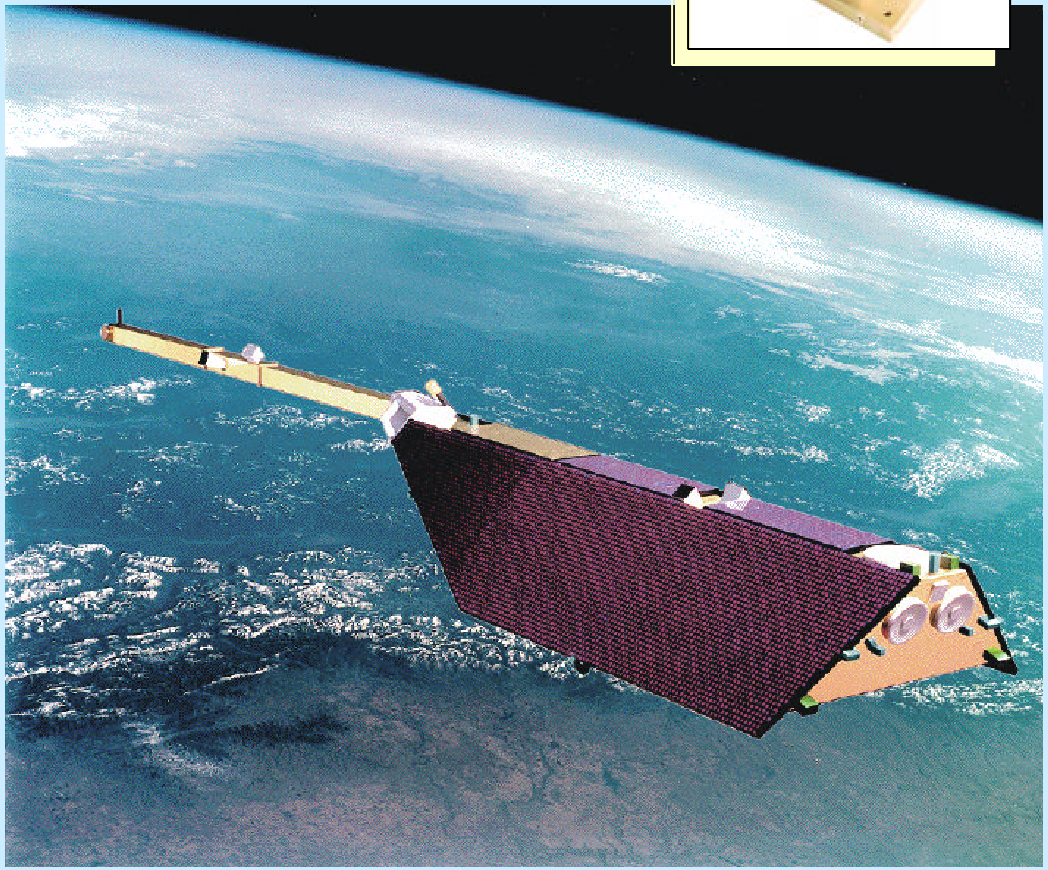


demonstration of GRADIO accelerometer concept and technology
but much larger range $10^{-3} m s^{-2}$ and so less resolution

Major specifications

- Measurement Range 10^{-3} m.s^{-2}
- Meas. bandwidth 10^{-4} Hz - 10^{-1} Hz
- Resolution 3 10^{-9} m.s^{-2} rms
- Integration period 1 second
  digital output 19 bits + sign
- Mass 9.7 kg (SU+EEU) 3.0 kg (ICU)
- Volume 13 litres (SU+EEU) 4 litres (ICU)
- Power 6.8 W

