CENTRE NATIONAL D'ÉTUDES SPATIALES

GROUPE DE RECHERCHE DE GEODESIE SPATIALE

Observatoire Midi-Pyrénées

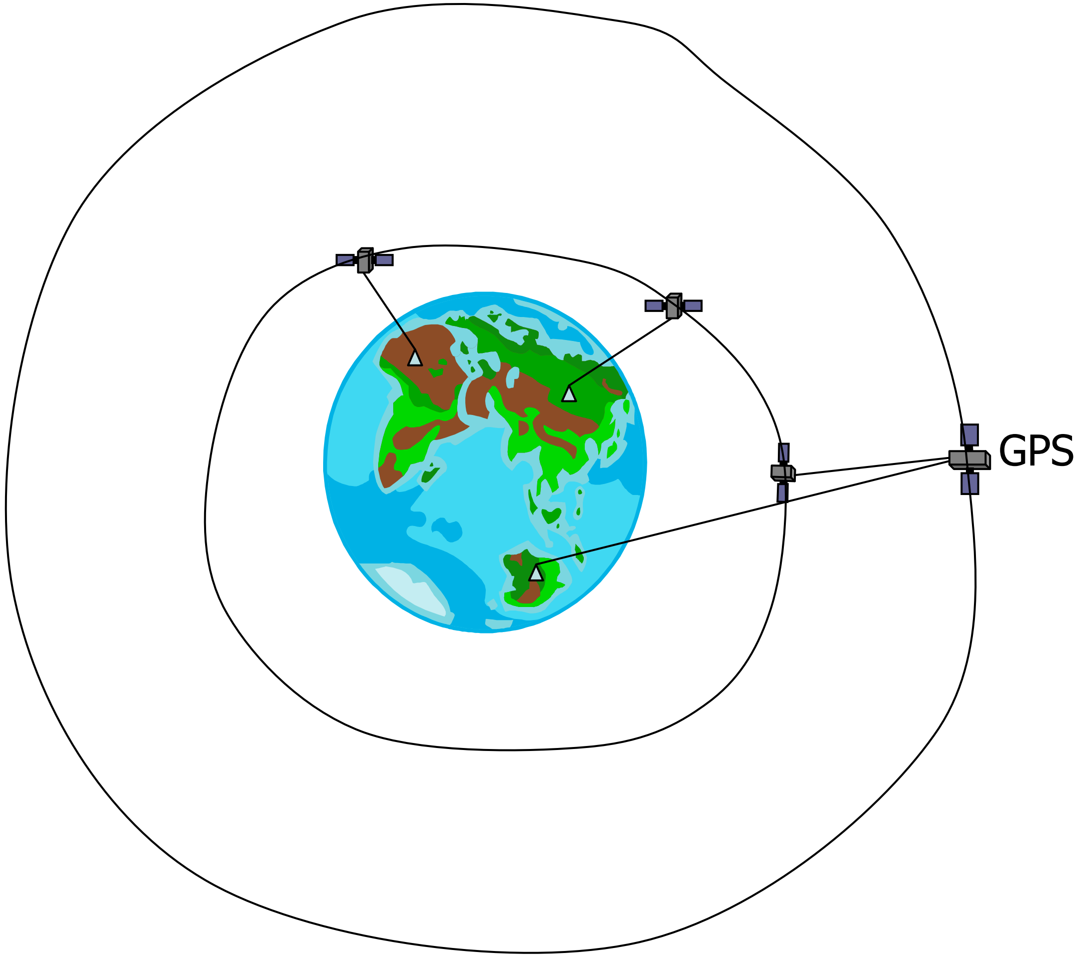
GÉODÉSIE PAR INTÉGRATIONS NUMÉRIQUES SIMULTANÉES

gins training 2013

4-7 Juin 2013, Toulouse

Rappels de géodésie générale

Rappels de géodésie générale
orbitographie, rotation, déformations, ITRF, mesures



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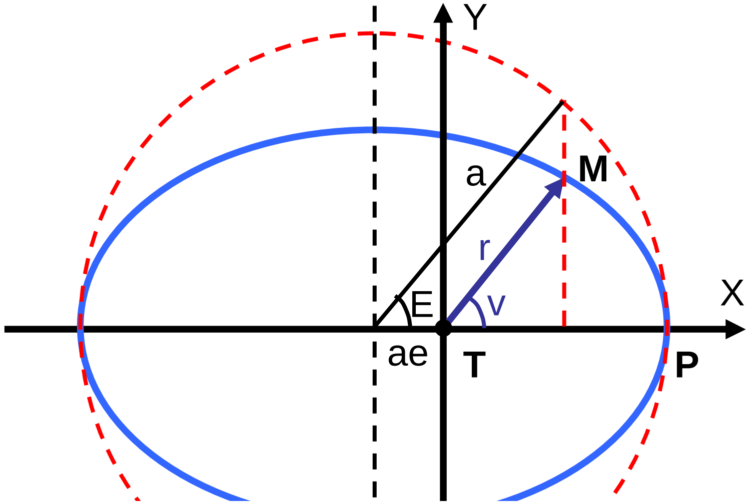
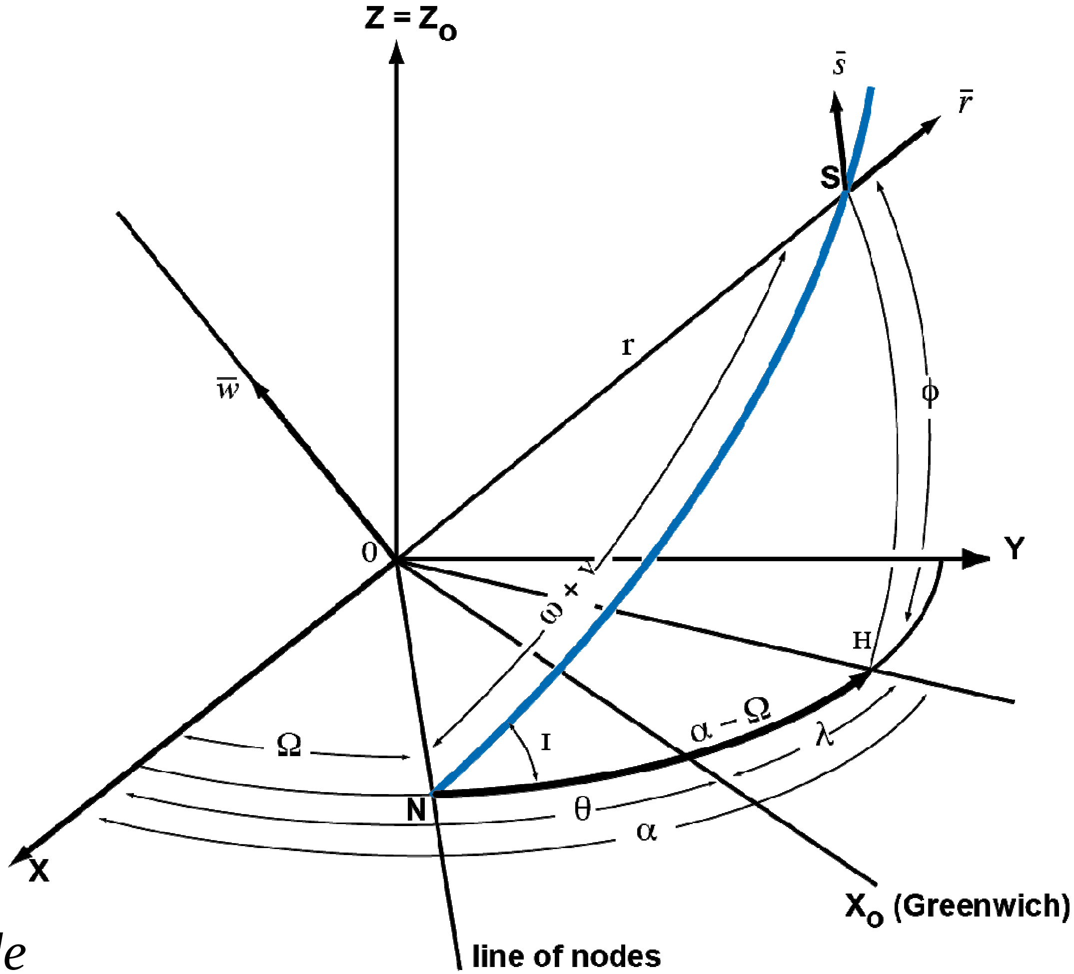
Rappels de géodésie générale

I. Orbitographie

The Keplerian elements





Keplerian orbital elements:

a: semi-major axis

e: eccentricity $\left(b = a\sqrt{1-e^2}\right)$

i: inclination

Ω: argument of the ascending node

ω: argument of perigee

M=n(t-t$_0$): mean anomaly $\left(n = dM/dt : mean\ motion\right)$

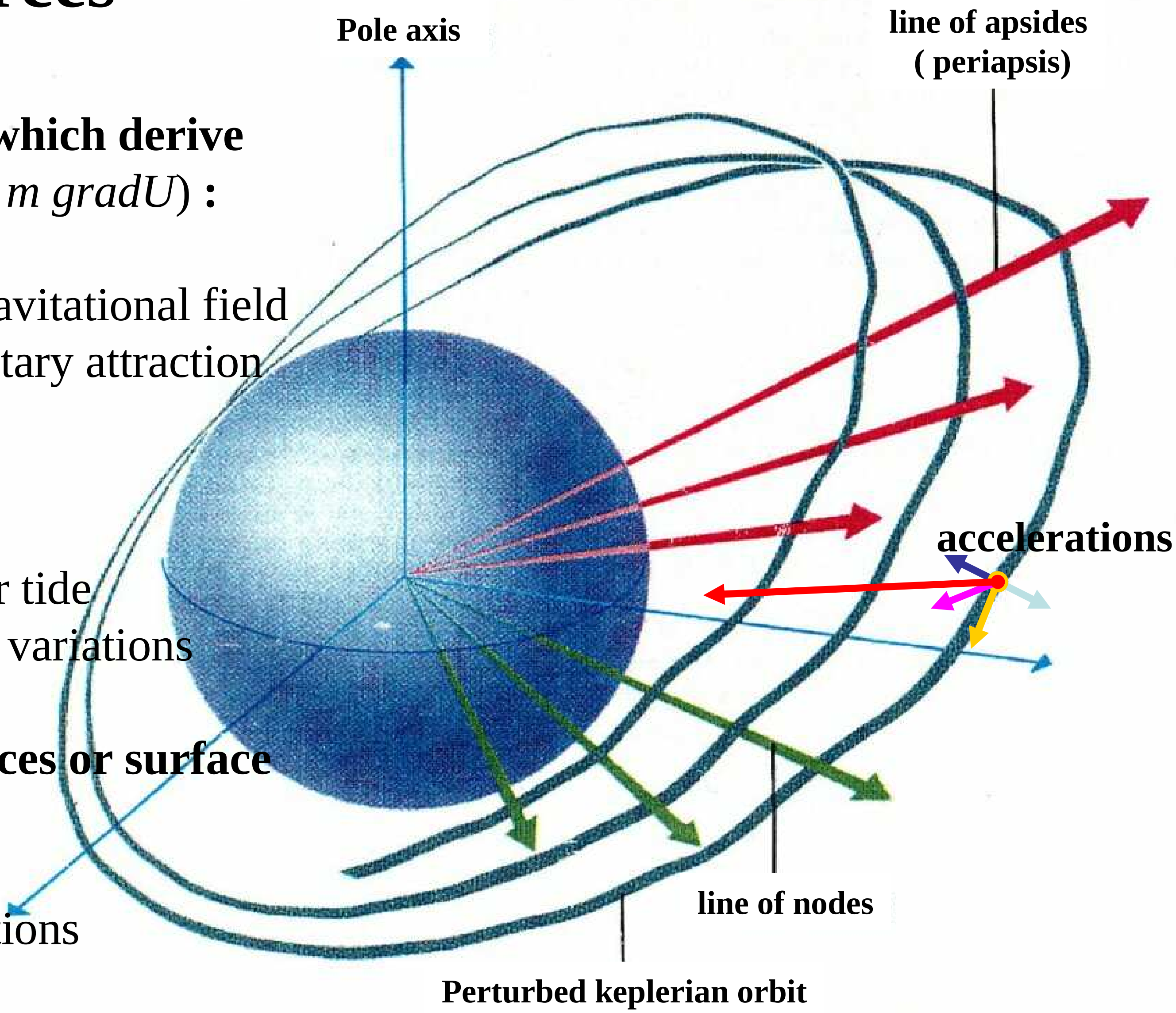
Disturbing forces

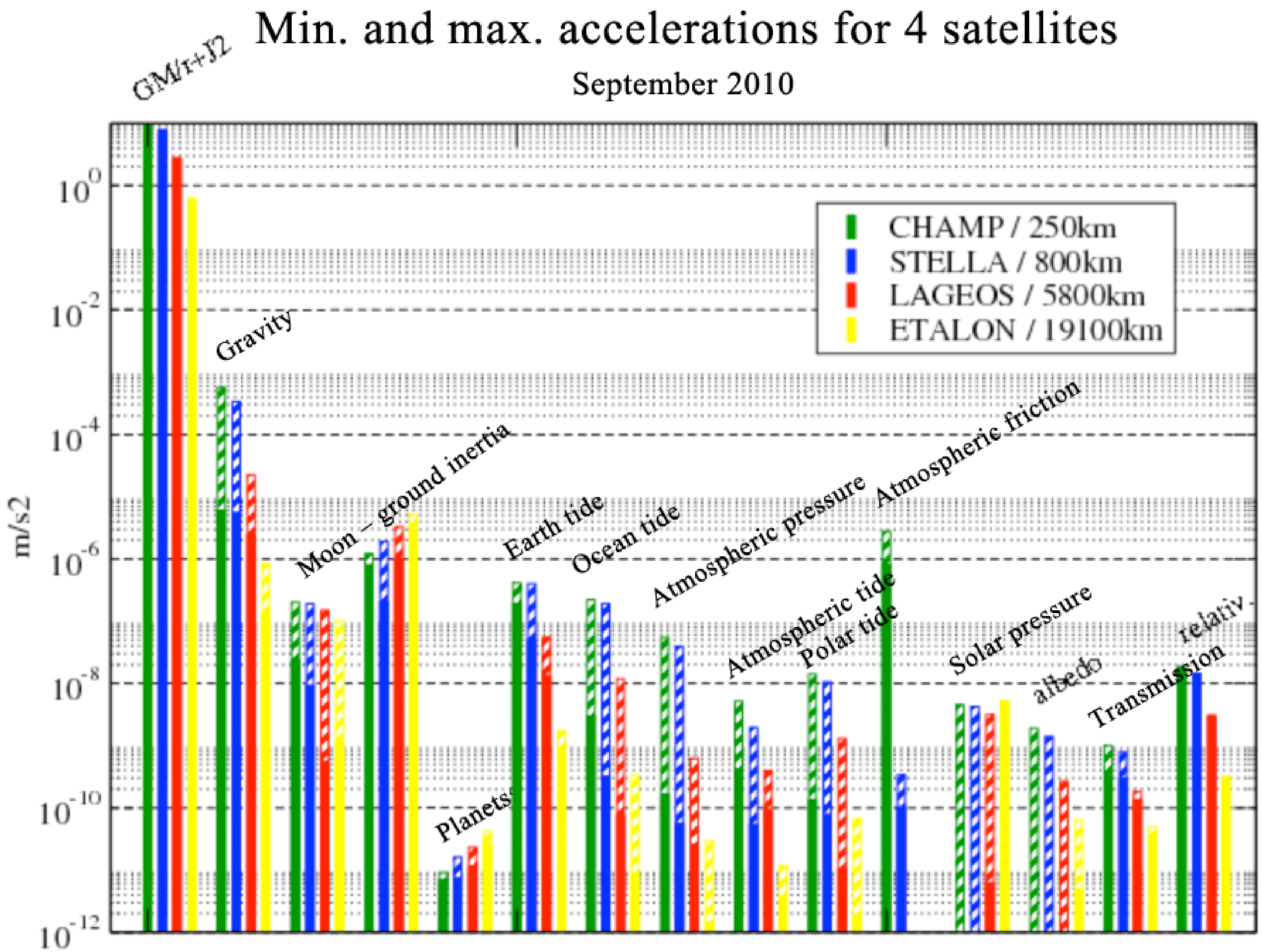
Gravitational forces which derive from a potential ($F = m\ gradU$) **:**

- GM/r^2
- Earth's perturbing gravitational field
- Moon, Sun and planetary attraction
- Earth tides
- ocean tides
- atmospheric tides
- Earth and ocean polar tide
- atmospheric pressure variations

Non gravitational forces or surface forces ($F = m\ \gamma$) **:**

- thermospheric drag
- solar and Earth radiations
- thermal diffusion
- relativistic corrections





Some examples of the amplitude of the acceleration taken into consideration for the numerical integration of the movement (the min. and max. values in the course of the arc are entered for each satellite).

Numerical vs. analytical methods

Numerical

• Numerical integration (of second order) of the fundamental equation of dynamics in Cartesian coordinates:

$$r = \iint \ddot{\bar{r}}\,dt \quad ; \quad \ddot{\bar{r}} = \sum_n \bar{A}(\bar{r}, \dot{\bar{r}}, \alpha_i)$$

from a set of initial conditions at t_0 (orbit and acceleration model parameters):

$$\bar{r}_0, \dot{\bar{r}}_0, \alpha_i = (C_{lm}, S_{lm}, \ldots, p_{dyn.})$$

• Adjustment of initial orbit parameters as well as of model parameters according to tracking observations:

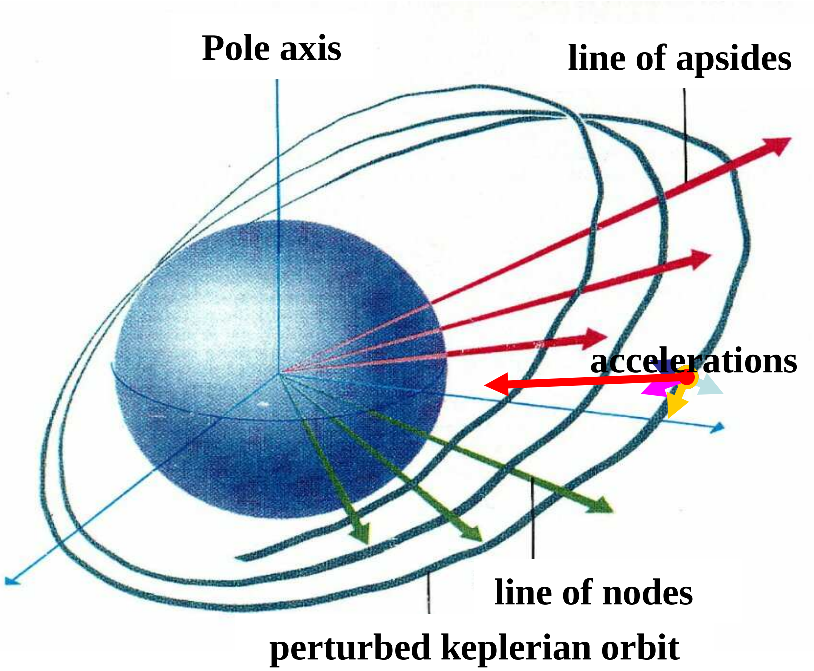
$$\Delta Q = Q_{obs} - Q_{calc} = \frac{\partial Q}{\partial \bar{r}_0}\Delta\bar{r}_0 + \frac{\partial Q}{\partial \dot{\bar{r}}_0}\Delta\dot{\bar{r}}_0 + \sum_i \frac{\partial Q}{\partial \alpha_i}\Delta\alpha_i$$

• Iterative method

Analytical

• Integration (of first order) of Gauss / Lagrange's equations in Keplerian elements

