CATREF Software

Zuheir ALTAMIMI
Laboratoire de Recherche en Géodésie
Institut Géographique National
France
E-mail: altamimi@ensg.ign.fr

Ecole d'été GRGS – Forcalquier, 5 Août, 2006

Plan

- **Rappels**
  - Définition d'un système de référence terrestre
  - Transformation entre 2 systèmes de référence
  - Réalisation d'un système de référence

- **CATREF**
  - Modèle de combinaison
  - Fonctionnalités
  - Exemple

Ideal Terrestrial Reference System

A tridimensional reference frame (mathematical sense)
Defined in an Euclidian affine space of dimension 3:

Affine Frame (O,E) where:

O: point in space (Origin)
E: vector base: orthogonal with the same length:
- **unit vectors co-linear to the base (Orientation)**
- **unit of length (Scale)**

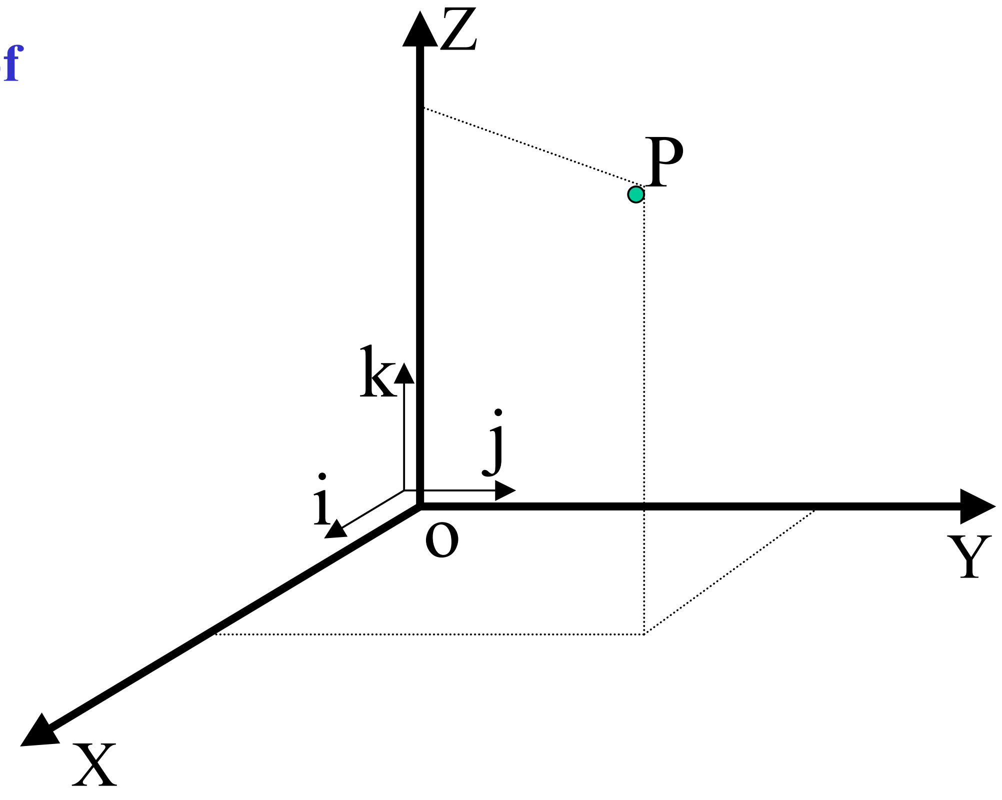
$$\lambda = \left\| \vec{E}_i \right\|_{i=1,2,3}$$

$$\vec{E}_i . \vec{E}_j = \lambda^2 \delta_{ij}$$

$$(\delta_{ij} = 1, \quad i = j)$$

Affine Frame

- **Origin**:
  - Barycentric (Center of Mass of the solar system)
  - Geocentric: CoM of the Earth
- **Orientation**:
  - Ecliptic
  - Equatorial
- **Unit of length (Scale)**: Same norm for the 3 vectors