- Launched on july 15, 2000
- From switch / on, continuous operation in Orbit



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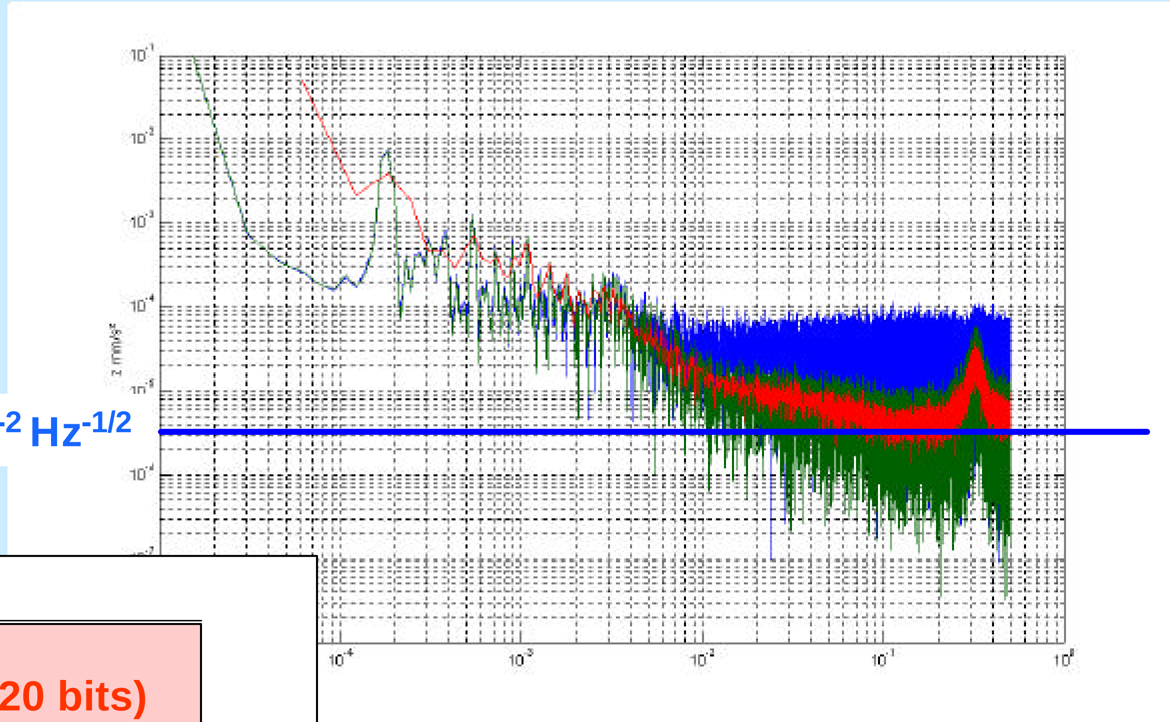
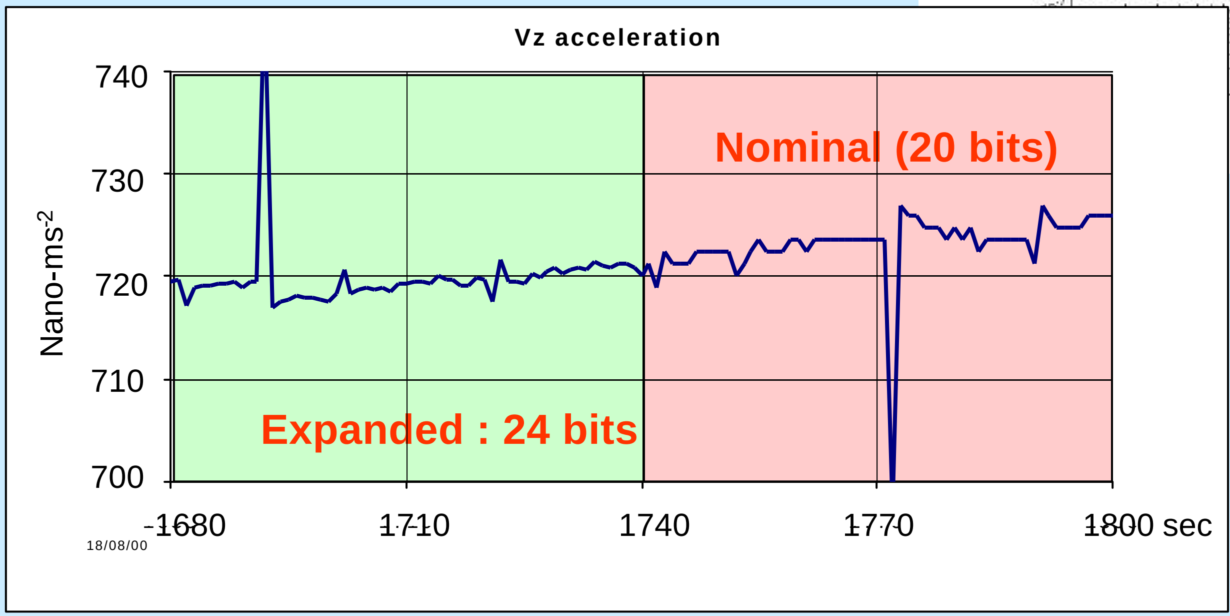
SU + EEU