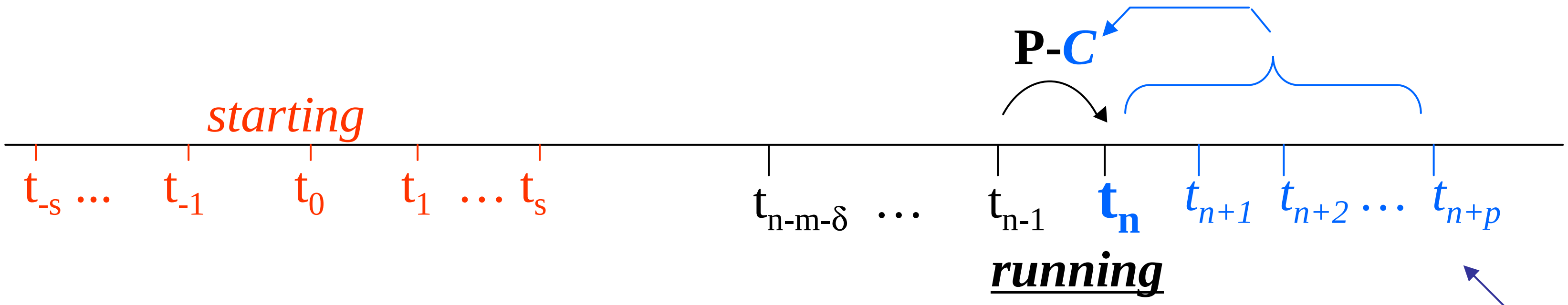
$$\left(\frac{da}{dt}, \frac{de}{dt}, \frac{di}{dt}, \frac{d\Omega}{dt}, \frac{d\omega}{dt}, \frac{dM}{dt}\right)$$



The Cowell integration method

System : $\ddot{x} = f(x, \dot{x}, t) \quad with \quad x(t_0), \dot{x}(t_0)$

Numerical integration stepsize : $h \leq$ (Orb. Period / l_{max})/4



Normal :

$$x_n = 2x_{n-1} - x_{n-2} + h^2 \sum_0^m A_j^\delta f_{n-j-\delta}$$

$$\dot{x}_n = \dot{x}_{n-1} + h \sum_0^m A_j'^\delta f_{n-j-\delta}$$

P (Predictor) : $\delta = 1$

C *(Corrector)* : $\delta = 0$
(with pseudo-correction after 1.st iteration)

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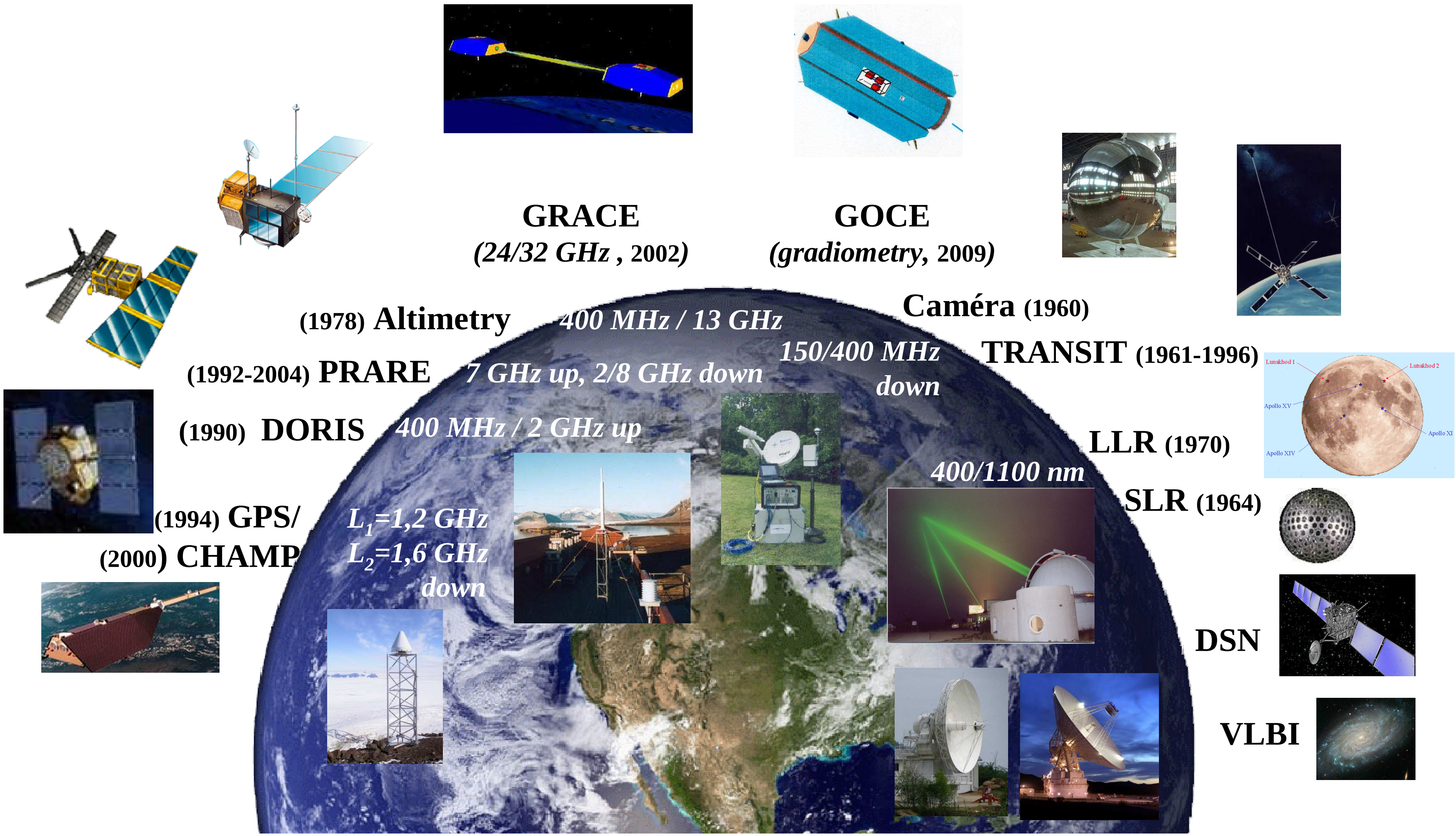
Rappels de géodésie générale

II. Mesures

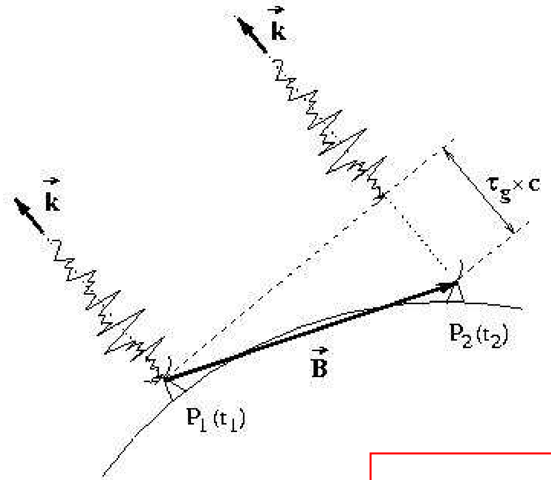
Space techniques measurement

used in geodesy



Space geodetic techniques

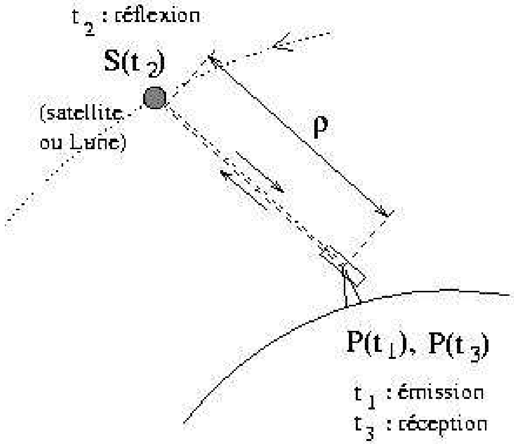
VLBI



fonction de mesure
géométrique simplifiée :

$$\tau_g = -\frac{\vec{B}.\vec{k}}{c}$$

SLR/LLR

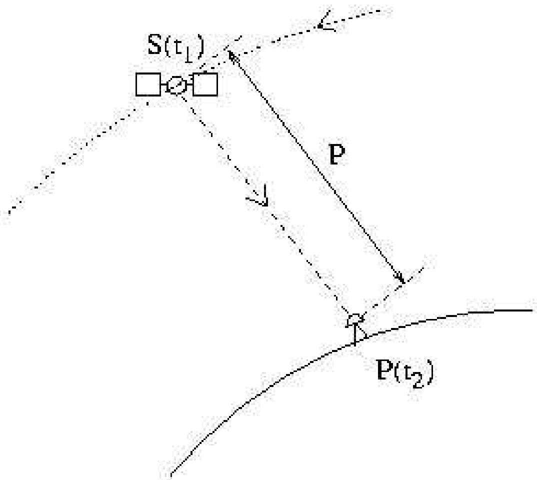


fonction de mesure
géométrique simplifiée :

$$\Delta t_g = \frac{2\rho}{c}$$

Δt_g = temps de vol A/R

GNSS



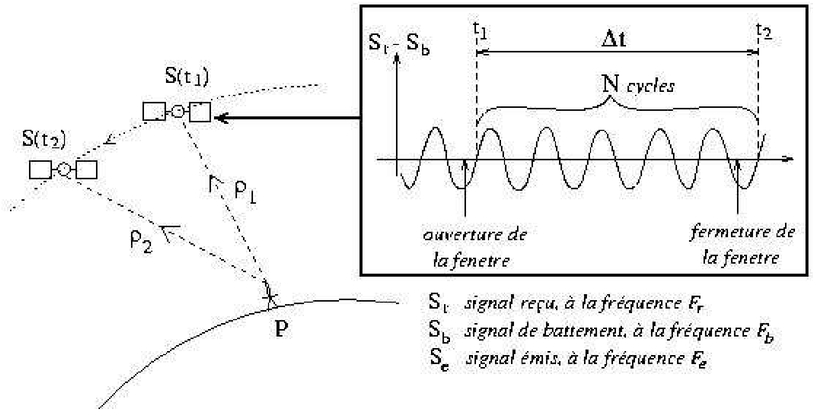
$\Delta t = (t_2 - t_1)$

fonction de mesure
géométrique simplifiée :

$$\begin{cases} P = c\,\Delta t \\ \lambda\Phi = c\,\Delta t + \lambda N \end{cases}$$

(P : pseudo-distance / Φ : phase)

DORIS

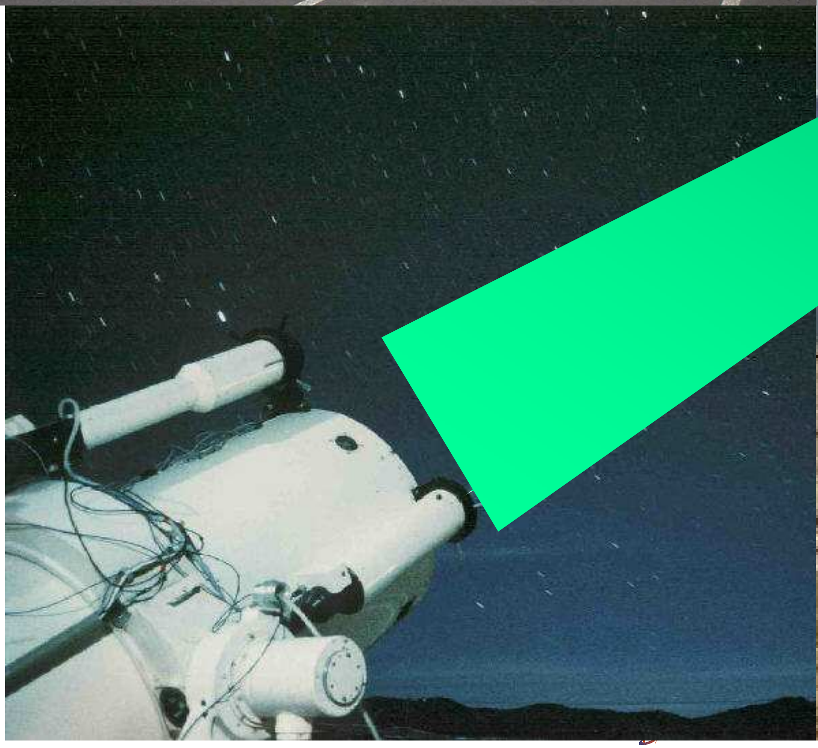


fonction de mesure
géométrique simplifiée :

$$N = [F_e(1 - \frac{\rho_2 - \rho_1}{c}) - F_b]\,\Delta t$$

Satellite Laser Ranging

Lageos 1/2 (Ø 1,2m; 407 kg)
426 rétro-réflectors



Détermination d'**O**rbite et **R**adiopositionnement **I**ntégrés par **S**atellite
Doppler **O**rbitography and **R**adiopositionning **I**ntegrated by **S**atellite

On board receiver, 18 kg
390 x 370 x 165 (mm)

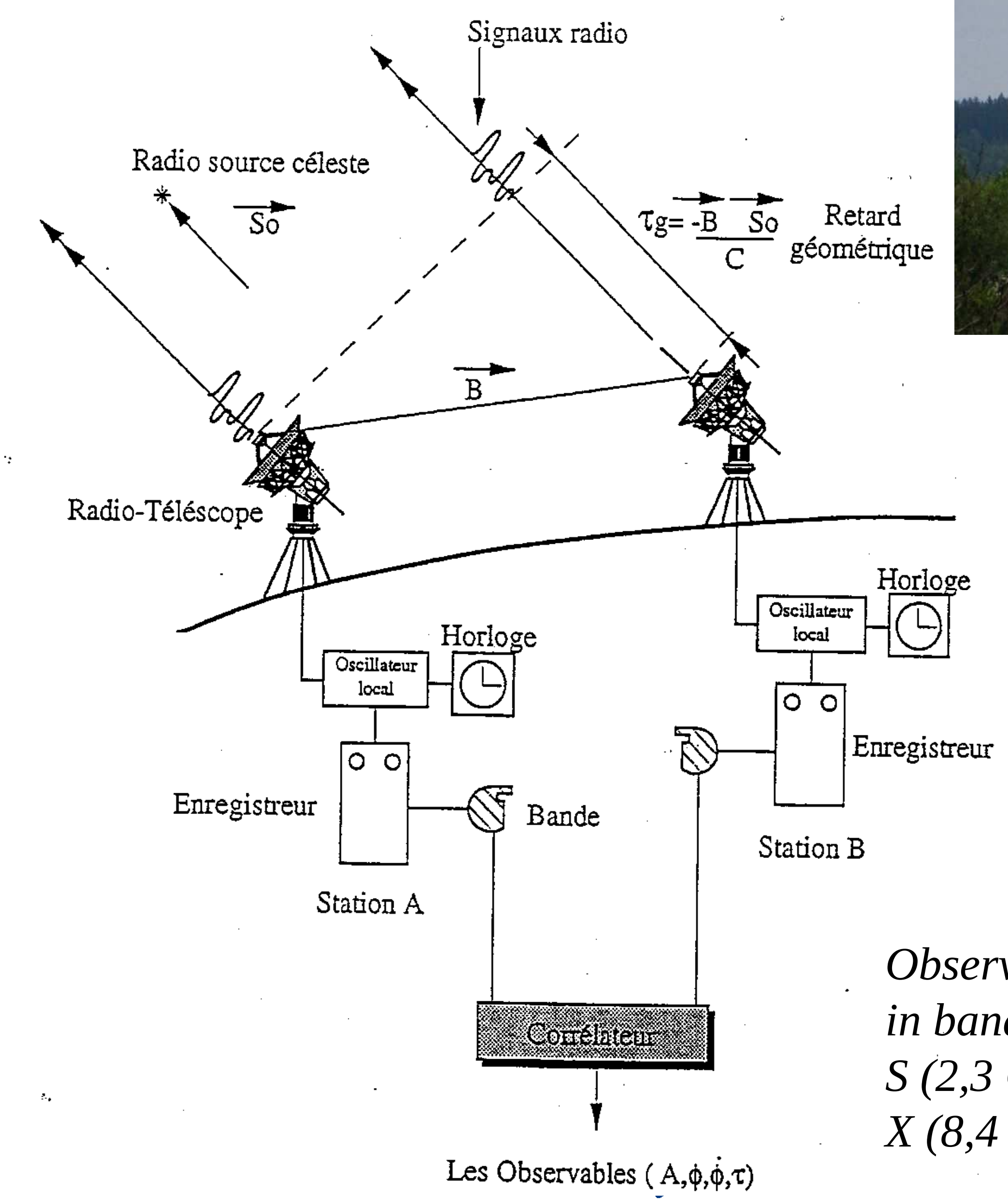
Omni-directional DORIS antenna, 2 kg
h 420 x Ø160 (mm)

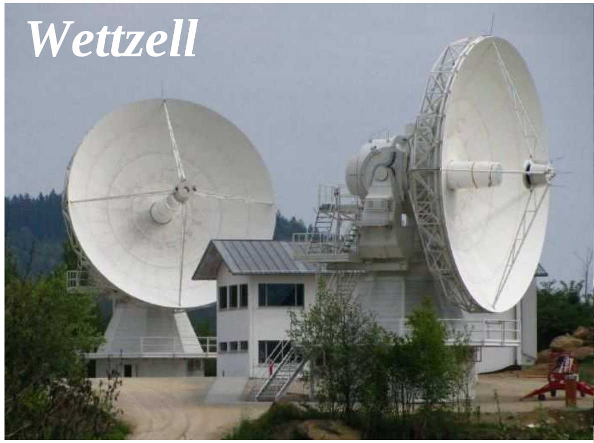
Transmitter beacon on 2 frequencies: 401.25 and 2036.25 MHz

STAREC transmitter beacon on ground

DORIS ultra-stable oscillator (USO):
Frequency short term stability : 2.10^{-13} over 10 seconds

VLBI



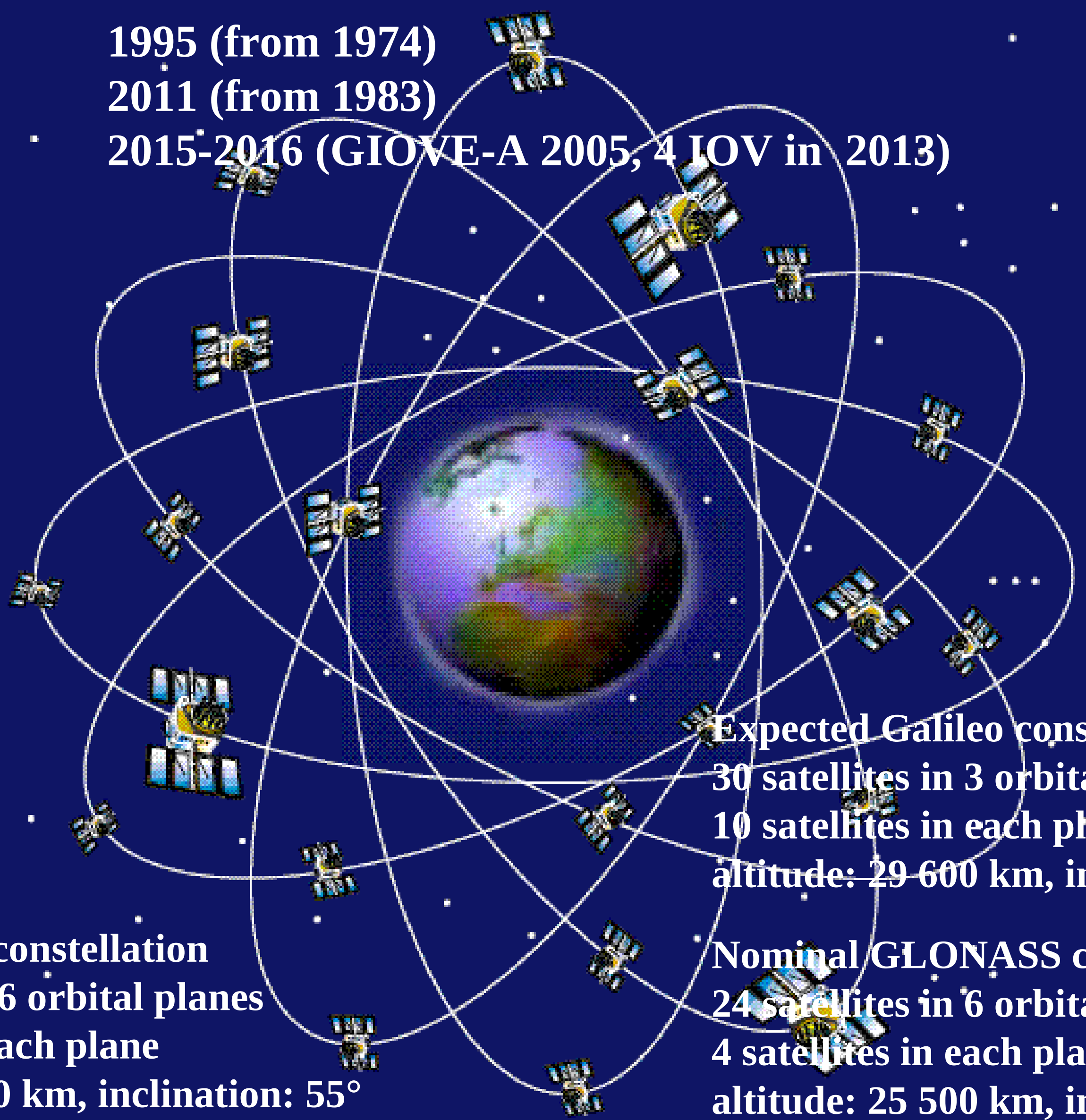


Observations in bands :
S (2,3 Ghz)
X (8,4 Ghz)

use

Global Navigation Satellite System (GNSS)