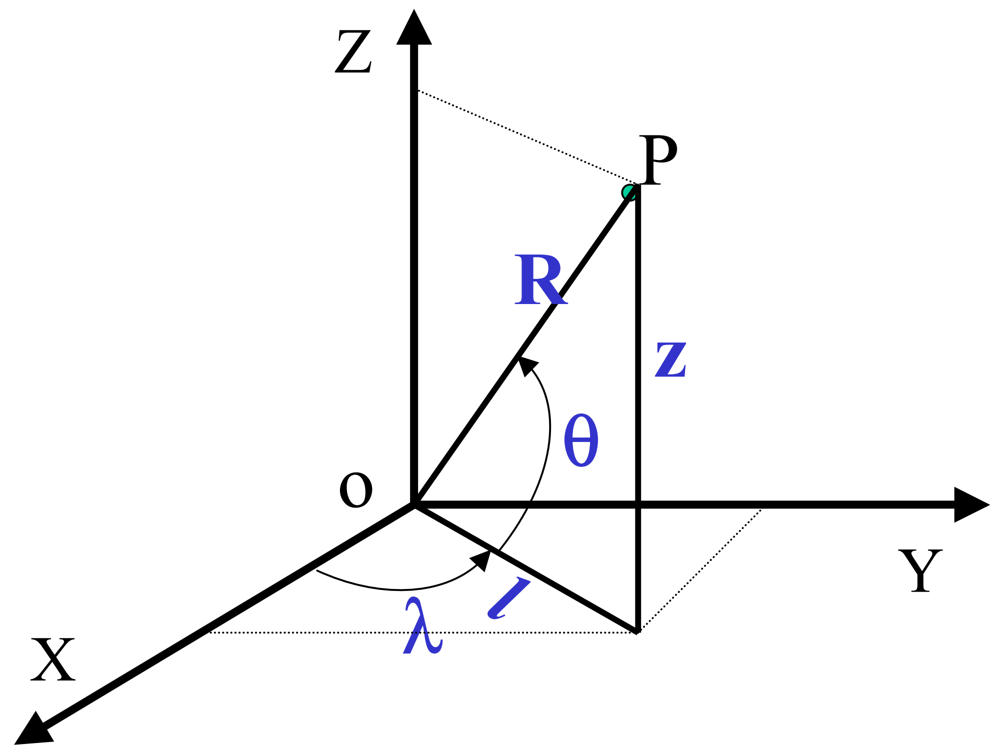


Ideal Terrestrial Reference System in the Context of Space Geodesy

- **Origin**: **Geocentric: Earth Center of Mass**
- **Scale**: **SI Unit**
- **Orientation**: **Equatorial (Z axis is the direction of the Earth pole)**

Coordinate Systems

- **3D**:
  - Cartesian: X, Y, Z
  - Ellipsoidal: λ, φ, h
  - Mapping: E, N, h
  - Spherical: R, θ, λ
  - Cylindrical: *l*, λ, Z
- **2D**:
  - Geographic: λ, φ
  - Mapping: E, N
- **1D** : Height system: H



$$OP \left| \begin{array}{l} l\cos\lambda \\ l\sin\lambda \\ z \end{array} \right.$$

Cylindrical

$$OP \left| \begin{array}{l} R\cos\theta\cos\lambda \\ R\cos\theta\sin\lambda \\ R\sin\theta \end{array} \right.$$

Spherical

Transformation Générale entre 2 SR (1/2)

Similitude Euclidienne
à 7 paramètres:

$$\boxed{X_2 = T + \lambda.\mathcal{R}.X_1}$$

Translation $T = (T_x, T_y, T_z)^T$

Facteur d'échelle λ

Rotation $\mathcal{R} = R_x.R_y.R_z$

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos R1 & \sin R1 \\ 0 & -\sin R1 & \cos R1 \end{pmatrix}$$

$$R_y = \begin{pmatrix} \cos R2 & 0 & -\sin R2 \\ 0 & 1 & 0 \\ \sin R2 & 0 & \cos R2 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos R3 & \sin R3 & 0 \\ -\sin R3 & \cos R3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation Générale entre 2 SR (2/2)

A partir de la forme générale: $X_2 = T + \lambda.\mathcal{R}.X_1$

en géodésie spatiale on utilise la forme linéarisée:

$$X_2 = X_1 + T + DX_1 + R.X_1$$

avec: $T = (T_x, T_y, T_z)^T$**,** $\lambda = (1 + D)$**, et** $\mathcal{R} = (I + R)$

où $R = \begin{pmatrix} 0 & -R3 & R2 \\ R3 & 0 & -R1 \\ -R2 & R1 & 0 \end{pmatrix}$