STAR resolution

- Difficult to demonstrate in orbit
- Time and / or spectral analysis

Day 290, Z axis, normal to the orbit plane





Blue: rough data
Green: corrected data, FFT 65.10^3pts
Red: corrected data, FFT 16.10^3pts, mean (9)

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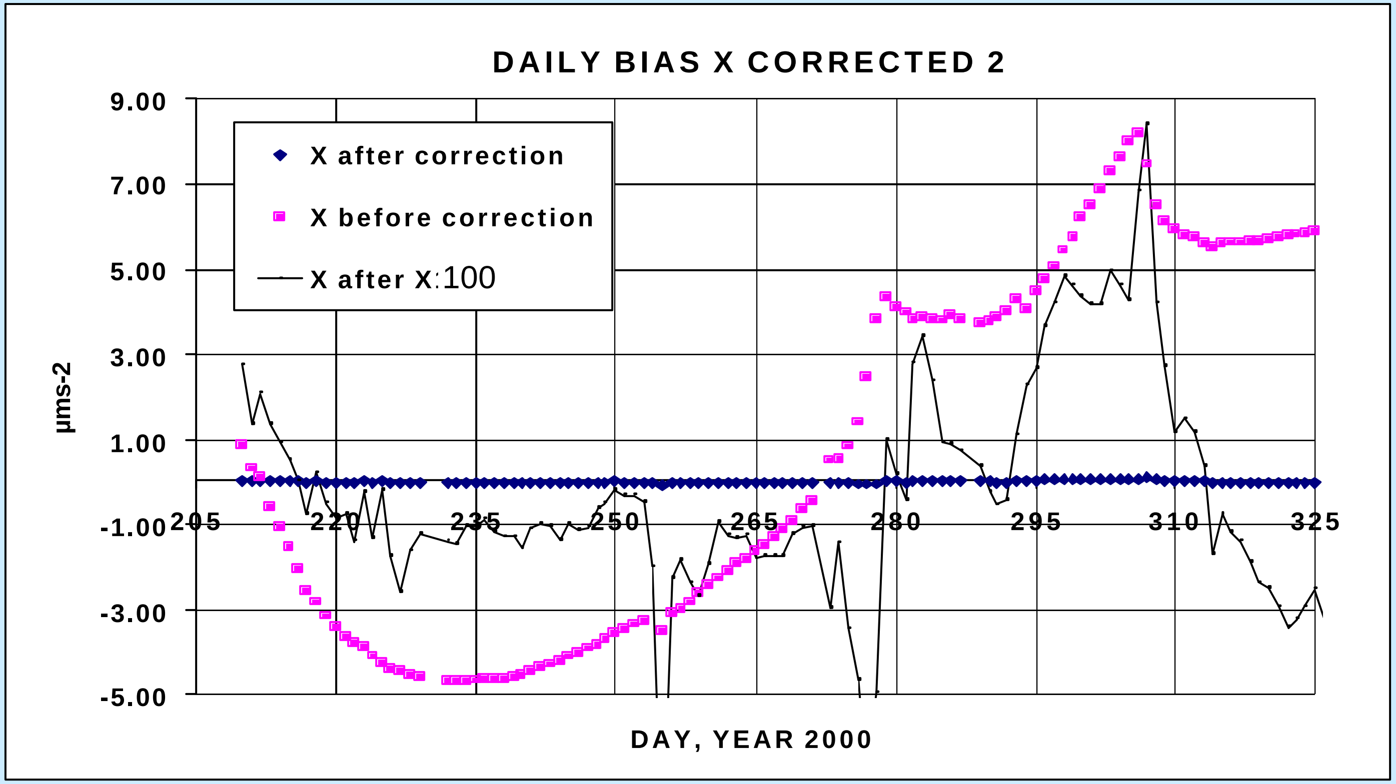
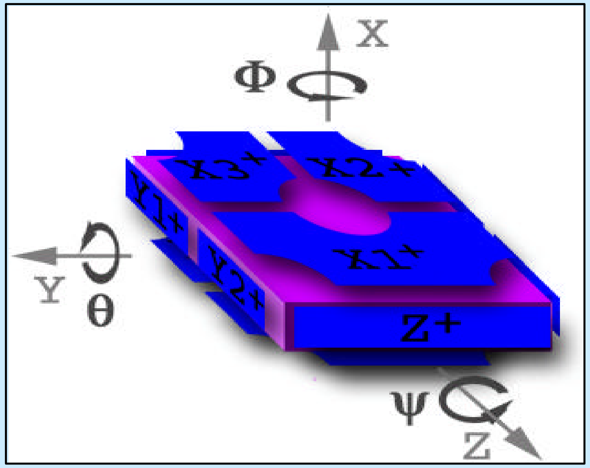
STAR / CHAMP : in flight thermal variations induced by X3