GPS : 1995 (from 1974)
GLONASS : 2011 (from 1983)
Galileo : 2015-2016 (GIOVE-A 2005, 4 IOV in 2013)
…

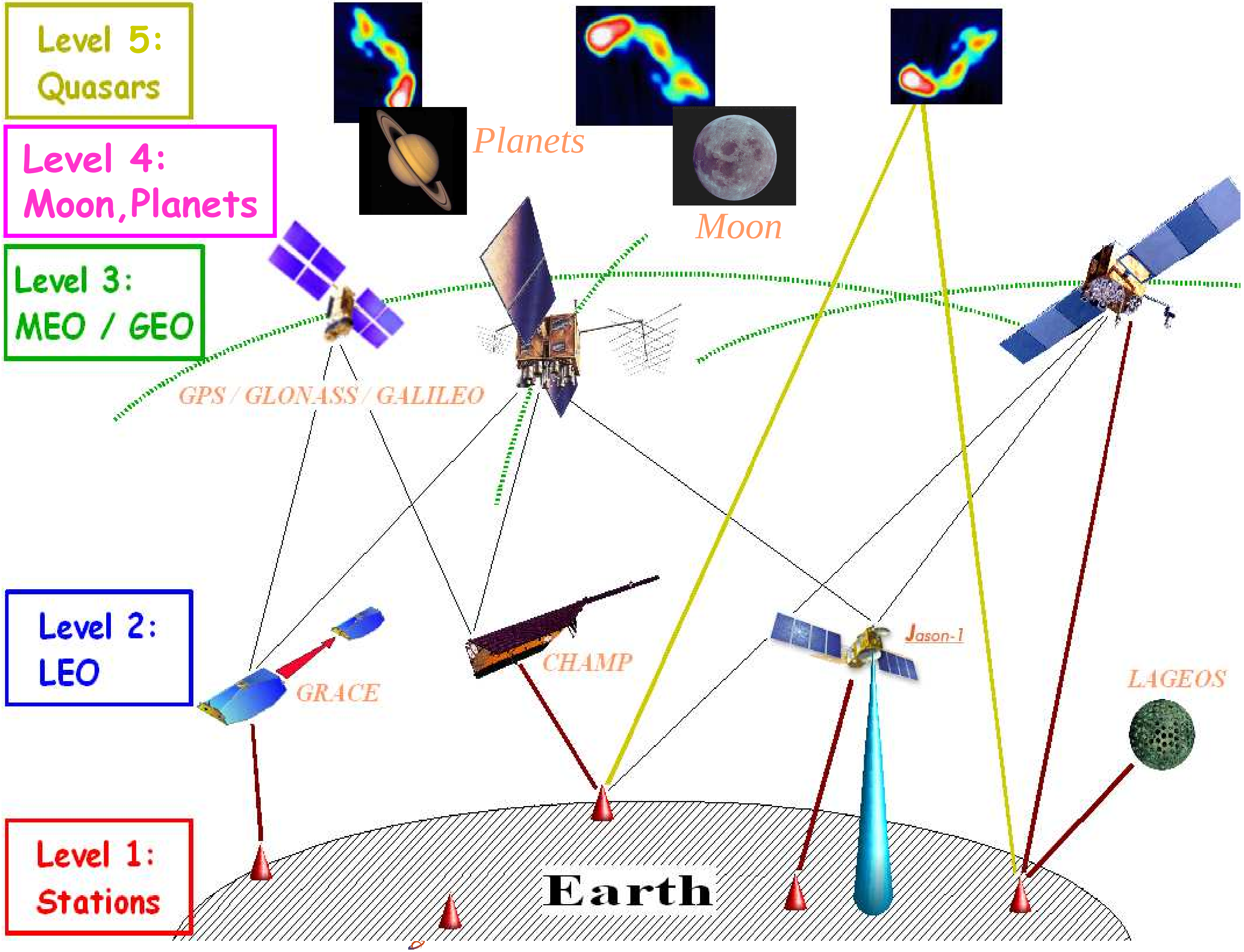


Expected Galileo constellation
30 satellites in 3 orbital planes
10 satellites in each plane
altitude: 29 600 km, inclination: 56°

Nominal GPS constellation
24 satellites in 6 orbital planes
4 satellites in each plane
altitude: 20 200 km, inclination: 55°

Nominal GLONASS constellation
24 satellites in 6 orbital planes
4 satellites in each plane
altitude: 25 500 km, inclination: 65°

The 5 levels of observation



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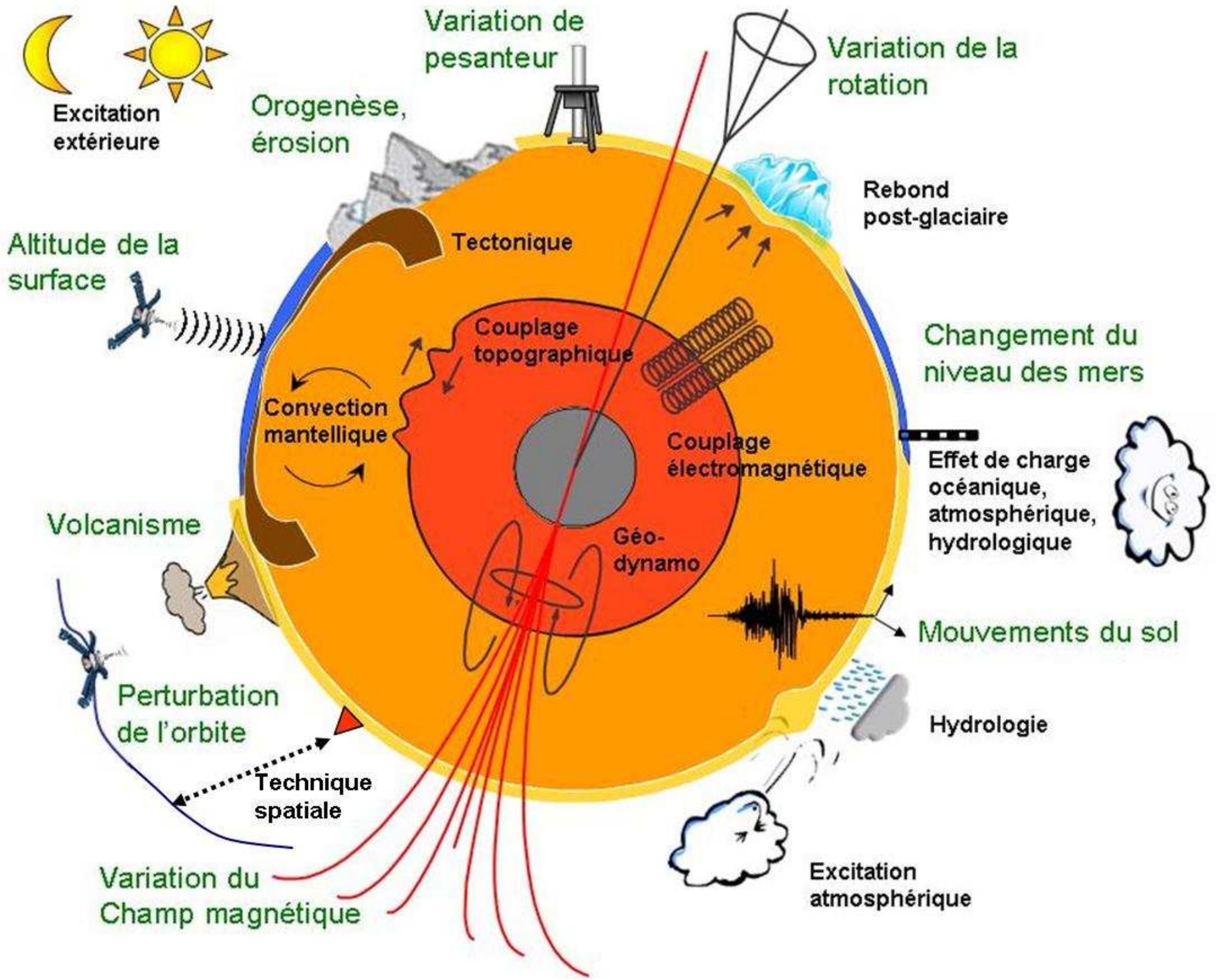
Rappels de géodésie générale

III. Rotation de la Terre

Internal structure of the Earth and dynamical processes



Earth rotation is modified by internal, surface and astronomic phenomena

IERS Terms of Reference (June 21, 2004)

The IERS was established as the International Earth Rotation Service in 1987 by the International Astronomical Union and the International Union of Geodesy and Geophysics and it began operation on 1 January 1988. In 2003 it was renamed to International Earth Rotation and Reference Systems Service.

The primary objectives of the IERS are to serve the astronomical, geodetic and geophysical communities by providing the following:

- The **International Celestial Reference System (ICRS)** and its realization, the International Celestial Reference Frame (ICRF).

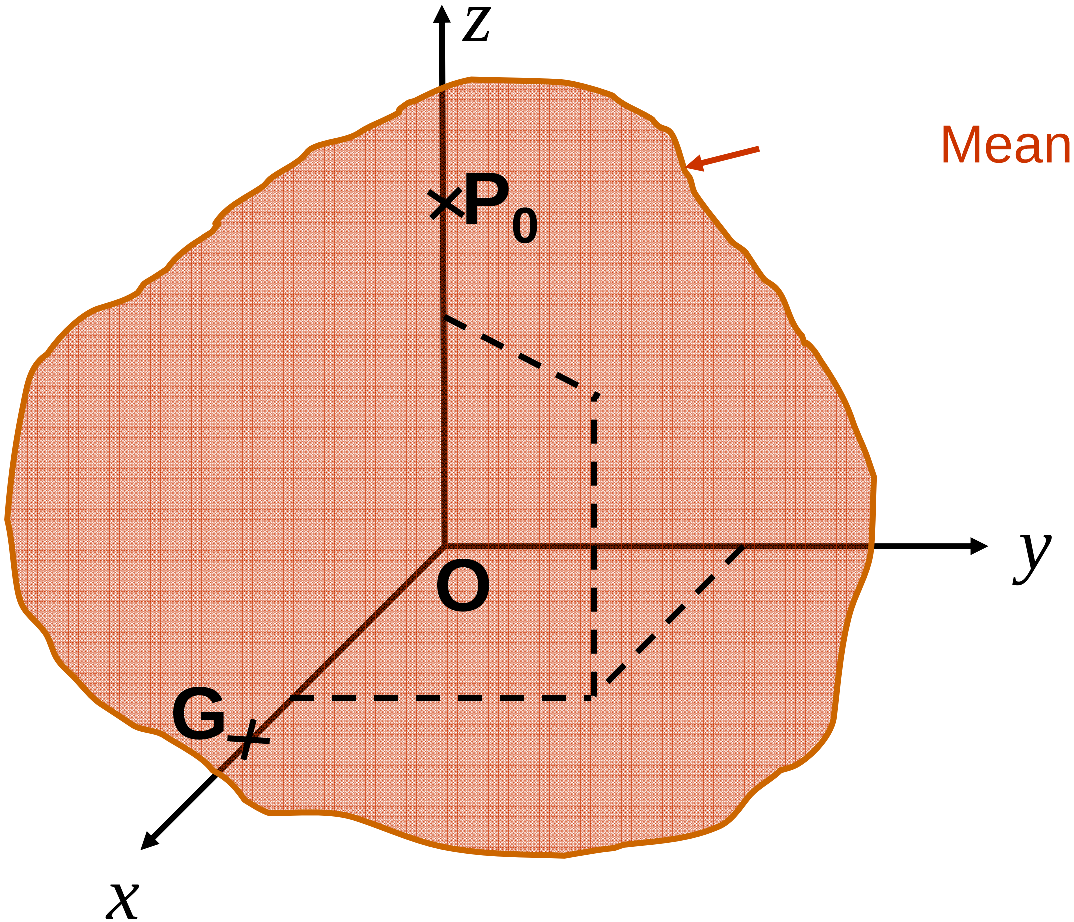
- The **International Terrestrial Reference System (ITRS)** and its realization, the International Terrestrial Reference Frame (ITRF).

- **Earth Orientation Parameters** required to study earth orientation variations and to transform between the ICRF and the ITRF.

- Geophysical data to interpret time/space variations in the ICRF, ITRF or earth orientation parameters, and model such variations.

- Standards, constants and models (i.e., conventions) encouraging international adherence.

ITRS: International Terrestrial Reference System



O: Earth barycenter (slow movements)

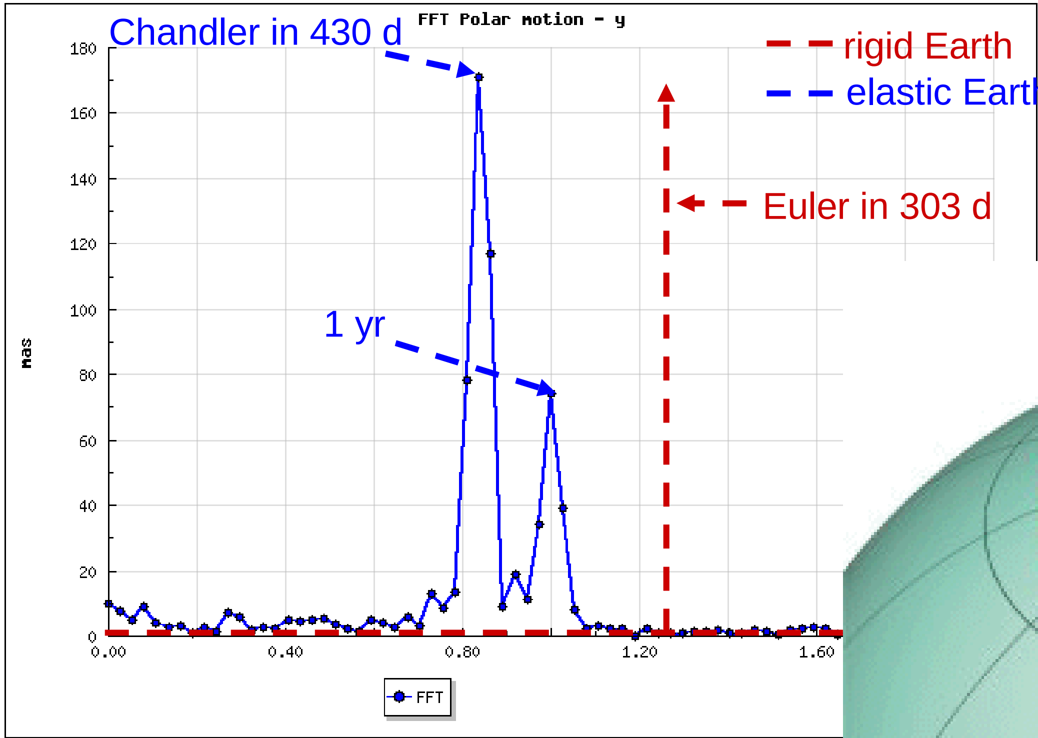
$\mathbf{P_0}$: conventional pole close to North pole

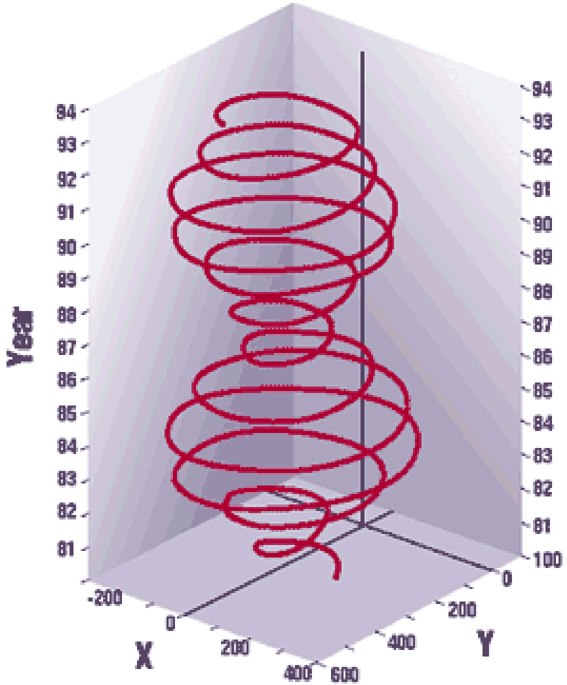
G: origin of longitudes

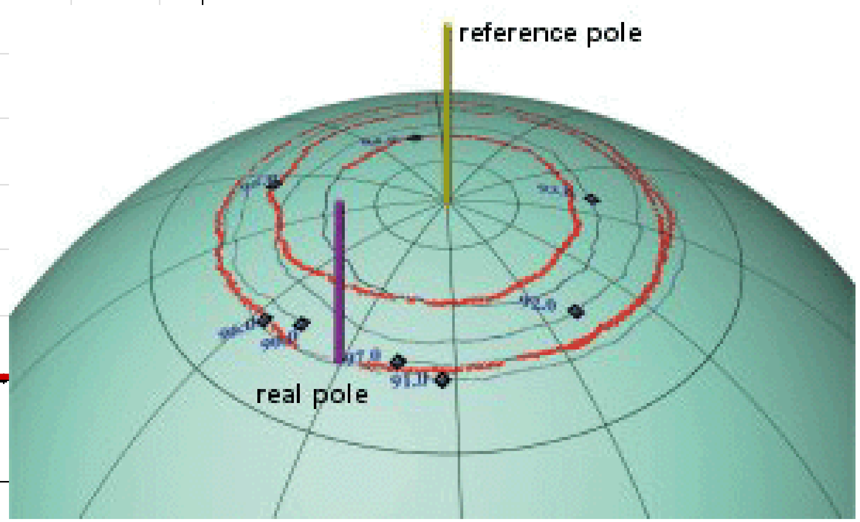
Polar motion

The polar motion has 3 principal components:

- the Chandler's oscillation at a 14 months period (~7 m)
- the annual oscillation caused by seasonal variations of the atmosphere and oceans (~3 m)
- the polar wander toward Canada (~10 cm/yr)





