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ALGORITHMIC DOCUMENTATION OF THE GINS SOFTWARE









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2. GENERAL

2.1. Introduction

This document contains a general and complete description of the algorithms used to process space data in the GINS (Géodésie par Intégration Numérique Simultanée) orbitography software. The codes change more quickly than the equations and we have deliberately limited the software descriptions in order to focus on the description of the algorithms and models.

This is not a document about orbitography in general or about all the possible applications of space geodesy. It is the algorithmic documentation of the GINS software. We have limited the scope to the description of the algorithms that were actually present in GINS at the start of 2012. The algorithms described in this document have all been validated and used for precise applications in recent years.

The organization of the GPS data processing in GINS is partially linked to the history of the software. The GINS software was initially designed during the 1970s to process the space geodesy data that was available at that time. The software could only process one satellite at a time. In the early 1990s, the software was enhanced to include satellites of the GPS constellation and to process data from receivers both onboard and on the ground. At the end of the 1990s, it became "planetary" and acquired the capacity to process the DSN (Deep Space Network) tracking data of satellites orbiting around bodies other than the Earth (Mars, Venus, planetary satellites, asteroids). More recent changes, made since 2000, have attempted to improve the reliability and the calculation times of the software for the routine processes of space gravimetry missions (CHAMP, GRACE, GOCE) or the DORIS, GNSS, LASER and VLBI data processes for the kinematics and the terrestrial reference system as part of the GRGS's contribution to international services (IGS, IDS, ILRS, IVS). In the course of these 15 years, the software has constantly followed the changes in international standards and the algorithms have been improved constantly in order to improve the models. The main applications of the software include the use of space geodesy measurements to recreate the precise orbit of satellites around different bodies in the solar system, the processing and interpretation of satellite data from missions to observe the gravity field of bodies in the solar system and the precise positioning of stations in satellite-based tracking systems used to build reference systems and to determine rotation parameters.

2.2. General principle of the GINS software

The GINS software is used to calculate the precise trajectories of artificial satellites around a body in the solar system by the numerical integration of the fundamental equation of the dynamics, based on knowledge of the forces acting on the satellite. There are two types of forces:

Gravitational forces: gravity field, third bodies, solid and oceanic tides, the gravitational effect of variations in atmospheric pressure, etc.

Non-gravitational forces: atmospheric friction, direct and rediffused solar pressure, thermal emissions, etc.

These trajectories are expressed in a reference system related to the central body (e.g.: ITRF) or in the celestial reference system (e.g.: ICRF).

The trajectories calculated by GINS or provided by external sources are used to make comparisons with the satellite tracking measurements taken from ground stations or from other satellites. Theoretical measurements are then built using the very precise knowledge of the position of the ground stations and their movements due to plate tectonics and loading phenomena.



The reconstituted theoretical measurements are then compared with the actual measurements. GINS can process the following types of measurements: distance (Laser), Doppler (DORIS), angular, GNSS (GPS, GLONASS, GALILEO, etc.), interferometric (VLBI), altimetric, ephemerid, inter-satellite (GRACE), accelerometric and gradiometric (GOCE), plus interplanetary range and Doppler measurements (DSN or ESA networks), etc.

The deviations between the actual and theoretical measurements are minimized by adjusting the physical and empirical parameters in a least squares iterative process. The most common physical parameters are the rotation parameters of the Earth or the central body, the coordinates of the stations, the coefficients of the gravity field, etc. Since some of these parameters cannot be resolved with a single orbital arc, the linear system is calculated and stored in convergence in the form of a normal equation. The normal equations are then processed by the DYNAMO chain (accumulation, reduction, resolution, etc.).

The GINS software can also work in simulation mode to study the capacity of new missions to restore different physical parameters.

The main outputs of the software are:

- a list describing the calculations
- the ephemeris of the satellite(s)
- the normal equation
- a file of statistics used to produce graphs

2.3. Structure of the GINS software

The general structure of the GINS software is shown in Figure 1. The global residuals are reduced in the iterations until convergence is achieved. The partial derivatives of the free parameters are calculated in iteration 1 and upon convergence, at which point additional parameters can be included.





3. TIME and SPACE REFERENCE

3.1. Units, time scales and date systems

The GINS software uses the International System units. International Atomic Time (IAT) is used for Earth and Barycentric Dynamical Time (BDT) is used for all other solar system bodies. The modified Julian date 1950.0 (01/01/1950) is used as the origin of the dates. The origin of the dates is converted to the integer date at the start of the arc for internal calculations. In general, the input data are converted to IS and IAT units when being read. The values in the output files (orbits, normal equations, residual files) are also expressed in IS units. Certain numerical values in the listing file may not be in IS units for better readability.