Servo still possible because Ve <Vx3 < Vp
=> Correction of the measured acceleration

$$\begin{pmatrix} ?(?_x)_{cor} \\ ?(l?)_{cor} \\ ?(l?)_{cor} \end{pmatrix} ? \begin{pmatrix} 1 & \frac{6?_x}{8?_?} & \frac{1?_x}{8?_?} \\ 0 & \frac{2}{8} & ?\frac{1?_x}{8?_?} \\ 0 & ?\frac{3?_?}{2?_?} & \frac{3}{4} \end{pmatrix} \begin{pmatrix} ?(?_x)_{mes} \\ ?(l?)_{mes} \\ ?(l?)_{mes} \end{pmatrix}$$





Vx3

X3 +

DVA

Test-Mass : Vp = 10 Volts

X3 -

Ve ~ mV

~~DVA~~

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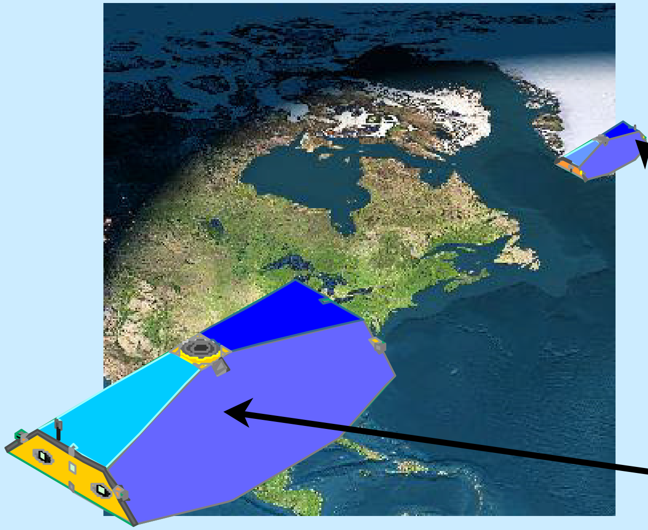
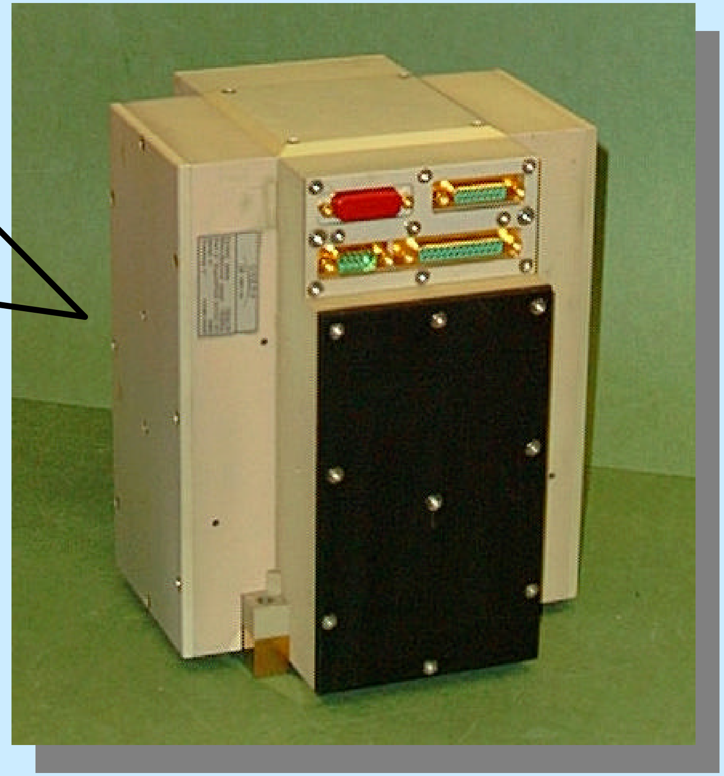
Accelerometer Specification Overview