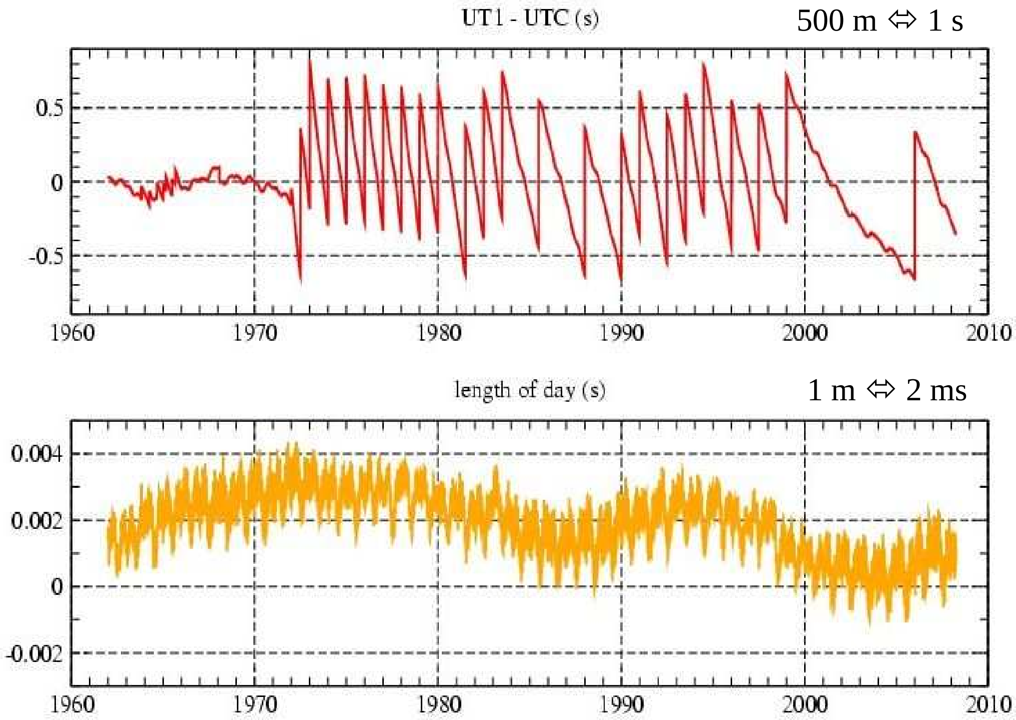


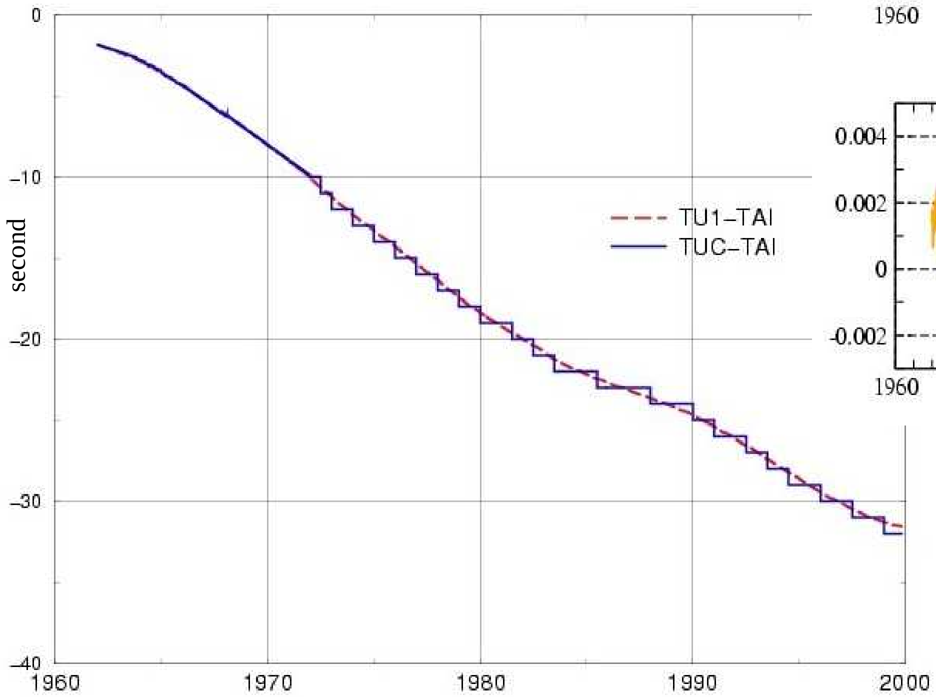
Earth's rotation

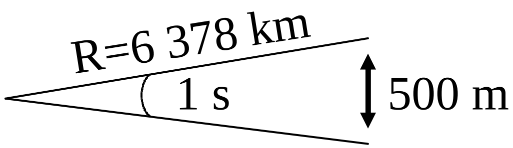
-0.9 s < UT1 – UTC < +0.9s

UTC = TAI + n

(TAI=UTC + 34s 01/01/09)





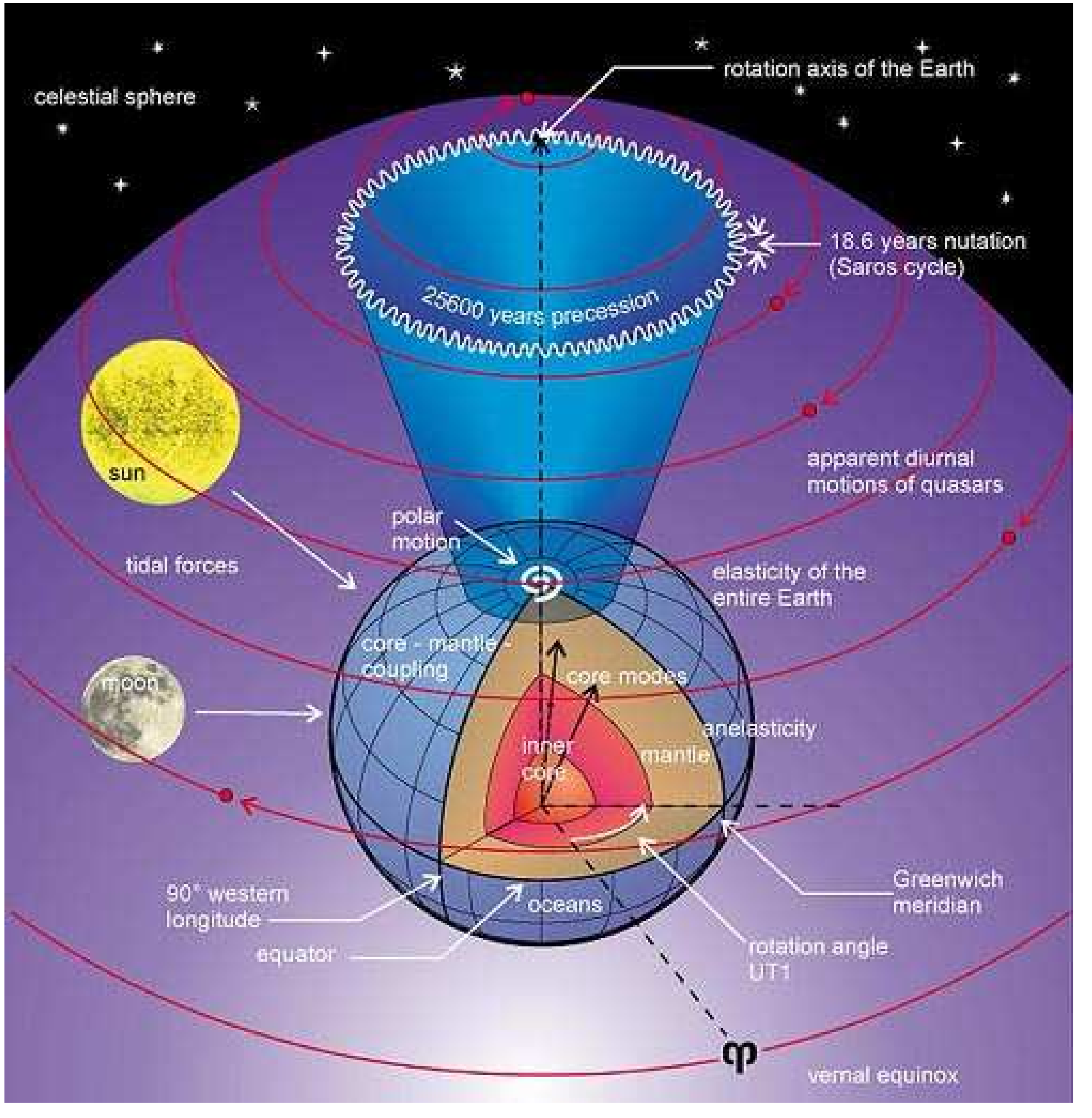


2 billions years ago: 1 day = 10 hrs

400 millions years ago: 1 year = 400 days of 21 hrs

53 millions years ago: 1 year = 370 days of 24 hrs

The Earth in space



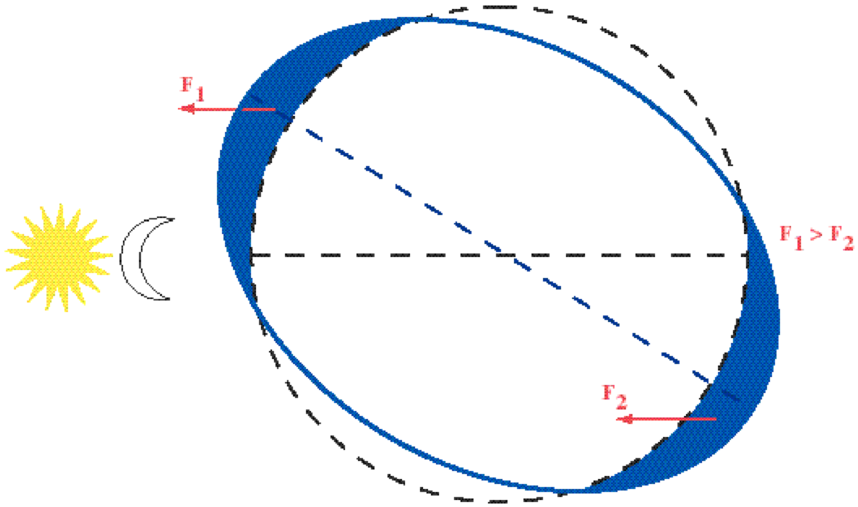
Earth's precession

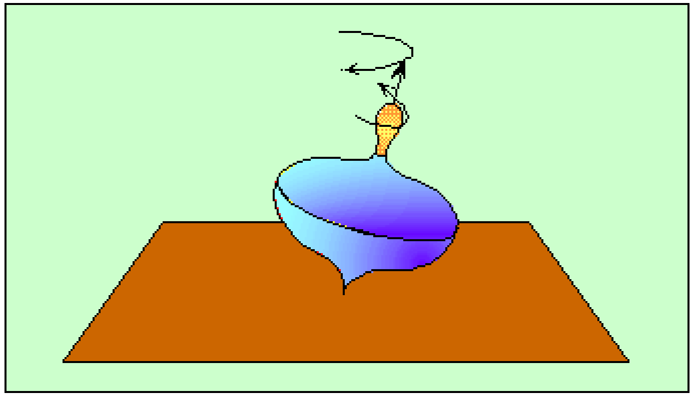
• discovered by Hipparchos (≈ - 100 BC)

• difference between the tropic year and the sidereal year

• intersection between the equator and the ecliptic with a retrograde motion in 26 000 years ≈ 50.3"/yr (⇔ 1500 m/yr) (Hipparchos : 40"/yr according to observations from Timocharis)

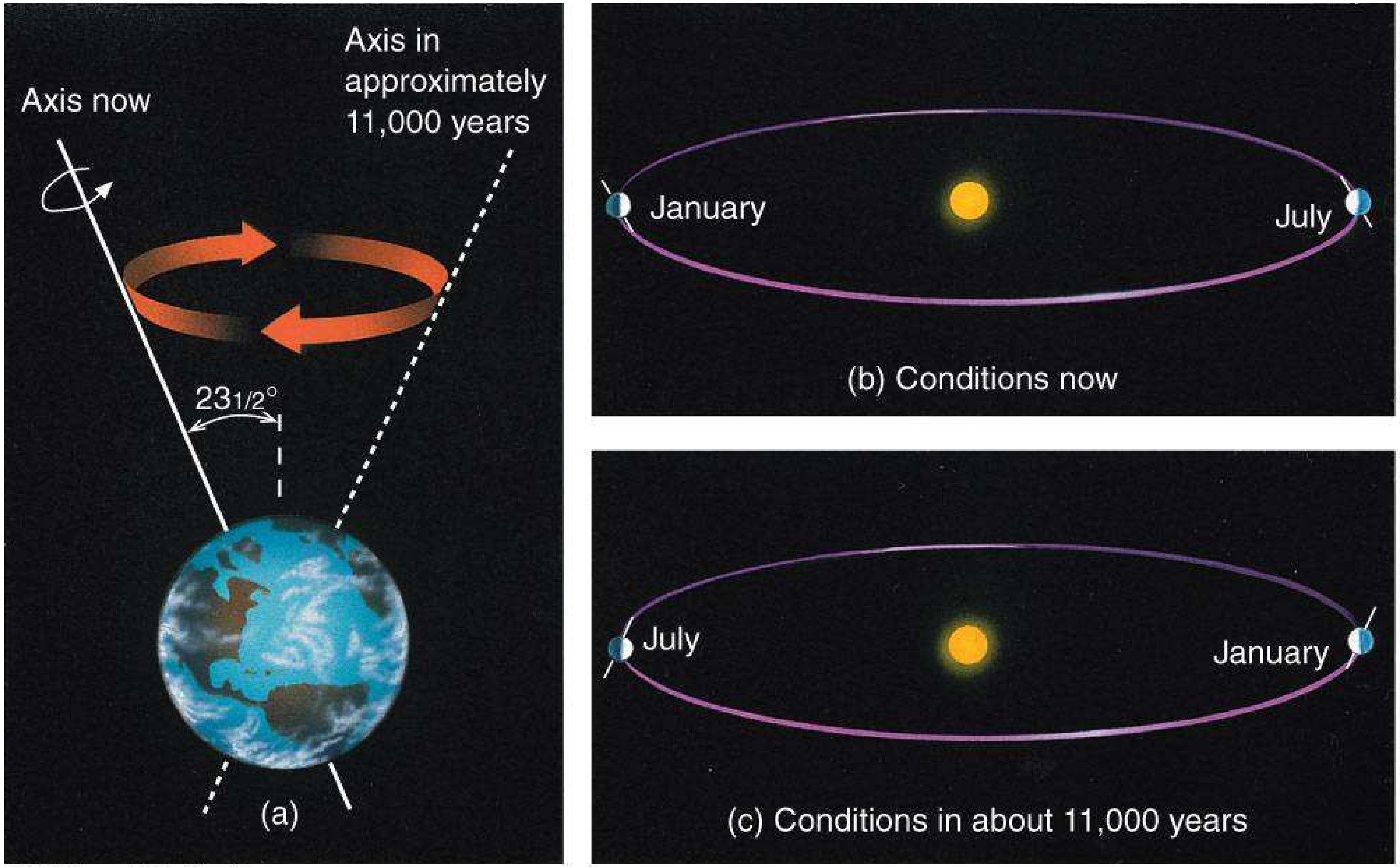
• origin: gravitational attraction force of the Sun and Moon on the Earth's equatorial bulge thwarted by the centrifugal force of rotation of the Earth





Analogy in spinning a top

The Earth on its orbite

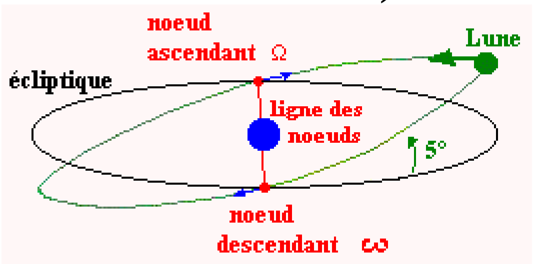


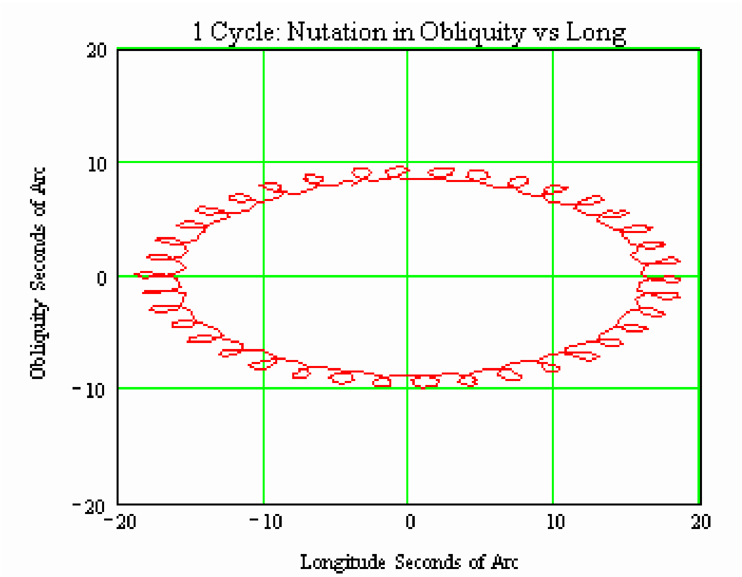
Earth’s nutation

• discovered by Bradley (1737)

• main elliptical oscillation in 18.6 years (6.86” × 9.21” ⇔ 300 m) of the Earth under the effect of the lunar gravitational attraction

• origin: variation of the lunar gravitational attraction on the Earth’s equatorial bulge due to the periodic motion of the lunar inclination (5° 9’) with respect to the Earth’s equator, consequence of the 18.6 yr precession of the node of the Moon’s orbit with respect to the ecliptic (motion caused itself by the Sun’s gravitational attraction)





other techniques

Techniques		*EOP fournis de façon opérationnelle*
VLBI		x_p, y_p, $UT1$, dψ et dϵ
Satellites	**Système GPS**	x_p, y_p, $\Delta(LOD)$, $\dot{x}_p$ et $\dot{y}_p$
	Système DORIS	x_p, y_p et $\Delta(LOD)$
	Télémétrie laser	x_p, y_p et $\Delta(LOD)$
Laser-Lune		$UT1$

In practice

- We need:
- A precession/nutation model
  - cf. IERS standards
- pole coordinates (x_p, y_p) and UT1
  - daily files including predicted values (IERS)
- sub-diurnal correction of pole coordinates and UT1
  - cf. IERS standards

Timescales

- **Atomic time**, with the use of the International System unit (the second), is the duration of 9.192.631.770 periods of the radiation corresponding to the transition between two levels of the Cesium 133 atom.

- **Sidereal time** is the rotation period of the Earth respect to fixed point among the stars. It varies with Earth rotation.

- **Universal time** is the length of the mean solar day. It tends to be as uniform as possible although the Earth rotation changes.

- **Universal Time Coordinated** (UTC) is based on international atomic timescale but can be a few seconds apart. UTC is set to be less than 0.9 seconds from universal time UT1.

UTC = TAI + n (TAI=UTC + 36s 01/07/2012)

TGPS = TAI - 19s from 6/1/1980 at 0h

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Rappels de géodésie générale

IV. ITRF

WGS84 / GRS80

The **World Geodetic System** of the US Department of Defence, called **WGS 84,** is currently the reference system being used by the **Global Positioning System**. It is geocentric and globally consistent within ±1 m with current geodetic realizations of the geocentric reference system family **International Terrestrial Reference System** (ITRS) maintained by the IERS.

The **WGS 84** is the realization of the **GRS80** (**1980 Geodetic Reference System**) reference ellipsoid.

Defining features:

Semi-major axis: a = 6 378 137.0 m

Semi-minor axis: b = 6 356 752.3 m

Flattening: 1/f = 298.257222

Defining physical constants

Geocentric gravitational constant, including mass of the atmosphere: $GM = 398600.5\ m^3/s^2$

Dynamical form factor: $J2 = 108263.\ 10^{-8}$

Angular velocity of rotation $\omega = 7292115.\ 10^{-11}\ s^{-1}$

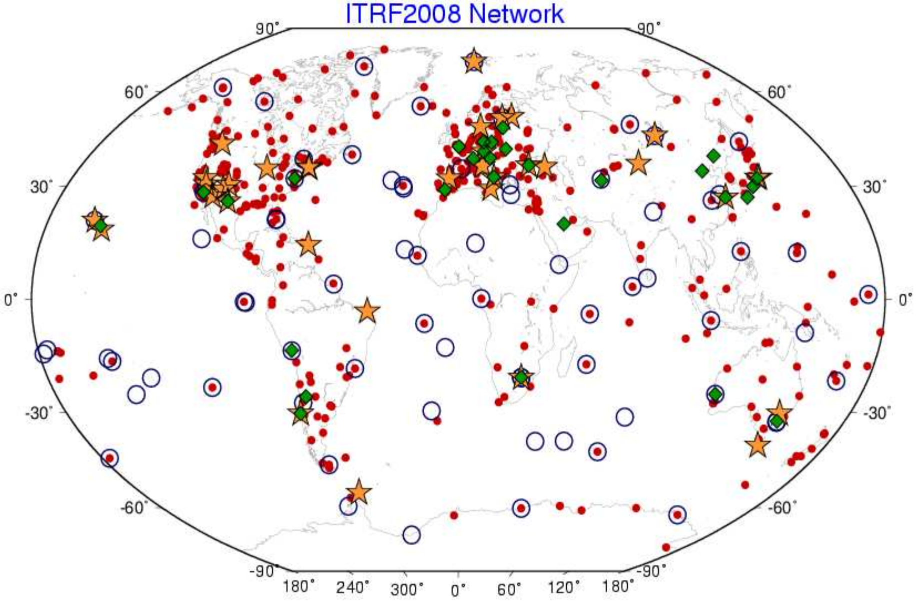
Longitudes, which are used by satellite navigation systems, differ slightly from traditional longitudes. The WGS84 zero meridian is 102.5 metres to the East of the line marked at Greenwich.