Car:

T **est de l'ordre de qq 100 mètres.** D **et** R **sont de l'ordre de** 10^{-5}
Les termes de 2ème ordre sont négligés et sont de l'ordre de 10^{-10}
≈ 0.6 **mm.**

Réalisation d'un système de référence

- Réalisation en utilisant les données d'une ou plusieurs techniques
  - Traitement des observations brutes
  - Logiciels: GINS, Gipsy, Bernese, GAMIT,...
- Réalisation par combinaison
  - Combinaison de jeux de coordonnées et EOPs
  - Logiciels : CATREF, ...

==> Repère de Référence Terrestre (RRT)

==>14 degrés de liberté à définir

Position instantanée d'un point

$$X(t) = X_0 + \dot{X} \times (t - t_0) + \sum_i \Delta X_i(t)$$

$\dot{X}$: Vitesse linéaire due au mouvement des plaques tectoniques

Correction diverses $\sum_i \Delta X_i(t)$**:**

- **Marées terrestres : $\approx$ 30 cm**
- **Surcharges Océaniques: qq cm**
- **Pression atmosphérique : qq mm**
- **Mouvement du Géocentre: qq mm , périodique et séculaire**
- **...**

Définition du RRT

$$X_2 = X_1 + T + DX_1 + R.X_1 \tag{1}$$

Supposant des vitesses constantes et en différenciant l'Eq. (1) par rapport au temps:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \overbrace{D\dot{X}_1}^{\approx 0} + \dot{D}X_1 + \overbrace{R\dot{X}_1}^{\approx 0} + \dot{R}X_1 \tag{2}$$

$$\dot{T} = \begin{pmatrix} \dot{T}1 \\ \dot{T}2 \\ \dot{T}3 \end{pmatrix}, \quad \dot{R} = \begin{pmatrix} 0 & -\dot{R}3 & \dot{R}2 \\ \dot{R}3 & 0 & -\dot{R}1 \\ -\dot{R}2 & \dot{R}1 & 0 \end{pmatrix}$$

=> 14 degrés de liberté pour définir un RRT.

Implementation of a TRF

- Definition at a chosen epoch, by selecting 7 transformation parameters, tending to satisfy the theoretical definition of the corresponding TRS
- A law of time evolution, by selecting 7 rates of the 7 transformation parameters, assuming linear station motion!
- ==> 14 parameters are needed to define a TRF

How to define the 14 parameters ?
« Datum definition »

- **Origin & rate:** CoM (Dynamical Techniques)
- **Scale & rate:** depends on physical parameters
- **Orientation:** conventional
- **Orient. Rate:** conventional: Geophysical meaning (Tectonic Plate Motion)

- ==> Lack of information for some parameters:
  - Orientation & rate (all techniques)
  - Origin & rate in case of VLBI
  - ==> Rank Deficiency in terms of Normal Eq. System

Datum Definition

	Satellite Techniques	VLBI
Origin	**Center of Mass**	-
Scale	GM, c & **Relativistic Corrections**	c **Relativistic Corrections**
Orientation	**Conventional**	**Conventional**

TRF implementation in practice

The initial NEQ system of space geodesy observations could be written as:

$$N_{unc}(\Delta X) = K$$

Where $\Delta X = X - X_{apr}$ **are the linearized unknowns**

N_{unc} **Normal matrix is singular having a rank deficiency Equal to the number of TRF parameters not reduced by the observations. Some constraints are needed:**

- **Tight constraints ($\sigma \leq 10^{-10}$) m**
- **Removable constraints ($\sigma \cong 10^{-5}$) m**
- **Loose constraints ($\sigma \geq 1$) m**
- **Minimum constraints (applied over the TRF parameters and not over station coordinates)**

Datum Definition: Minimum Constraints (1/3)

Application of Minimum Constraints (MC) approach based on

theoretical works by many authors, since the 70's on, e.g.:

- Free Network Adjustment
- S-transformation
- Minimum/Inner Constraints

Main Goals:

- "Best" TRF datum definition
- TRF internal consistency: no distortion
- Preserve actual quality of space geodesy observations

Datum Definition : Minimum Constraints (2/3)