SuperSTAR

- Measurement range $5\ 10^{-5}\ m.s^{-2}$
- Bandwidth $5\ 10^{-5}$ Hz - $4\ 10^{-2}$ Hz
- Noise $S(f)=10^{-20}(1+5\ 10^{-3}/f)\ m^2.s^{-4}.Hz^{-1}$
- Bias $C_0 < 2\ 10^{-6}\ m.s^{-2}$
- Scale factor $C_1=1.0$ +/- 2%
- Non linear term (quadratic) $C_2 < 20\ m^{-1}.s^2$
  (cubic) $C_3 < 10^4\ m^{-2}.s^4$

Orbit 480 km (↘) , 89°

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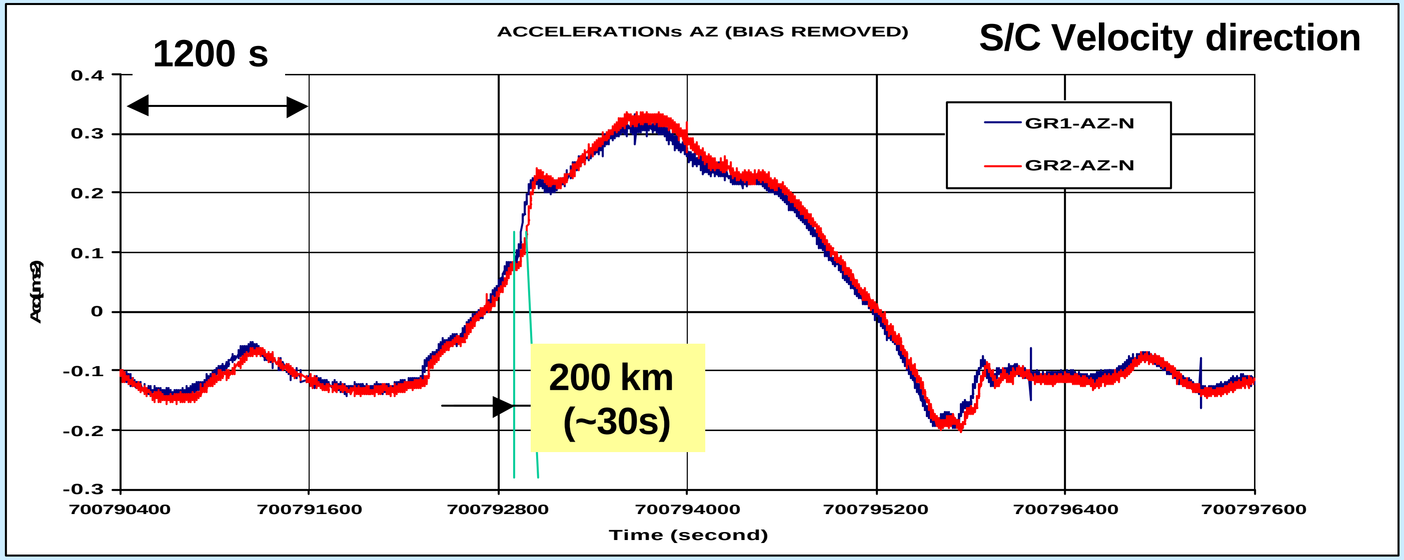
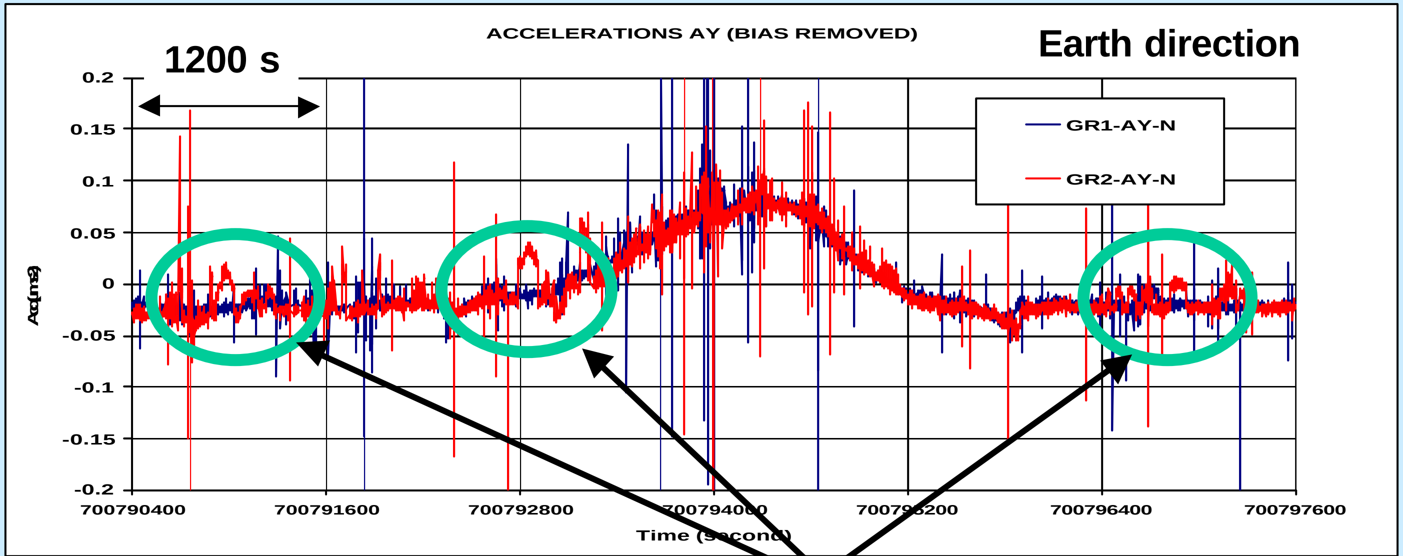
GRACE Mission : first day data