ITRF2008 realization

ITRF solutions (ITRF88, ITRF92, ITRF93, ITRF94, ITRF96, ITRF97, ITRF2000, ITRF2005, ITRF2008) consist in sets of station positions and velocities with their variance/covariance matrices.
In the ITRF2008 release, Earth Orientation Parameters (EOPs) have been combined simultaneously with the station coordinates.

Input of ITRF2008:

Technique	Nb of sites	Time span
VLBI	84	1980.0 - 2009.0
SLR	89	1984.0 - 2009.0
GPS	492	1997.0 - 2009.5
DORIS	67	1993.0 - 2009.0

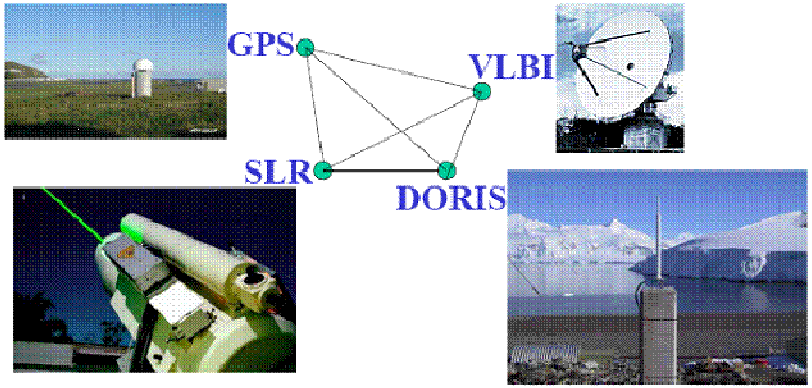


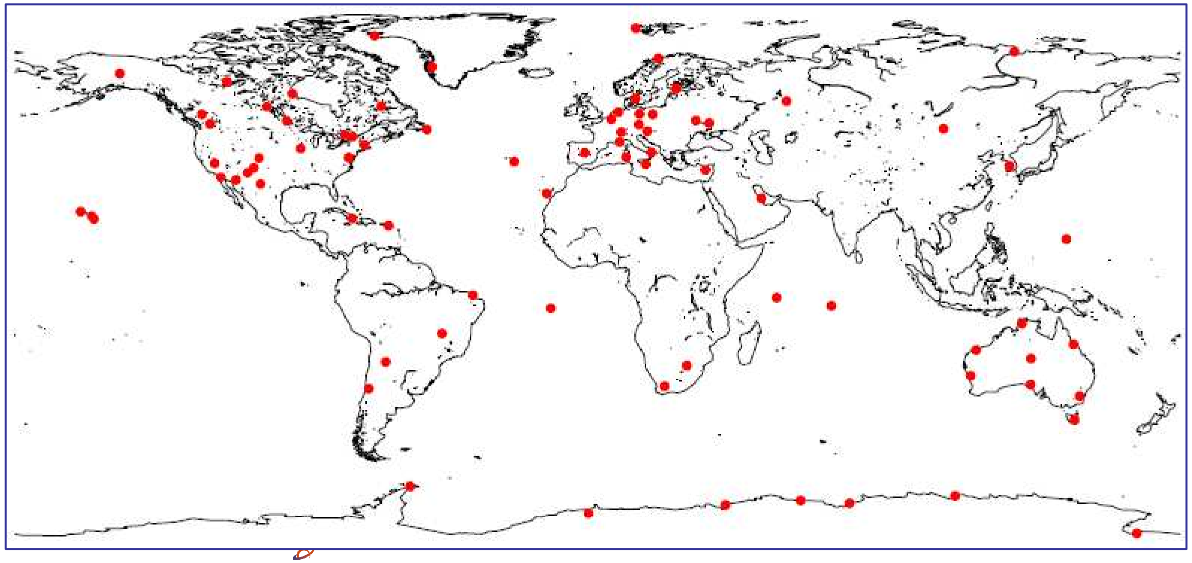
579 sites (920 stations)

ITRF2008 Datum Specification

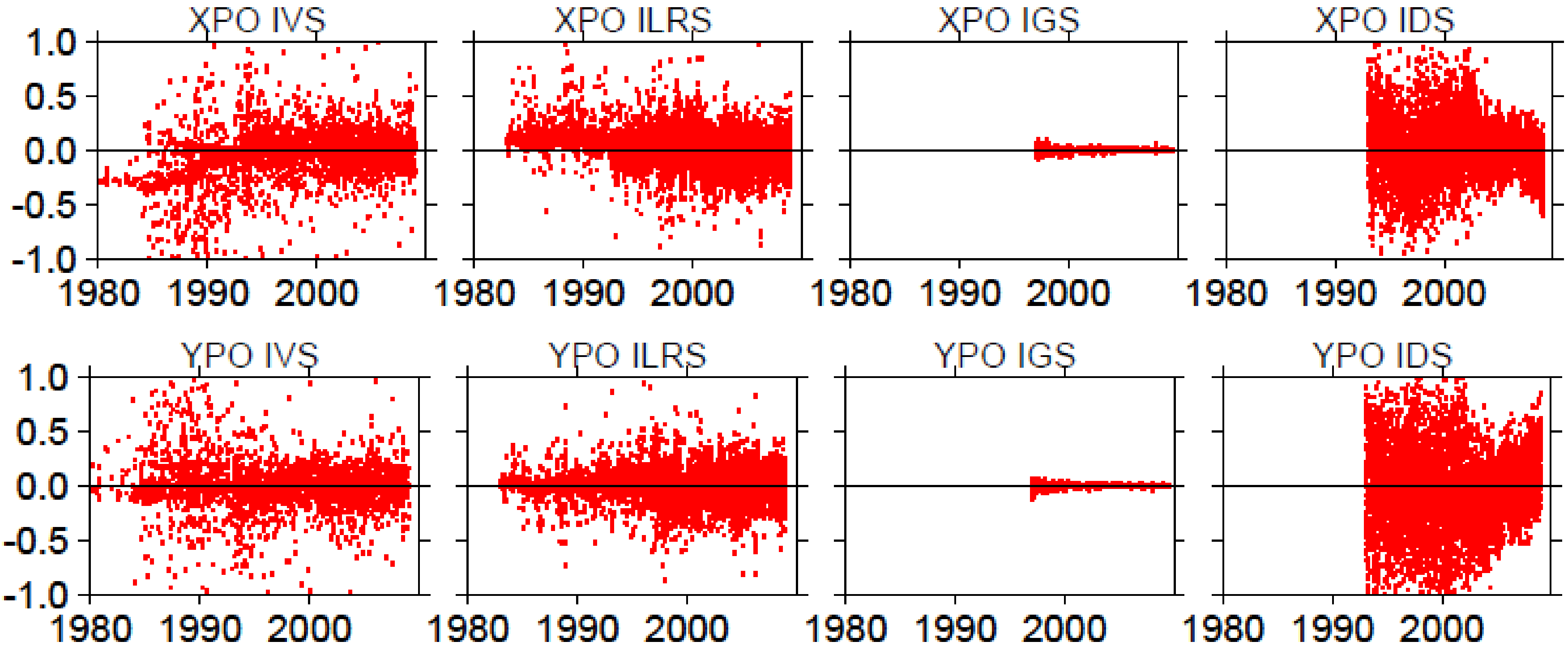
- Origin: SLR
- Scale : mean of SLR &VLBI
- Orientation : aligned to ITRF2005 (and rates)

  using 95 stations located at 79 sites:
  - 55 at northern hemisphere
  - 24 at southern hemisphere





ITRF2008P PM residuals



Source: Zuheir Altamimi

WRMS in µas

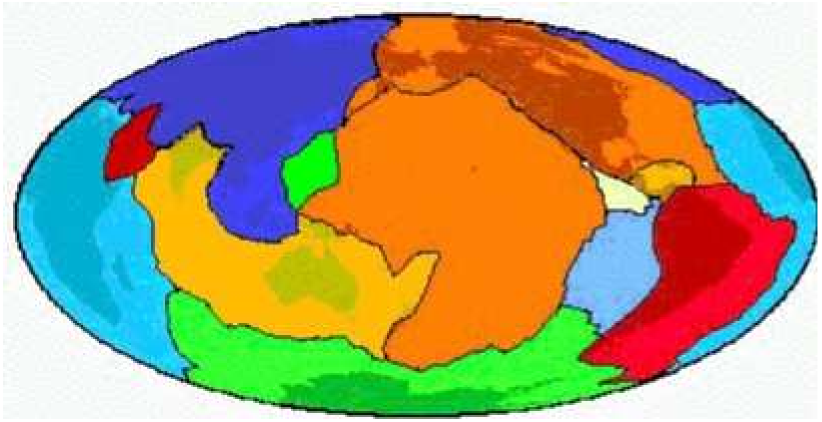
	X-pole	Y-pole
GPS	10	10
DORIS	239	353

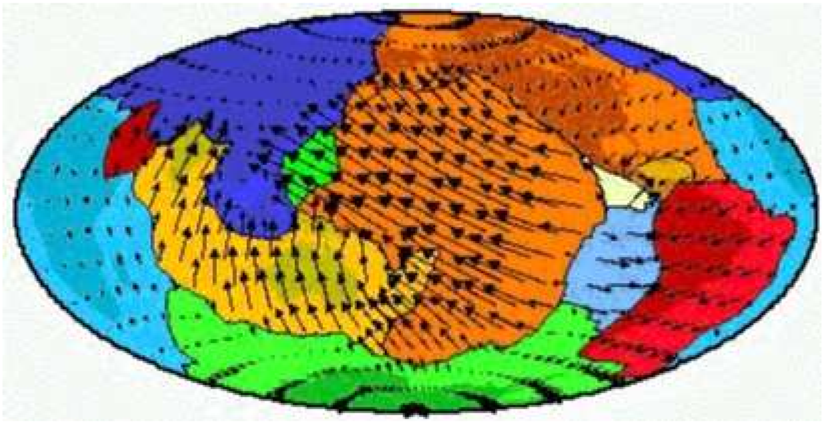
	X-pole	Y-pole
VLBI	142	120
SLR	144	128

Tectonic motion (in the hot spot reference frame)

NNR-Nuvel-1a (DeMets et al., 1994)

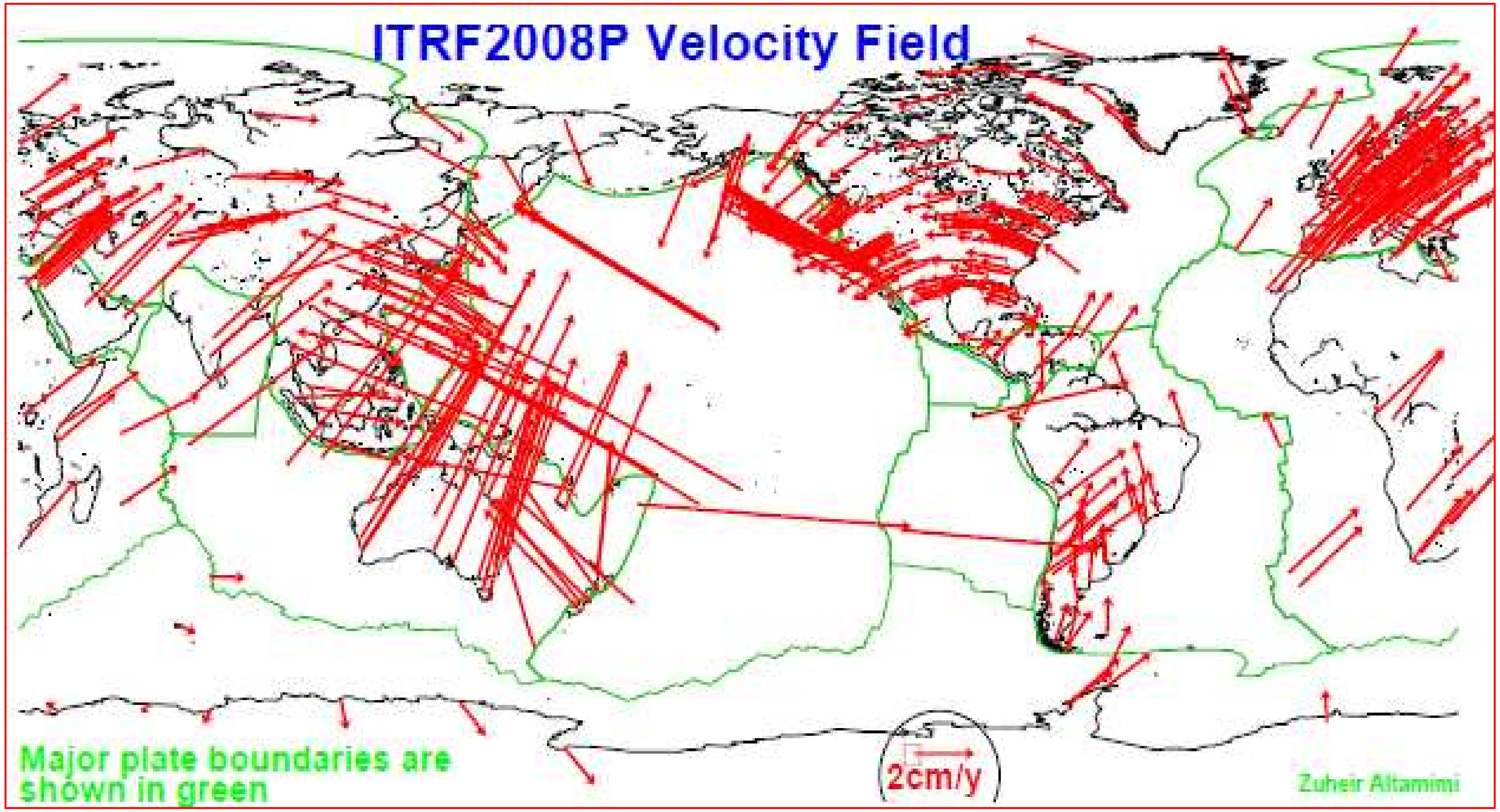
Plate		Velocity (cm/a)
1	Pacific	10 cm/a North-West
2	Eurasia	1 cm/a East
3	Africa	2 cm/a North
4	Antarctic	Rotates around itself
5	Indo-Australia	7 cm/a North
6	North America	1 cm/a West
7	South America	1 cm/a North
8	Nazca	7 cm/a East
9	Philippine	8 cm/a West
10	Arabia	3 cm/a North-East
11	Coco	5 cm/a North-East
12	Caribbean	1 cm/a North-East



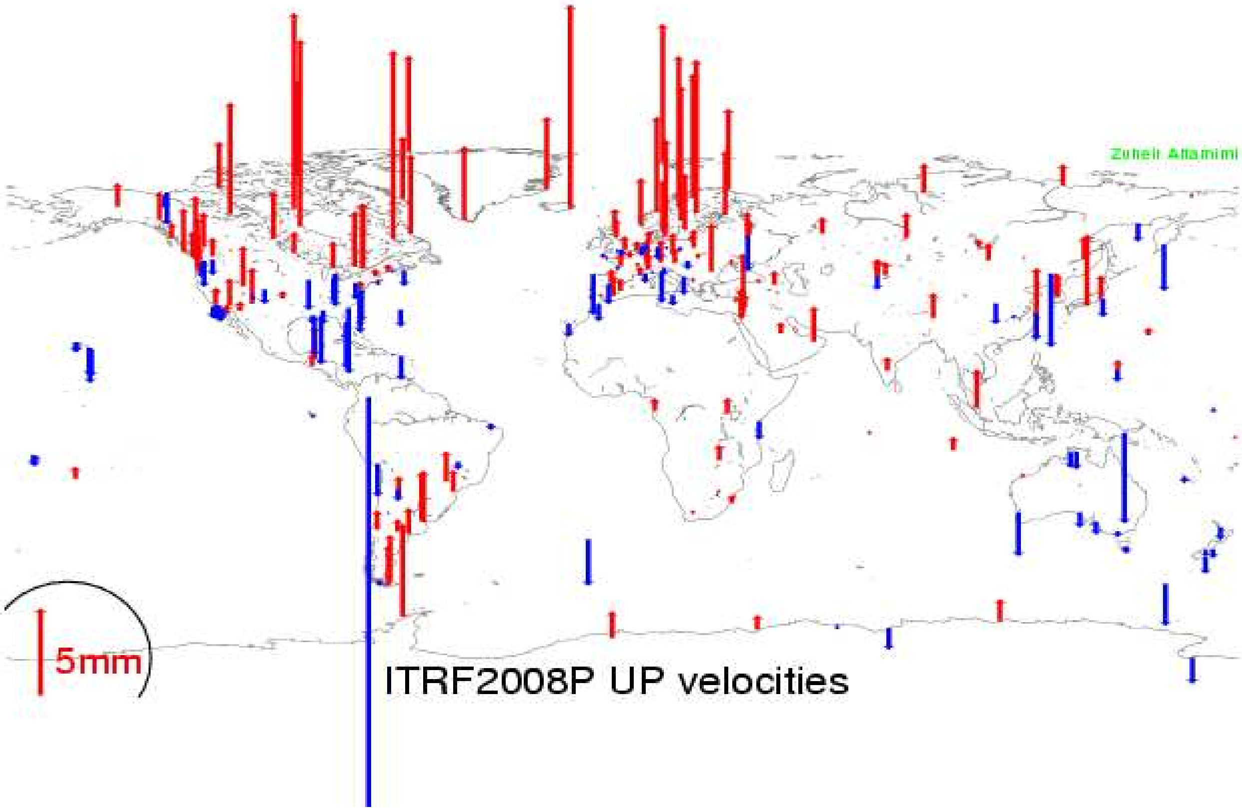


(source: www.geologie.ens.fr)

ITRF2008P horizontal velocity field



ITRF2008P Vertical Velocities σ < 3mm/y



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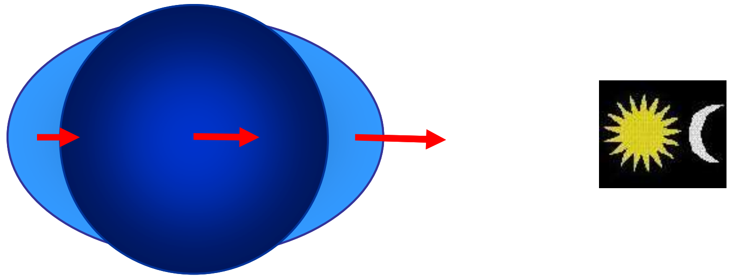
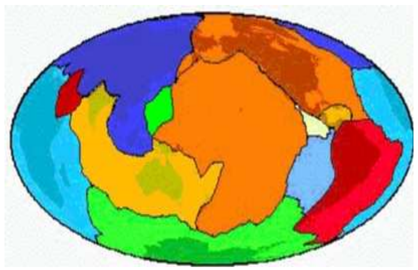
Rappels de géodésie générale

V. Déformations

Main physical deformations

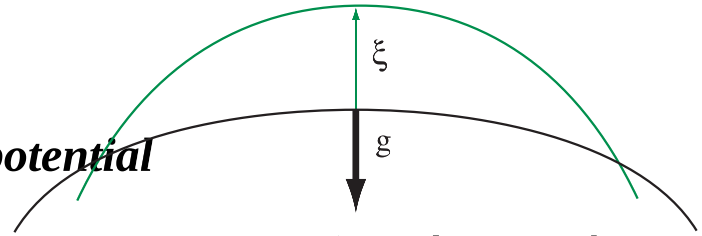


- Earth tide
- surface loading
  - ocean tides
  - ocean currents
  - atmospheric pressure
  - hydrology
  - post-glacial rebound
- polar tide
- tectonics
- earthquakes



Solid Earth tide : theory of Love (1909)

Equipotential at the surface : $U(r, \varphi, \lambda) = C$



The tide potential introduces a ***deformation of the equipotential***

as : $U(r+\xi,\varphi,\lambda)+U_S(r,\varphi,\lambda)=C$

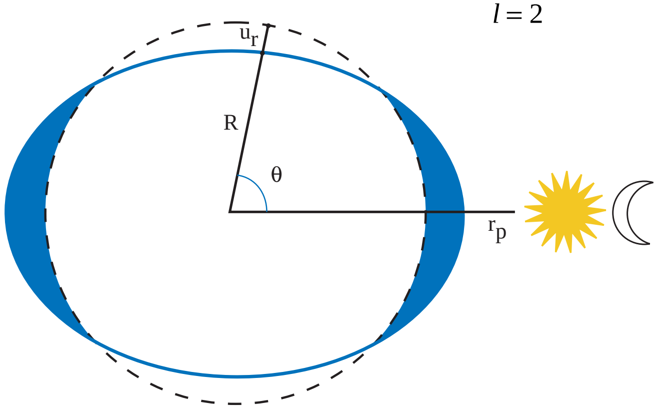
hence : $U(r,\varphi,\lambda)+\dfrac{\partial U}{\partial r}\xi+U_S(r,\varphi,\lambda)=C$ *where :* $\xi=-\dfrac{U_S}{\frac{\partial U}{\partial r}}=\dfrac{U_S}{g}$

ΔU : tide potential
U : Earth potential

In a hypothesis of elastic Earth, the displacement of the crust must be proportional to the excitation.