The starting point is the standard relation between two TRF's:

$$X_2 = X_1 + A\theta$$

$$\theta = (T1, T2, T3, D, R1, R2, R3, \dot{T}1, \dot{T}2, \dot{T}3, \dot{D}, \dot{R}1, \dot{R}2, \dot{R}3)^T$$

$$A = \begin{pmatrix} . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 & & & & & & & \\ 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 & & & & 0 & & & \\ 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 & & & & & & & \\ & & & & & & & 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\ & & & \approx 0 & & & & 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\ & & & & & & & 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . \end{pmatrix}$$

Datum Definition : Minimum Constraints (3/3)

L.S. yields for θ: $$\theta = \overbrace{(A^TA)^{-1}A^T}^{\mathbf{B}}(X_2 - X_1)$$

To have X_1 and X_2 be expressed in the same frame (i.e. $\theta = 0$), a "datum definition" equation at Σ_θ level could be written as

$$B(X_2 - X_1) = 0 \qquad (\Sigma_\theta)$$

and in terms of normal equation:

$$B^T\Sigma_\theta^{-1}B(X_2 - X_1) = 0$$

Adding the above equation to the initial NEQ system, we have:

$$(N + B^T\Sigma_\theta^{-1}B)(\Delta X) = K + B^T\Sigma_\theta^{-1}B(X_R - X_{apr})$$

Σ_θ is a diagonal matrix of small variances over the 14 transformation parameters.

Time series combination (Rigourosly stacking)

- **Input:**
  - Weekly Station Positions: X(t)
  - Daily Polar motion (& rates), UT1, LOD
- **Output: Long-Term Solution (LTS):**
  - Station positions at a reference epoch t_0
  - Station Velocities
  - Daily EOPs
  - Time series of the transformation parameters between each week and the LTS

Stacking
TRF & EOP time series Combination

CATREF Software

INPUT: $X(t)$, **EOP(t) in daily/weekly/monthly SINEX files**

time series

OUTPUT: $X(t_0)$, $\dot{X}$, **EOP(t)**, $(\underbrace{T_x, T_y, T_z}_{\text{Geocenter}}, D, R_x, R_y, R_z)$

$$\left\{\begin{array}{l} X_s^i = X_{itrf}^i + (t_s^i - t_0)\dot{X}_{itrf}^i + T_k + D_k X_{itrf}^i + R_k X_{itrf}^i \\ \qquad + (t_s^i - t_k)\left[\dot{T}_k + \dot{D}_k X_{itrf}^i + \dot{R}_k X_{itrf}^i\right] \\ \dot{X}_s^i = \dot{X}_{itrf}^i + \dot{T}_k + \dot{D}_k X_{itrf}^i + \dot{R}_k X_{itrf}^i \end{array}\right.$$

Datum Definition with Minimum Constraints Over a Reference Set of stations

$$(A^T A)^{-1} A^T (X_{RS} - X_c) = 0$$

$$\left\{\begin{array}{ll} x_s^p & = x^p + R2_k \\ y_s^p & = y^p + R1_k \\ UT_s & = UT - \frac{1}{f} R3_k \\ \dot{x}_s^p & = \dot{x}^p + \dot{R}2_k \\ \dot{y}_s^p & = \dot{y}^p + \dot{R}1_k \\ LOD_s & = LOD + \frac{\Lambda_0}{f} \dot{R}3_k \end{array}\right.$$

- **Matching common EOP parameters at UT noon**
- **Propagate at UT noon if rates are available**

Datum definition: current principles for time series stacking

- **(1) Define the frame at a given epoch t_0**

==> 7 degrees of freedom to be selected/fixed

- **(2) Define a linear (secular) time evolution**

==> 7 degrees of freedom to be selected/fixed

Assume linear station motion:

- **Add break-wise approach for discontinuities**
- **Investigate the non-linear part in the time series of the residuals**

Ways of implementation

- **(1) Select an external frame as a "reference" and apply minimum constraints approach:**

$$(A^T A)^{-1} A^T (X_R - X_c) = 0$$

or

- **(2) Considering that for any Transf. Param.** P

$$P(t) = P(t_0) + \dot{P} \times (t - t_0)$$

apply "inner" conditions:

$P(t_0) = 0$ **and** $\dot{P} = 0$