Thruster boost for attitude control

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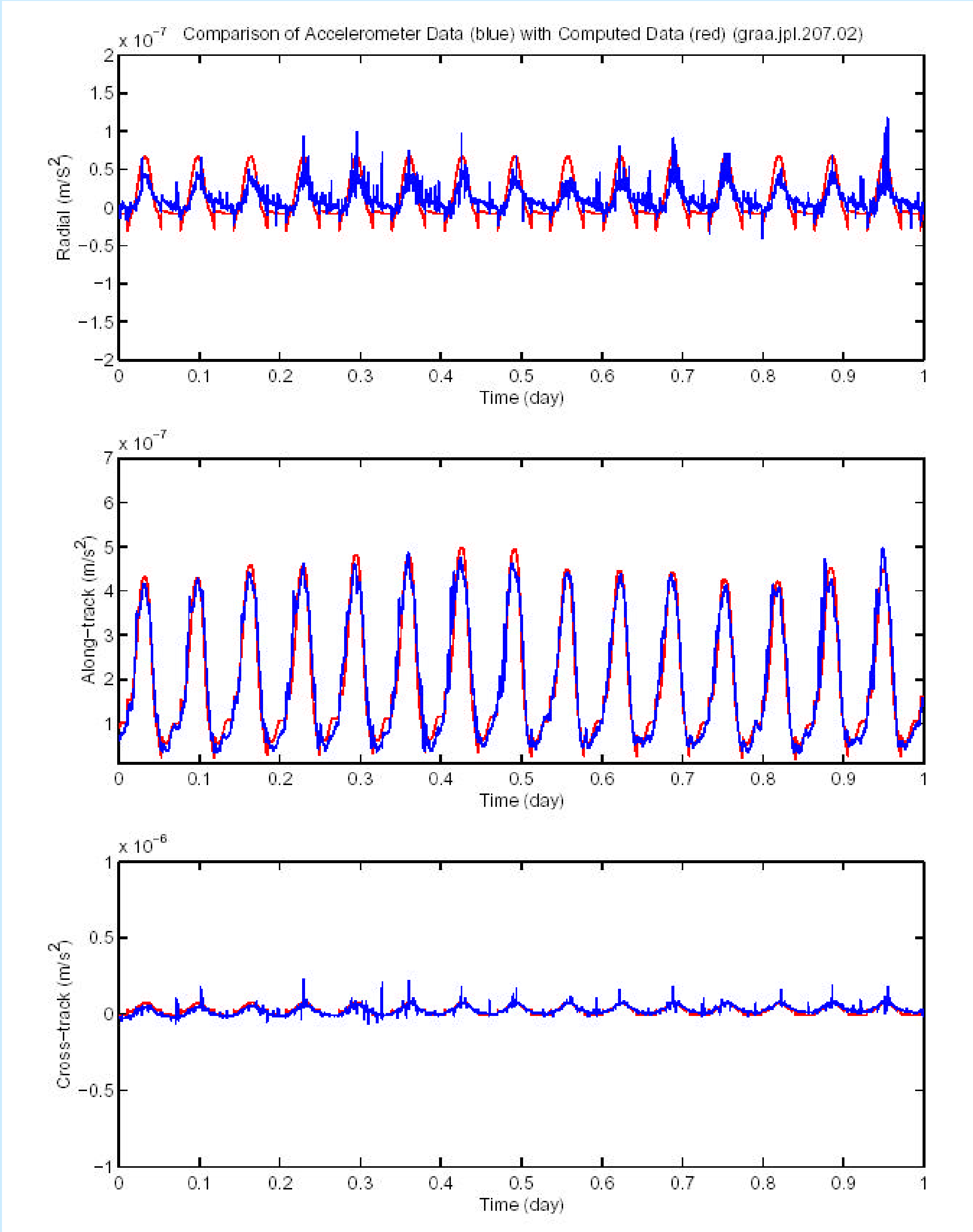
Model and actual measurement comparison GRACE mission



Preliminary analysis from Texas University

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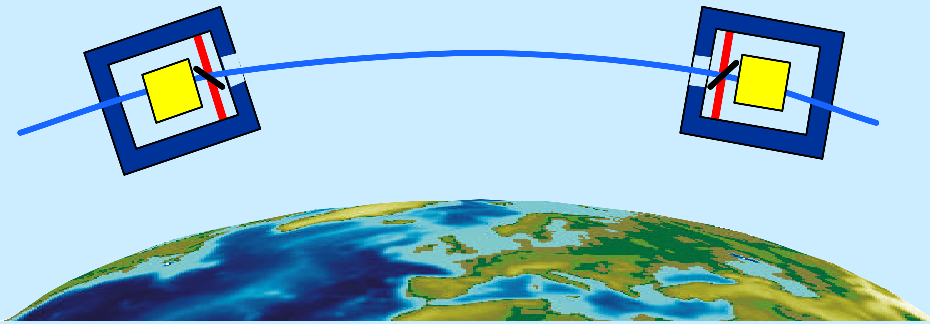
Space Gradiometry & Future Geodesie

✍ GOCE Mission

Instrument present STATUS

- ✍ Definition phase (B): Review April 2002
- ✍ Starting of production phase (C/O): July 2002

Launch expected at early 2006



✍ And After ?

Cryogenic gradiometry

with

- ✍ **higher resolution accelerometer**
- ✍ **higher drag-free satellite performance**
- ✍ **higher calibration performance**

Satellite Satellite tracking: GRACE follow-on

with

- ✍ **drag-free satellite**
- ✍ **lower altitude**
- ✍ **laser ranging**

References

[1] http://op.gfz-potsdam.de/champ/

[2] http://www.csr.utexas/grace/

[3] http://www.esa.int/export/esaLP/goce/

[4] http://www.onera.fr/dmph/accelerometre/

END

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