Excitation : $U_S=\dfrac{Gm_S}{R}\sum_{l=2}^{3}\left(\dfrac{R}{r_S}\right)^{l+1}P_{l0}(\cos\theta)$

Displacement : $u_r=\sum_{l=2}^{3}h_l\dfrac{U_S}{g}$, ***h_l : Love number (dimensionless) of vertical deformation***

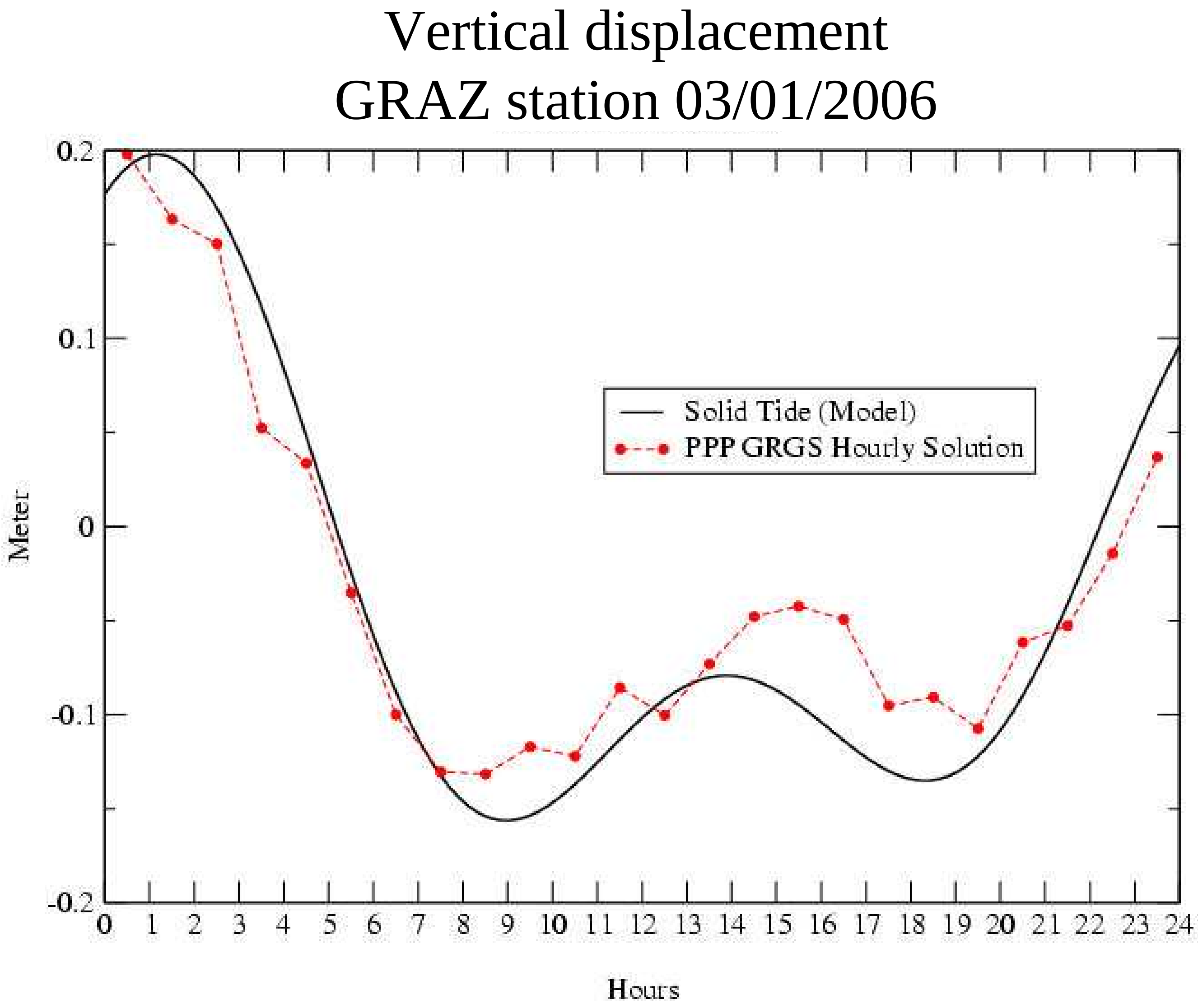


The increment of external potential generated by the elastic deformation of the Earth is proportional to the excitation potential and verified the Dirichlet principle. That is the ***Earth tidal potential:***

$$\Delta U=\sum_{l=2}^{3}k_l\left(\frac{R}{r}\right)^{l+1}U_{S_l}(r)$$

k_l : Love number (dimensionless) of potential

Solid earth tide deformation



Ocean tides modeling

The height of ocean tides is expressed by a sum over n waves :

$$\xi(\varphi,\lambda,t)=\sum_n Z_n(\varphi,\lambda)\cos\left(\theta_n(t)-\psi_n(\varphi,\lambda)\right)$$

Z_n is the amplitude of the wave n,

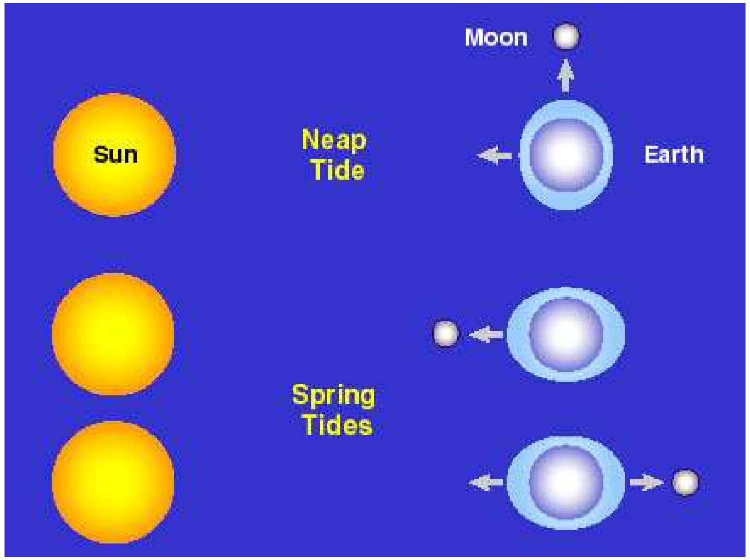
ψ_n is the phase,

θ_n is the Doodson argument which is expressed in linear combination of 6 variables

$$\theta_n(t)=n_1\tau+(n_2-5)s+(n_3-5)h+(n_4-5)p+(n_5-5)N'+(n_6-5)p_s$$

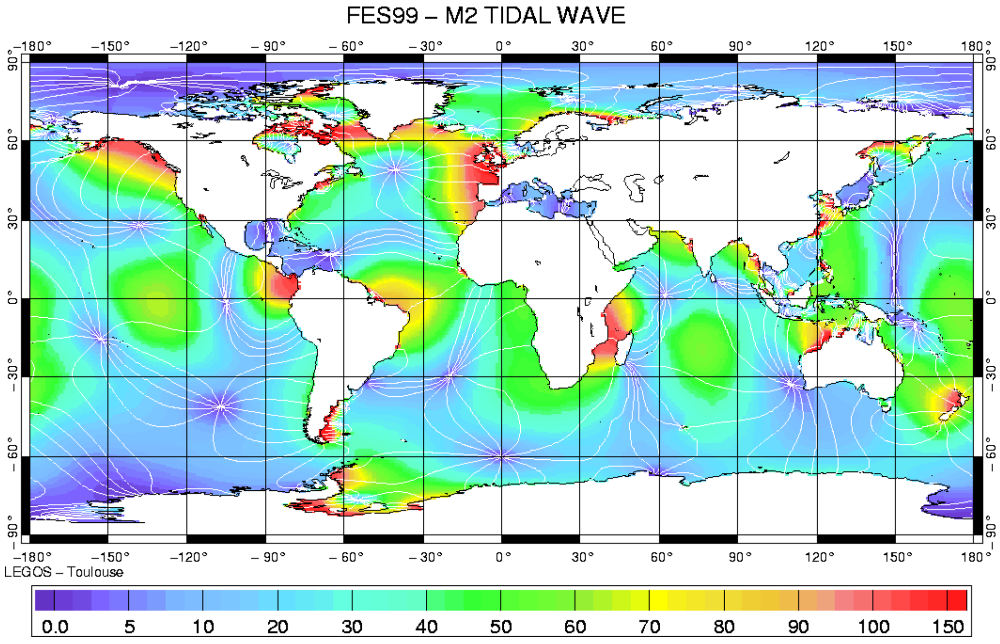
These 6 variables with decreasing frequencies represent the fundamental arguments according to Sun and Moon motions :

τ : angle of the mean lunar day (1.03505 d)
s : angle of the mean tropic month (27.32158 d)
h : angle of the mean tropic year (365.2422 d)
p : angle of the mean lunar perigee (8.8473 y)
N' : angle of the mean lunar node (18.6129 y)
p_s : angle of the perihelion (20940.28 y)



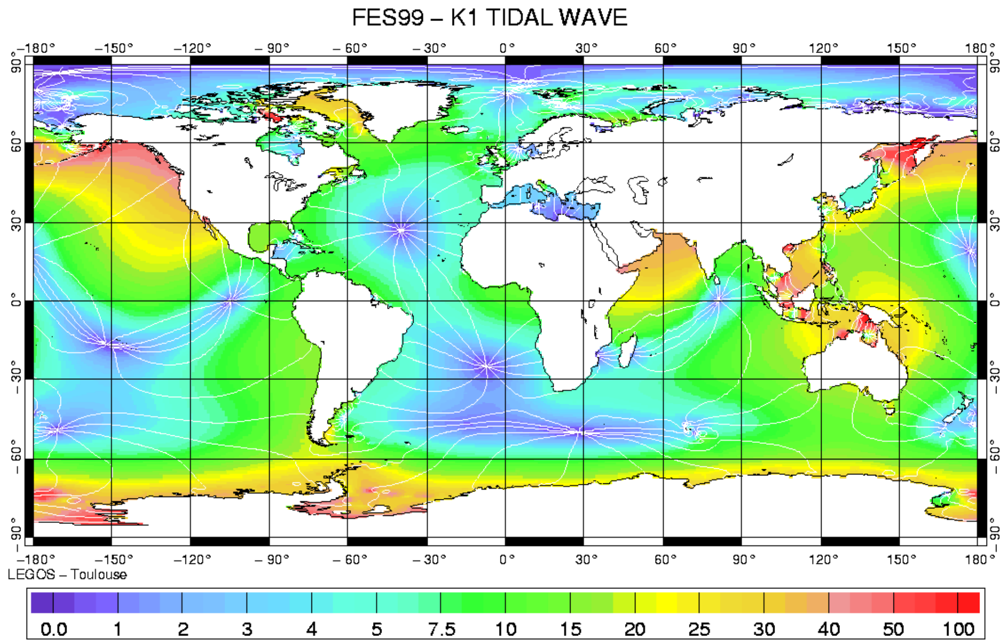
n_1 (= 0, 1, 2, 3...) defines the specie (long period, diurnal, semi-diurnal, ter-diurnal...),

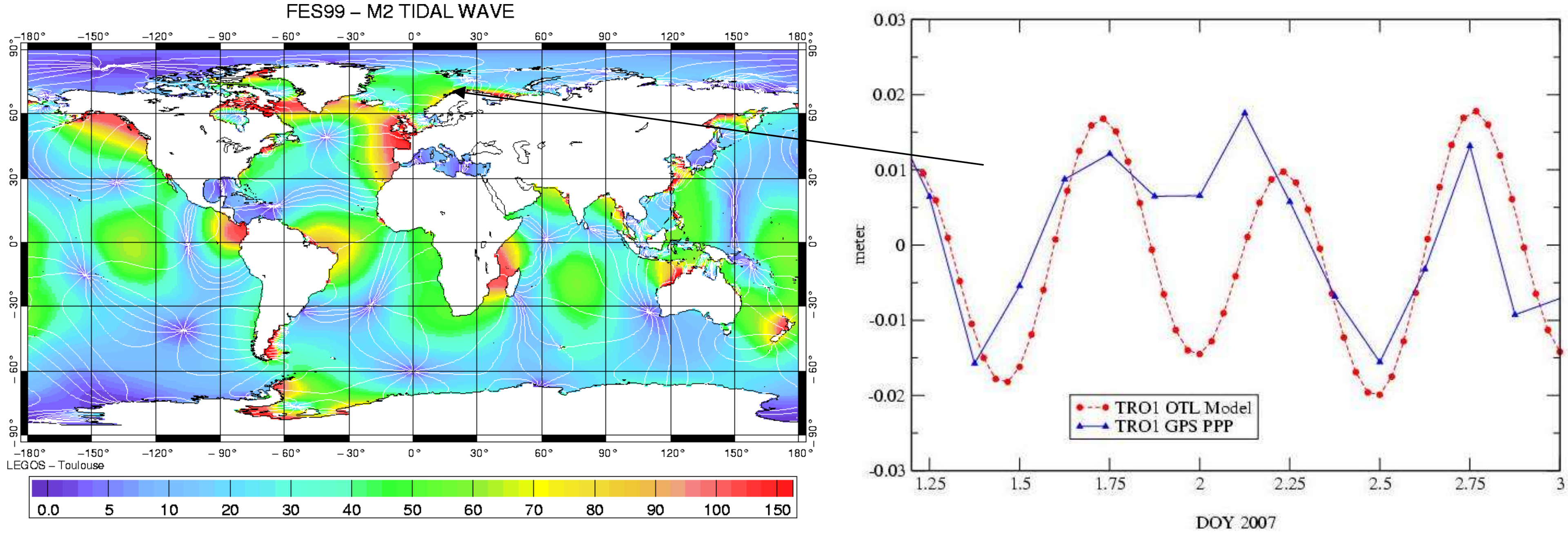
n_2 the group (in general :$1 \leq n_2 \leq 9$) and n_3 the constituent ($1 \leq n_3 \leq 9$).

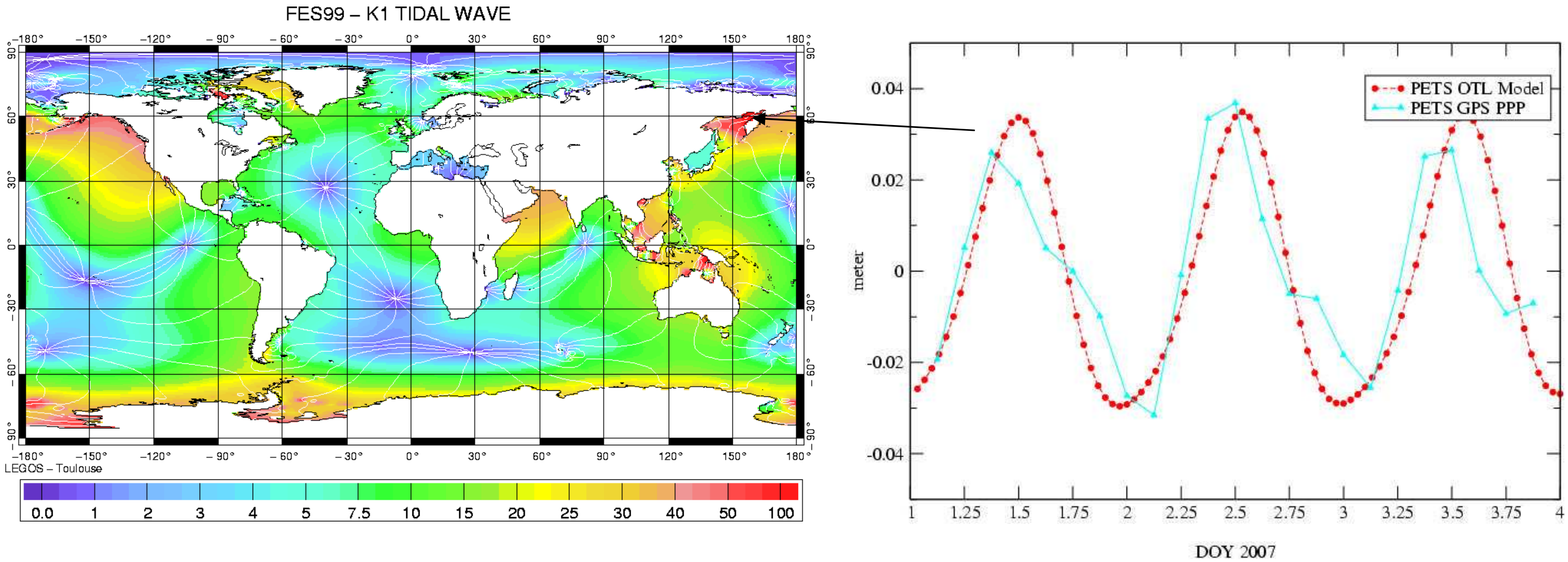


M2 wave (amplitude in cm)

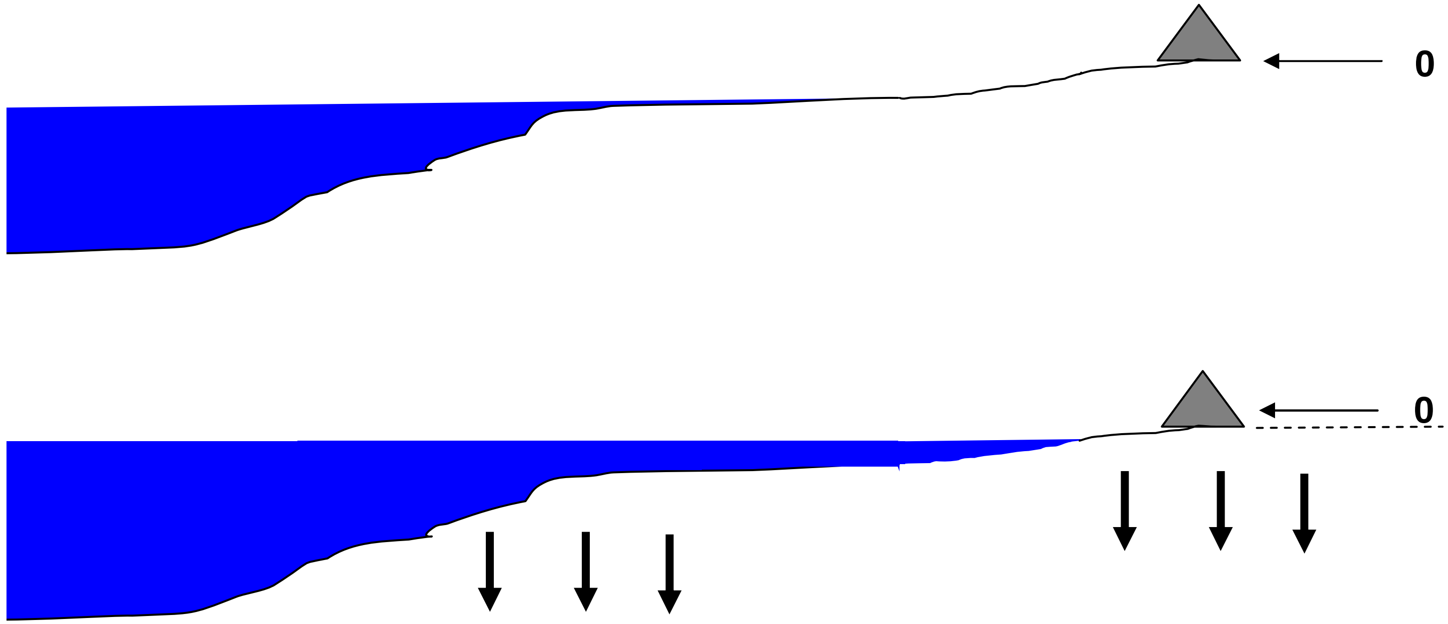
K1 wave (amplitude in cm)