The following date systems are found in the various input and output files:

calendar dates: DD/MM/YYYY or DD/MM/YY.
calendar dates: DOY/YYYY = Day of Year/Year
Julian dates 1950 (JUL50) = days after 01/01/1950 at 0h
Julian dates J2000 = days after 01/01/2000 at 12h
Julian/Gregorian dates = J2000 dates + 2451545 days.
modified Julian dates (MJD) = J2000 date - 51544.5 (days)
GPS dates: WWWW/D = GPS week and day of the week (from 0=Sunday to 6=Saturday, week 1 = week starting 13/01/1980).

and the various time scales:

UTC (Coordinated Universal Time). The relation between the UTC and the IAT depends on the date in question (UTC = IAT – 34 s at the start of 2011). IAT = International Atomic Time GPS time (GPST = IAT-19 s) BDT = Barycentric Dynamical Time TT = Terrestrial Time (TT=IAT +32.184 s)

Where necessary, GINS performs the conversions between the various time scales and date systems. The online **jjul** program can be used to switch from one date system to another in order to make the conversions.

3.2. Coordinates/Coordinate systems

The central body is the body around which the orbit calculations are to be made. The main reference system is the system the origin of which is at the centre of mass of the central body and whose axes are oriented according to the usual international conventions, usually J2000 when the central body is Earth (Petit and Luzum, 2010). The central body may be Earth or any other body in the solar system implemented in the software. GINS uses Cartesian coordinates (x,y,z) for most calculations, but some inputs (or outputs) use the other usual coordinate systems found in space geodesy that are used to model the motion of the satellite or to make specific calculations regarding the central body.





Figure 2: Geographical coordinate systems

<u>3.2.1.</u> Ellipsoidal coordinates φ , λ , h / reference ellipsoid

The closest mathematical surface to the surface of a rotating body (the Earth or a planet), ignoring the topography, is that of a revolution ellipsoid, or of a sphere that is flattened at its poles. Revolution ellipsoids are characterized by the lengths of its two main radii: **a** (equatorial semi major axis) and **b** (polar semi minor axis) or, in an equivalent manner, by the semi major axis and the oblateness $\mathbf{f} = (a-b)/a$. The ellipsoidal coordinates are the ellipsoidal latitude (φ measured positively from South to North), longitude (λ measured positively from West to East) and the height (**h**) above the ellipsoid. Subroutines convert Cartesian coordinates to ellipsoidal coordinates (*xyzflh* and *flhxyz*) whenever necessary. The values of the semi major axis **a** and the oblateness **f** are specified in the station file (with the list of coordinates).

Spherical coordinates

In the case of a sphere (zero oblateness), the spherical coordinates are usually linked to the rectangular coordinates according to (r = a + h):

 $\begin{aligned} x &= r \cos \varphi \ \cos \lambda \\ y &= r \cos \varphi \sin \lambda \\ z &= r \sin \varphi \\ \text{where } r &= a + h \text{ is the distance between the point in question and the origin of the reference.} \end{aligned}$

<u>3.2.2.</u> Local coordinates (azimuth / elevation)

The local coordinates are angles used to indicate the direction of a point in relation to an observer or a geodesic instrument. The **elevation** is the angle between the local normal and the direction of the observed object and the **azimuth** is the angle between the direction of the object projected in the local horizontal plane and a reference direction. For observers on the central body, the reference direction is North. Local **ellipsoidal** coordinates often use the normal of the reference ellipsoid as the local normal.

These coordinates are also used to describe the sight angles of the antennas of onboard instruments. In this case, the reference normal is the major axis of the antenna. The reference direction for azimuths is defined by a direction given in the satellite reference, which depends on the case in question.

The six keplerian orbital elements are used to describe the orbits of objects around a central body (see Figure 3).