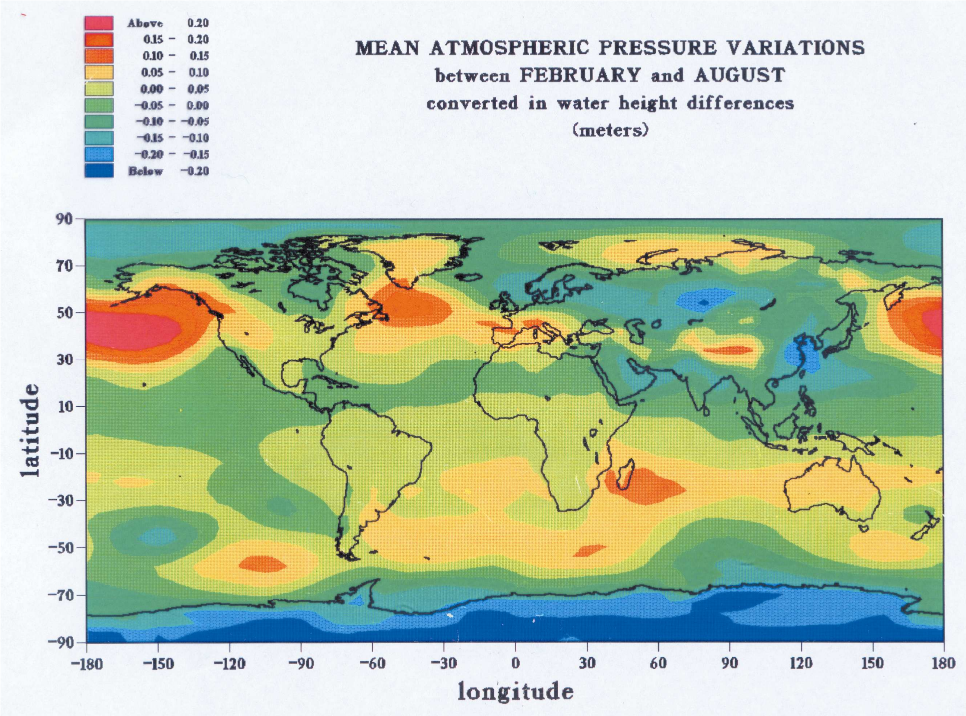

FES99 – M2 TIDAL WAVE
LEGOS – Toulouse
0.0 5 10 20 30 40 50 60 70 80 90 100 150
TRO1 OTL Model
TRO1 GPS PPP
meter
DOY 2007


FES99 – K1 TIDAL WAVE
LEGOS – Toulouse
0.0 1 2 3 4 5 7.5 10 15 20 25 30 40 50 100
PETS OTL Model
PETS GPS PPP
meter
DOY 2007

Vertical displacement due to ocean tide loading



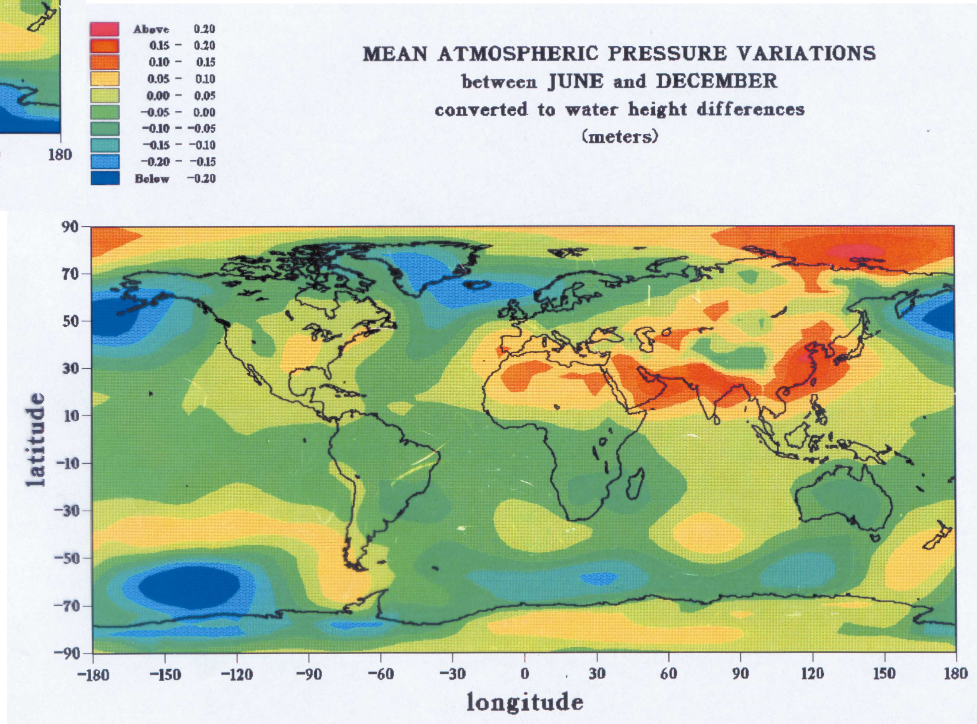
Atmospheric load



Amospheric pressure over continents in spherical harmonics functions:

$$\Delta P = \sum_{l=0}^{L} \sum_{m=0}^{l} P_{l,m}(\sin\varphi)\left(\Delta P_{l,m}^{C} \cos m\lambda + \Delta P_{l,m}^{S} \sin m\lambda\right)$$

$$= \sum_{l=0}^{L} \sum_{m=0}^{l} \Delta P_{l,m}(\varphi, \lambda)$$



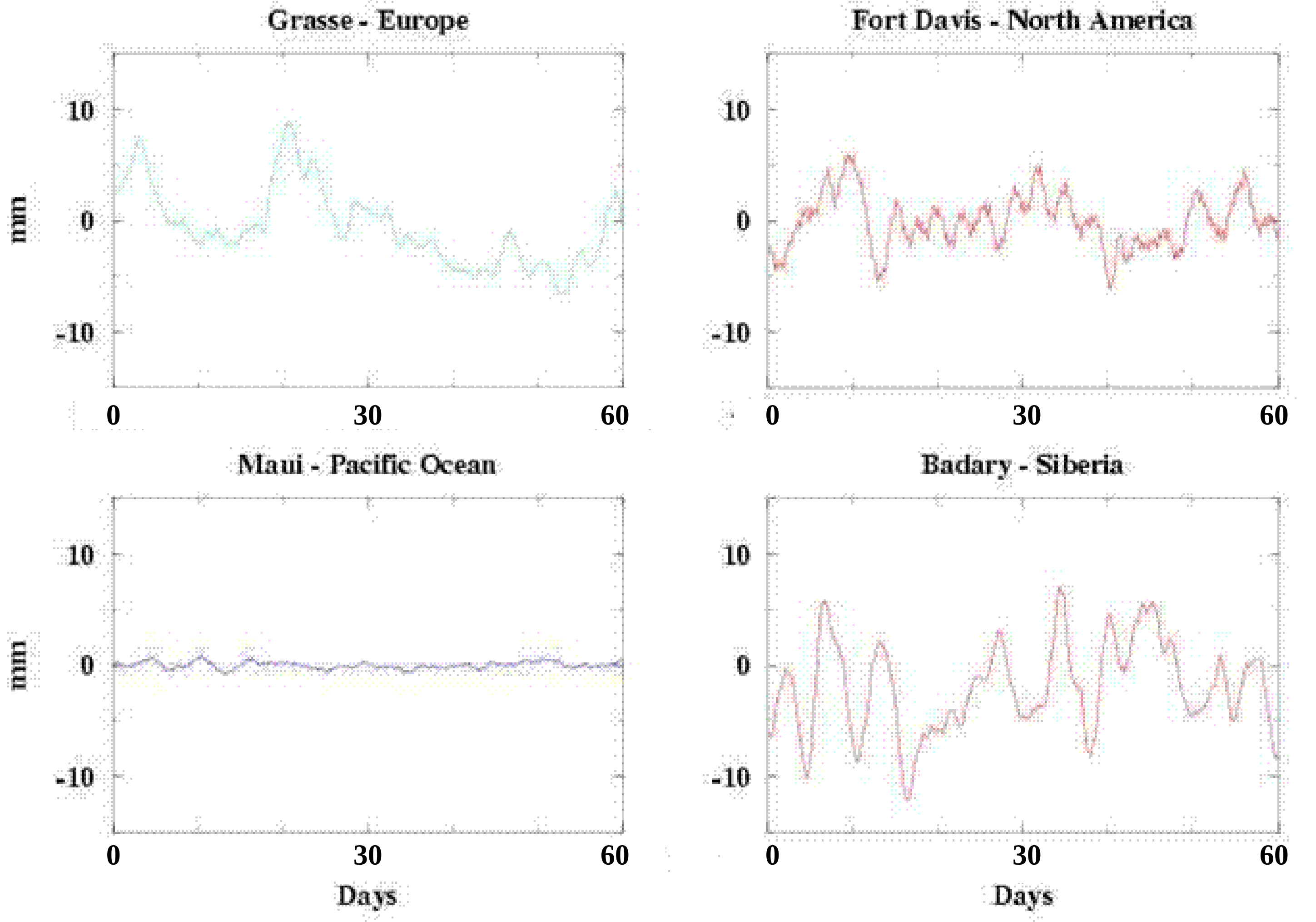
Load deformation from ground pressure over continents only:

$$u_r' = \frac{4\pi GR}{g^2} \sum_{l=2}^{L} \frac{h_l'}{2l+1} \sum_{m=0}^{l} \Delta P_{l,m}(\varphi, \lambda)$$

Load over ocean is supposed to be balanced by the hydrostatic (or inverse barometer) principle:

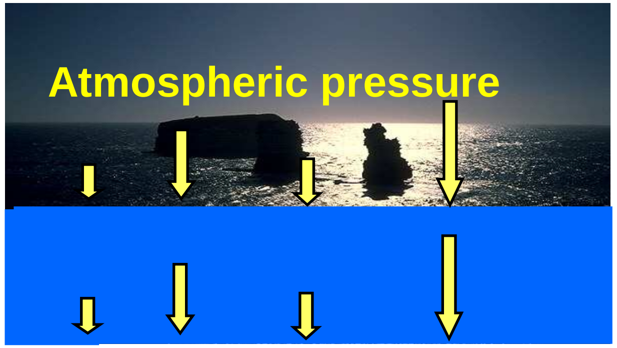
$$q = \frac{\Delta P}{g} = \rho_w h \quad (1\, mb \Leftrightarrow 1\, cm)$$

Vertical displacement at station induced by atmospheric loading (in mm)

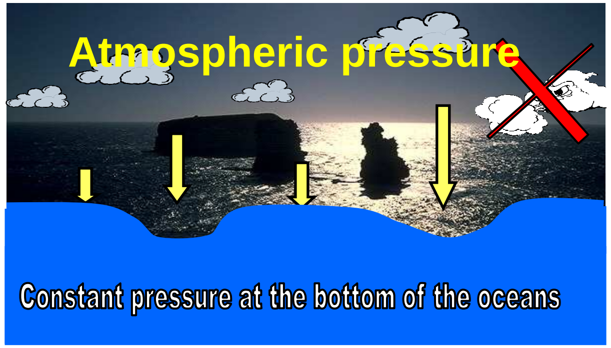


Atmosphere/ocean: the ocean reaction

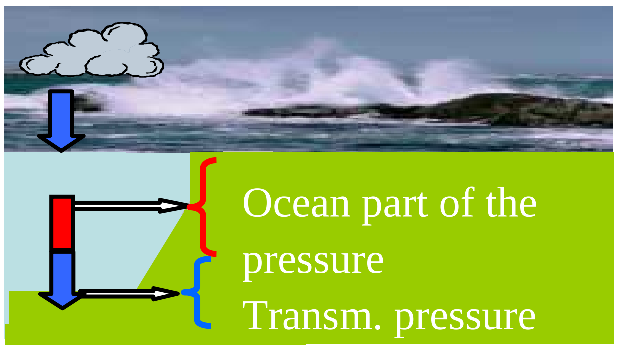
Non inverse barometer model :
The ocean surface is not changed ; atmospheric pressure is completely transferred to ocean floor



Inverse barometer model :
Ocean surface moves to stay in hydrostatic equilibrium with the atmosphere ($\Delta P=\rho_w gh$)



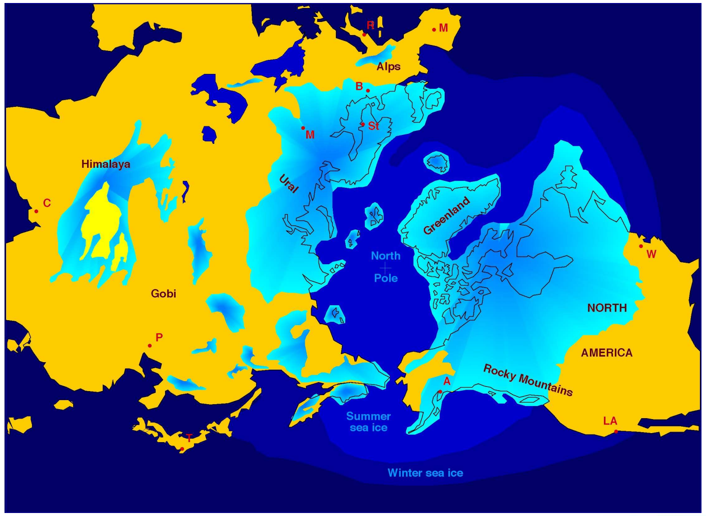
Dynamical model (ECCO, MIT,MOG2D, TUGO) : ocean surface changes but one part of atmospheric pressue is transmitted to ocean floor



Post-glacial rebound

Secular deformation (from GRACE):

$$\Delta \dot{N}_{lm} = R \sum_{l=2}^{L} \sum_{m=0}^{l} \overline{P}_{lm}(\sin \varphi)\left(\Delta \dot{\overline{C}}_{lm} \cos m\lambda + \Delta \dot{\overline{S}}_{lm} \sin m\lambda\right)$$

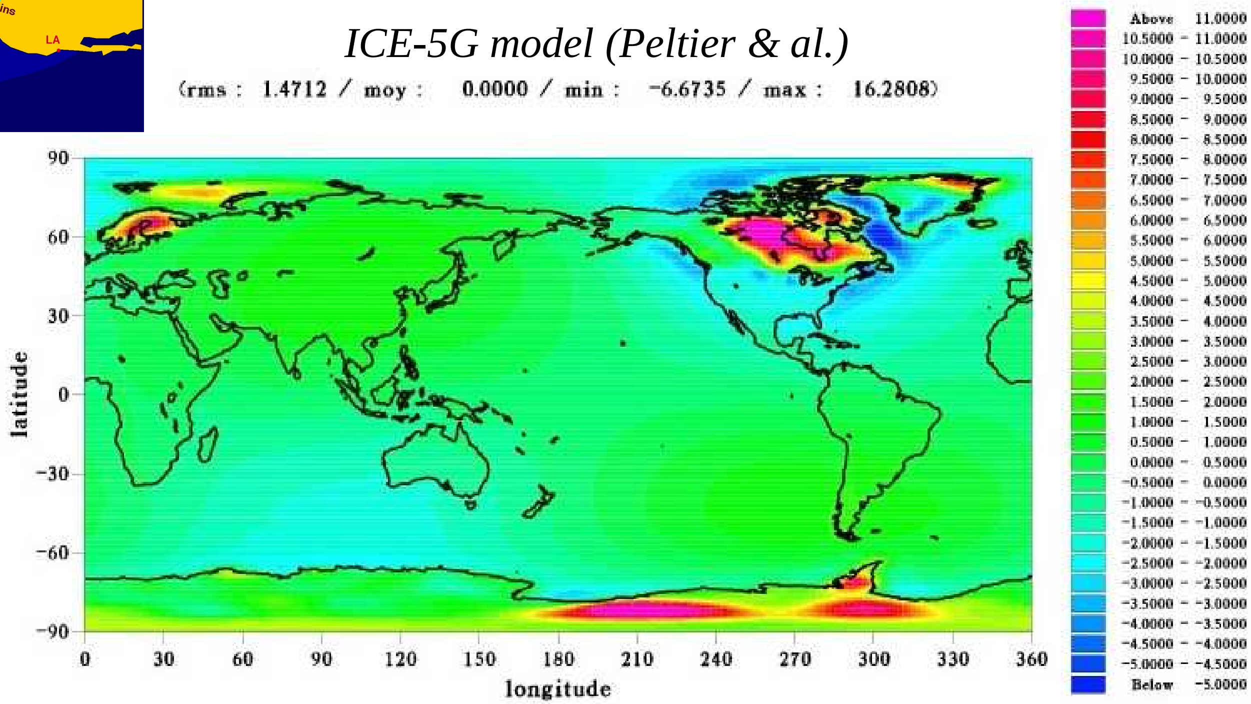


Ice coverage at the end of Pleistocen

Surface deformation (mm/yr)

ICE-5G model (Peltier & al.)

(rms : 1.4712 / moy : 0.0000 / min : -6.6735 / max : 16.2808)



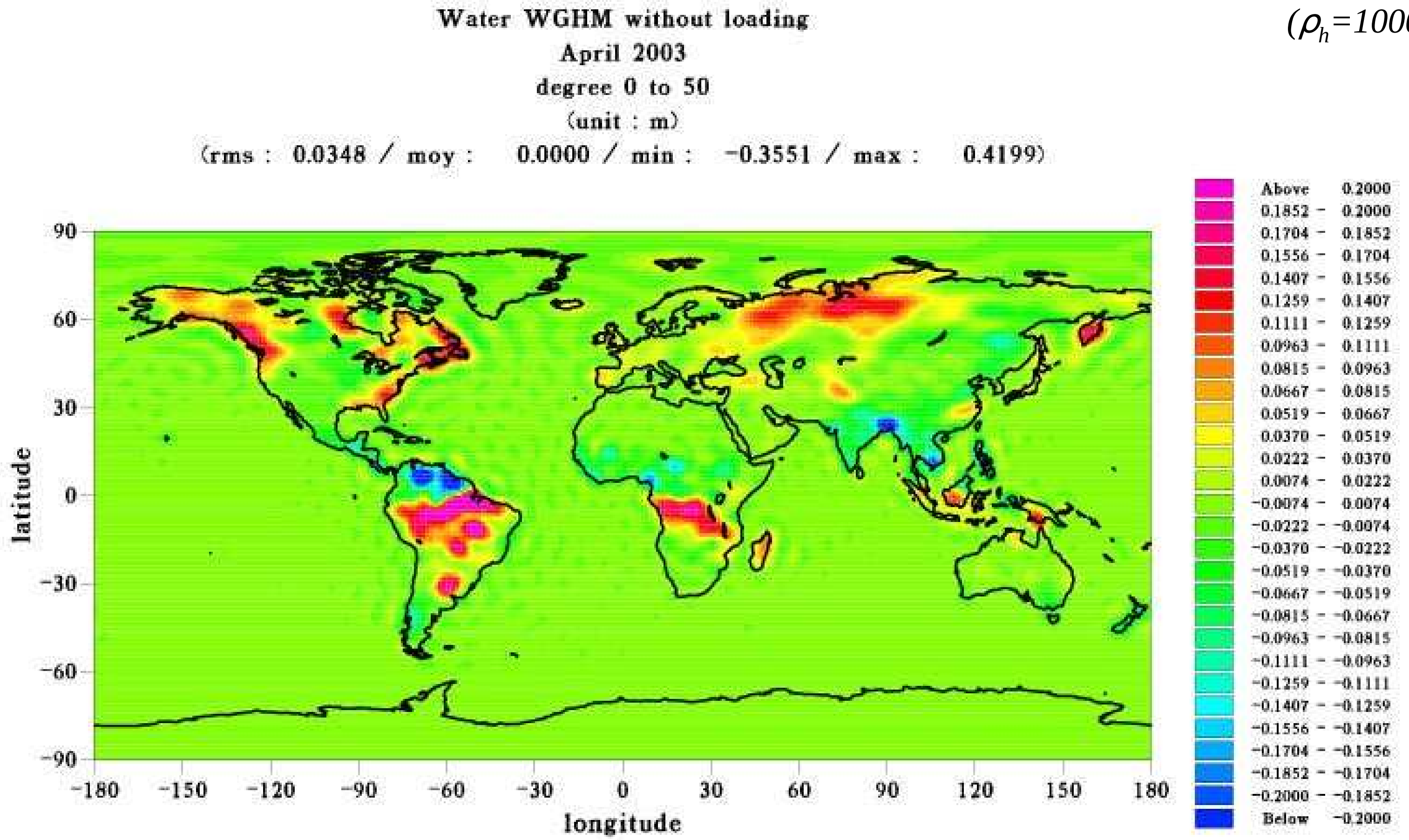
Empirically (*Wahr et al., 2002*):

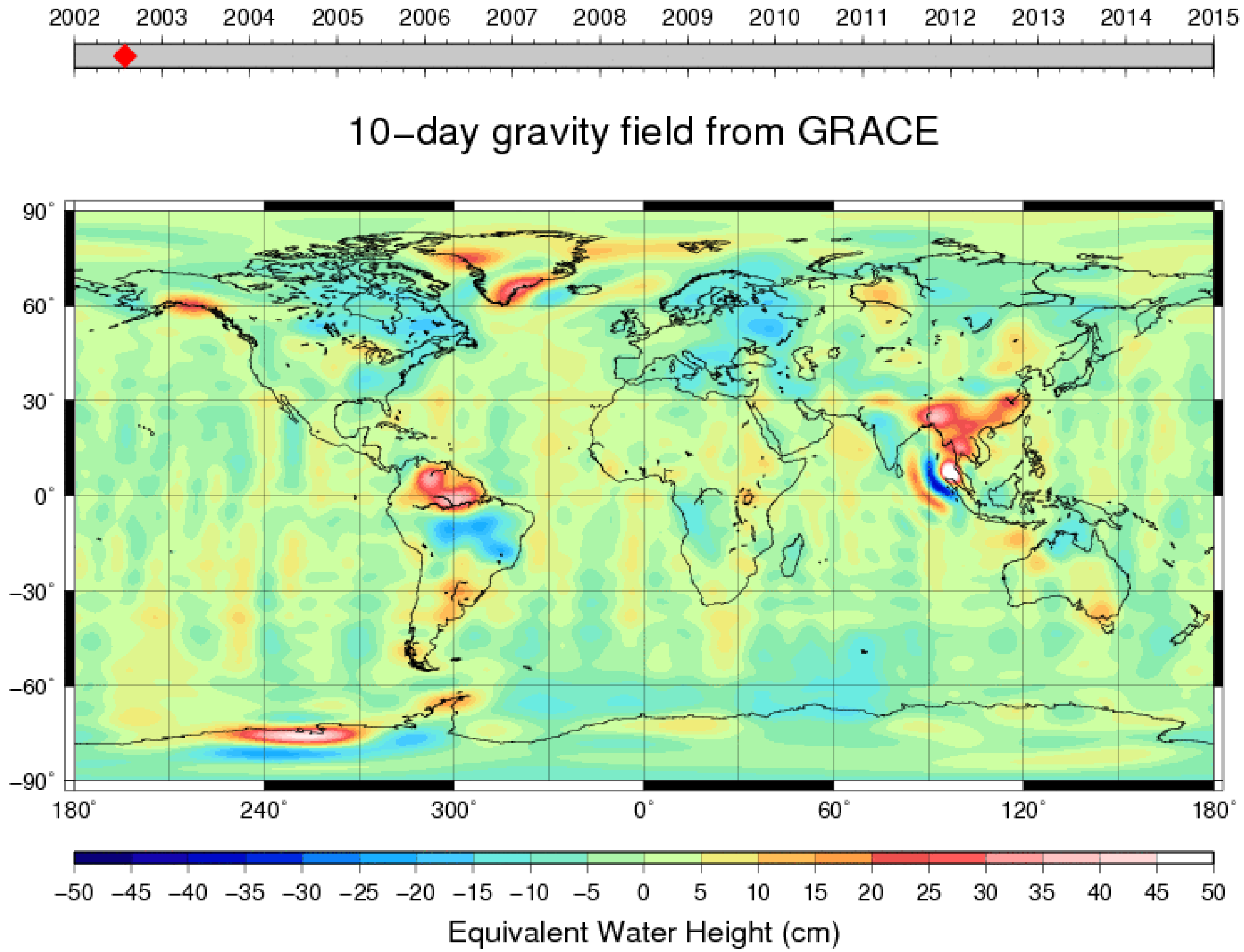
$$\dot{u}'_r = \sum_{l=2}^{L} \frac{2l+1}{2} \sum_{m=0}^{l} \Delta \dot{N}_{lm}$$

Hydrology (from climatology data)

Load deformation from water height (over continents only): $$u'_r = \frac{4\pi G R \rho_h}{g} \sum_{l=2}^{L} \frac{h'_l}{2l+1} \sum_{m=0}^{l} \xi_{l,m}$$

$(\rho_h = 1000\ kg/m^3)$

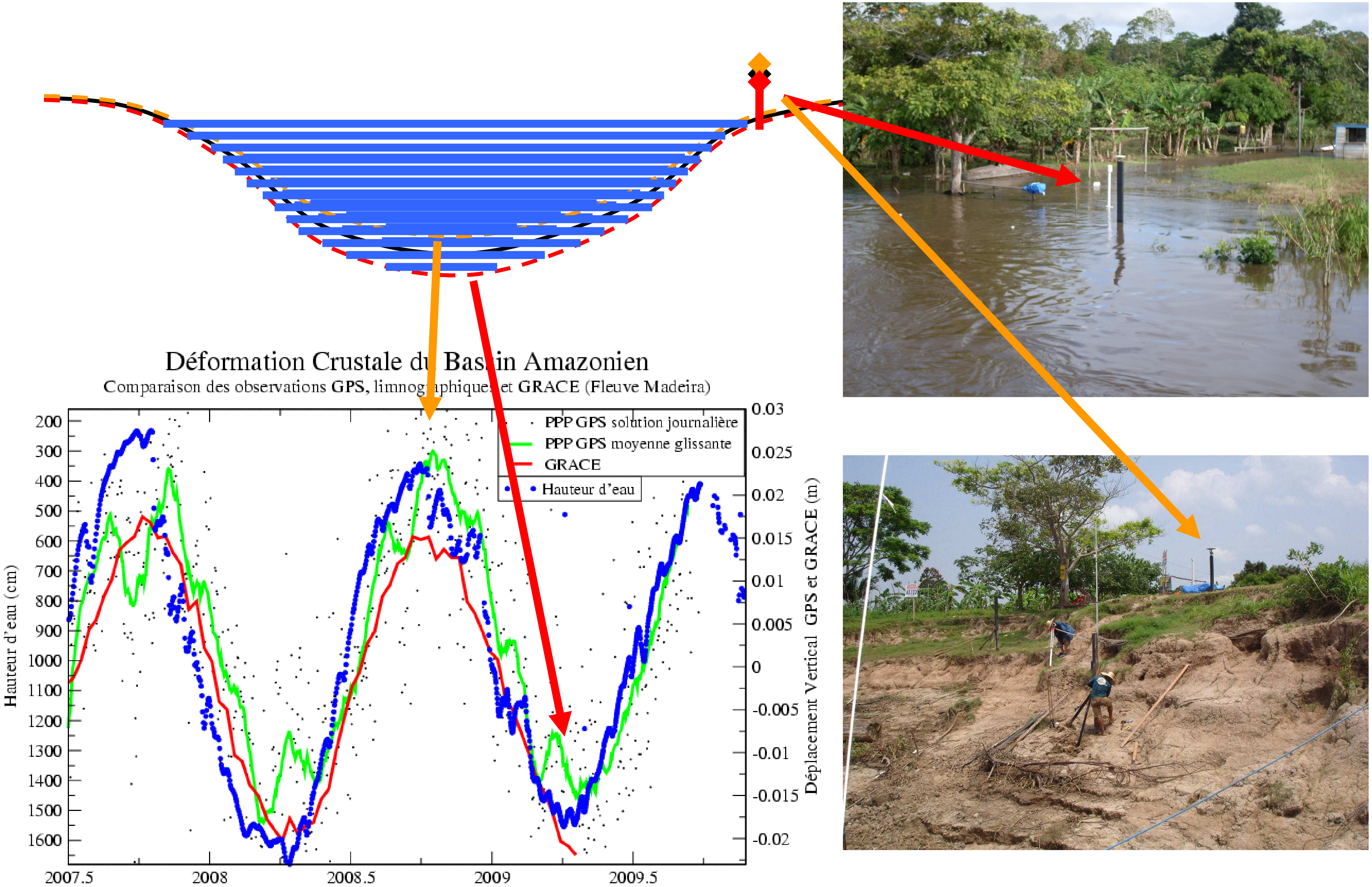




$$\Delta N = R\sum_{l=2}^{L}\sum_{m=0}^{l}\overline{P}_{lm}(\sin\varphi)\left(\Delta\overline{C}_{lm}\cos m\lambda + \Delta\overline{S}_{lm}\sin m\lambda\right)$$

$$\Delta h_w = \frac{g}{4\pi G R\rho_w}\sum_{l=2}^{50}\frac{2l+1}{1+k'_l}\Delta N_l^{geoid}$$

Hydrological loading in Amazonian bassin



D. Moreira, 2011

Synthesis of displacements

	Vertical	horizontal
Earth’s tide	30 cm	10 cm
Ocean tide loading	10 cm	3 cm
Atmospheric loading	2.5 cm	0.8 cm
Polar tide	2.5 cm	0.7 cm
Hydrology	20 cm	a few cm
Tectonic	1 cm/yr	10 cm/yr
Geocenter	~3 mm	

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Rappels de géodésie générale

VI. Méthode inverse

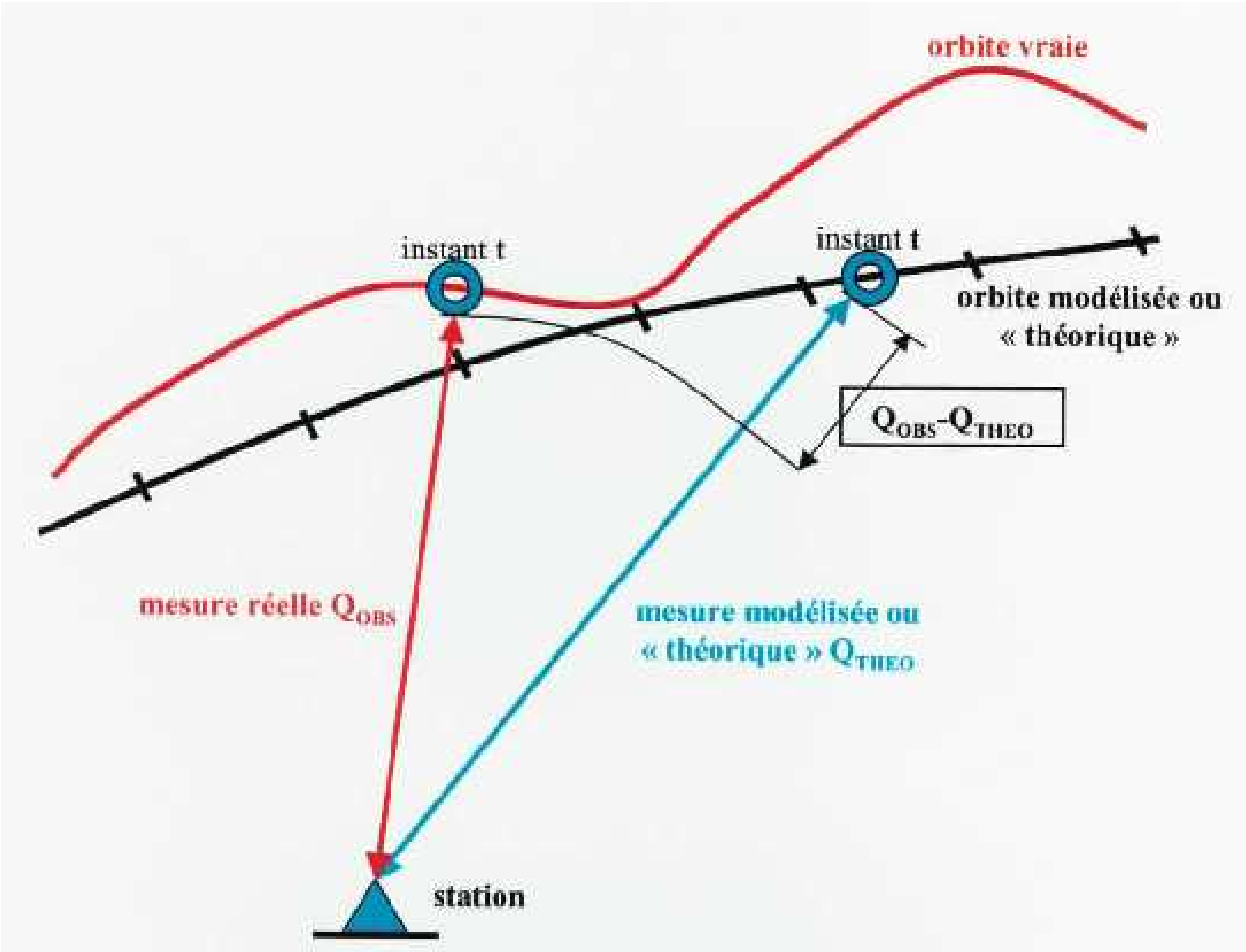

orbite vraie
instant t
instant t
orbite modélisée ou
« théorique »
Q_{OBS}-Q_{THEO}
mesure réelle Q_{OBS}
mesure modélisée ou
« théorique » Q_{THEO}
station

Illustration d'ajustement de paramètres par la méthode des moindres carrés

Ces équations d'observation proviennent d'un développement en série de Taylor au premier ordre (donc de la linéarisation) d'un système non linéaire :

$$\sum_{j=1,n} \frac{\partial Q_{theo}}{\partial p_j} * \Delta p_j = Q_{mes} - Q_{theo}$$

Avec :

Q_{theo} et Q_{mes} : les quantités théorique et mesurée,

p : le vecteur des paramètres,

Δp : la correction des paramètres (noté x dans l'équation (1))

$\frac{\partial Q_{theo}}{\partial p_j}$: le vecteur des dérivées partielles de la quantité théorique par rapport à chaque paramètre.

La matrice *A* dans l'équation d'observation est alors appelée "matrice des dérivées partielles", le vecteur *x* des inconnues "vecteur de correction des paramètres" et le vecteur *B* "vecteur des résidus d'observation" (quantité mesurée moins théorique).

L'équation d'observation (1) est une équation surdéterminée, elle n'a donc aucune solution exacte. Il faut introduire dans l'équation (1) le vecteur ε des erreurs. L'équation (1) devient : $A * x = B + \varepsilon \pm \Sigma \quad (2)$

Illustration d'ajustement de paramètres par la méthode des moindres carrés

Étant donné qu'il y a une infinité de solutions possibles à l'équation (2), il faut choisir un critère permettant de trouver la solution "optimale".

Le critère choisi est celui des moindres carrés : on cherche le vecteur x qui minimise $\varepsilon^T \Pi \varepsilon$, où $\varepsilon = A^*x\text{-}B$ et Π est la matrice diagonale de pondération des observations telle que $\Pi_{ii} = 1/\sigma_i^2$.

On peut montrer, en dérivant $\varepsilon^T \Pi \varepsilon$ par rapport à chaque inconnue x_i, que $\varepsilon^T \Pi \varepsilon$ minimum $\leftrightarrow$ $A^T \Pi \varepsilon = 0$.

En reportant dans l'équation (2), on aboutit à l'équation normale suivante :

$$N^*x = S \quad (3)$$

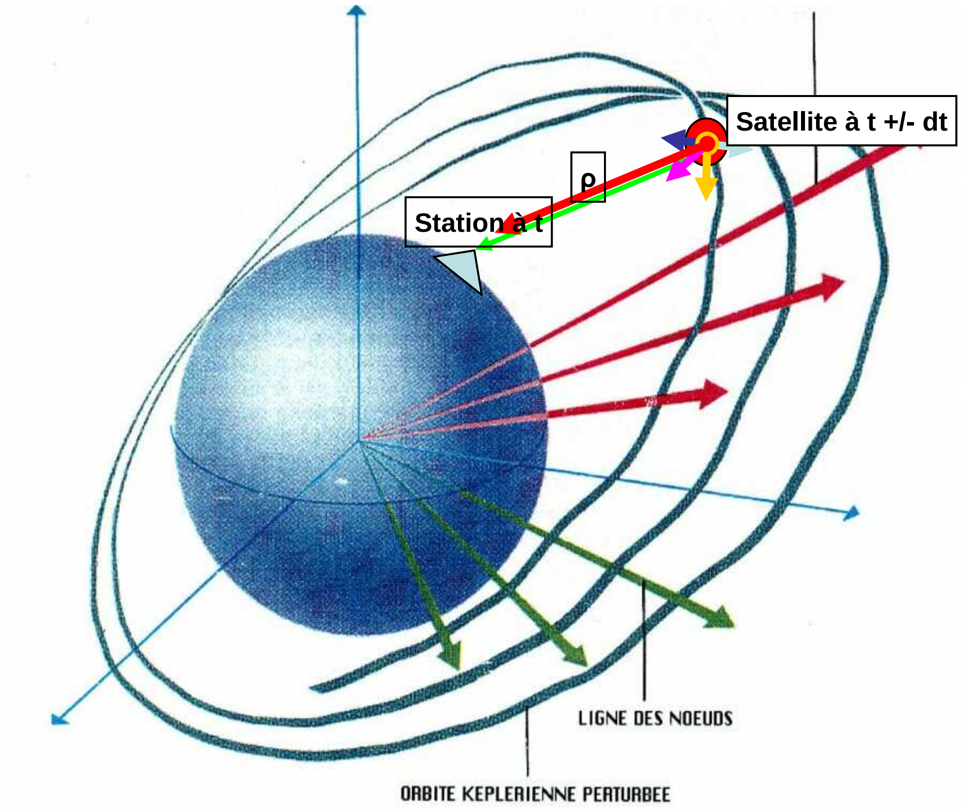
Où :

$N = A^T \Pi A$ est la matrice normale et $S = A^T \Pi B$ est le second membre

N est carrée, de dimension n^*n, et est symétrique définie positive.

La solution x, optimale au sens des moindres carrés, est obtenue en inversant la matrice normale N et en calculant le produit : $x = N^{-1} * S$

Synthèse :



Accélérations

$$\frac{\vec{F}}{m} = \overrightarrow{gradU} + \vec{\gamma}$$

Intégration

$$\frac{\partial^2 \vec{r}}{\partial t^2} = \sum \frac{\vec{F}}{m}\left(\vec{r}, \dot{\vec{r}}, t\right)$$

$$\left(\vec{r}_0, \dot{\vec{r}}_0\right)$$

Comparaison aux observations

$$\rho_{obs} - \rho_{calc} = \sum_i \frac{\partial \rho}{\partial p_i} \Delta p_i$$

Formation des équations normales

$$N \Delta p_i = S$$

Résolution

$$\Delta p_i = N^{-1} S$$

Détermination par méthode inverse des paramètres :

- dynamiques des modèles
- des observations

FIN

units

- milli arc seconde (mas) = deg/3600/1000

• 1 mas	= 3.1 cm	= 66 µsec
• 1 cm	= 0.32 mas	= 21 µsec
• 100 µsec	= 1.5 mas	= 4.6 cm