Figure 3: Definition of the keplerian elements

The elements are used to define the form of the ellipse that is tangential to the orbit (elements a and e), its orientation in the reference linked to the central body (elements i and Ω), the position of the perigee of the ellipse (ω)) and the position of the object on the ellipse (angle M, counting from the perigee). In unperturbed movements (the object in orbit is only subjected to a central attraction of the central body), all of these elements are constant, apart from the mean motion, which varies in linearly over time. In general, these elements vary slowly over time as a function of the perturbing forces. An osculating orbit can be defined at any point of the orbit using its instantaneous keplerian elements. With conventional formulae, the keplerian elements of the osculating orbit are replaced by the instantaneous vectors of position and speed of the orbiter, and vice versa (Brouwer, 1961). The orbits of most geodetic satellites are close enough to circular orbits (i.e. low eccentricity), but the altitudes and the inclinations may vary significantly, depending on the required properties (see Table 1).



satellite	a (metre)	e	i (degree)
Beacon-B	7500000.	.024	41.200
Champ	6823250.	.003000	87.274
Cryosat-2	7093646.	.001952	91.996
Diademe-1C	7190000.	.038000	39.940
Diademe-1D	7533000.	.076000	39.450
Envisat / ERS-1	7159500.	.001384	98.547
/ERS-2			
Etalon-1	25503150.	.000900	65.313
GPS	26400000.	.001000	55.000
Grace-A (Tom)	6865000.	.002400	89.01
Grace-B (Jerry)	6840700.	.002400	89.05
HaiYang-2A	7341800.	.001370	99.303
Jason 1/2	7714431.	.000599	66.073
Lageos	12273000.	.004000	109.800
Lageos-2	12162064.	.01400	52.000
Meteor-3	7572500.	.001800	82.560
Nova-3	7553000.	.003500	90.040
Spot (s)	7200514.	.00141	98.703
Starlette	7331000.	.02000	49.800
Stella	7176760.	.00106	98.683
Topex/Poseidon	7714278.	.000095	66.039
Westpac	7202076.	.001200	98.752

Table 1: Orbit parameters of a selection of geodetic satellites

3.2.4. Satellite RTN local orbital coordinates

These coordinates are defined locally on the basis of the trajectory of the satellite. The direction is aligned towards the centre of the central body, the normal direction is perpendicular to the orbit plane and the tangential direction completes the right-handed orthogonal frame (see Figure 2). This coordinate system is used in particular to project the differences between two computations of a trajectory.



Figure 4: Graphical representation of the local orbital coordinate system of a satellite. The radial, normal (cross-track) and tangential (along-track) directions are defined at each point of the orbit.

3.2.5. Coordinates linked to the satellite

Each satellite has its own specific coordinate system that defines the coordinates of the various components. The origin and the major axes of this reference system depend on the satellite and are closely linked to the satellite's attitude law. The coordinates (instruments, orientation of the satellite's surfaces, etc.) are specified



in the file of satellite macro models (see section 3.3). In some cases, the satellite's barycentre is at the origin of this coordinate system.

<u>3.2.6.</u> Central body

The central body (planet in the solar system, planetary satellite, asteroid or the Sun) is specified in the **Director** file. It defines the reference used for the calculations. The possible bodies are the Earth (the default central body), Mercury, Venus, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto, the Moon, Phobos, Eros, Wirtanen, Chury, and the Sun.

<u>3.2.7.</u> Changes of reference

A set of position and speed coordinates (P_1, V_1) expressed in reference frame 1 will be converted into a set of coordinates (P_2, V_2) expressed in reference frame 2 by transformation relations involving the rotation matrix M(t) between the two references:

 $P_{2} = M (t) P_{1}$ $V_{2} = M (t) V_{1} + dM (t)/dt M (t)^{-1} P_{2}$

In most of our cases, we use two main reference frames: a reference linked to the central body and an inertial reference (J2000), in which the dynamic calculations are made, thereby avoiding the need to model the rotational accelerations. In some cases, other coordinate systems have to be taken into consideration simultaneously, for example in order to process the DSN data between an orbiter around a planet and the ground stations, where three references, plus the laws of transformation (translation, rotation) between the references, are used: the reference linked to the planetary body, the terrestrial and planetary inertial references and the terrestrial reference, in which the coordinates of the DSN stations are expressed. The precise modeling of very long base interferometry (VLBI) measurements and planetary measurements uses the barycentric reference of the solar system.

<u>3.2.8.</u> Celestial reference system for the Earth and Earth rotation

GINS uses the J2000 celestial reference system defined by the IERS Conventions (McCarthy and Petit, 2004, Petit and Luzum, 2010). The transformation matrix between the terrestrial reference and the celestial reference is used to pass between the celestial reference system (CRS) and terrestrial reference system (TRS). This matrix is usually written as follows:

[TRS] = M(t) [CRS] = W(t) R(t) Q(t) [CRS]

W(t): polar motion matrix. This depends on the observed coordinates of the terrestrial pole x_p, y_p

R (t): rotation matrix of the Earth around the axis linked to the pole. This depends on the observed corrections for the angle of Earth rotation (dUT1)

Q(t): rotation matrix based on the motion of the celestial pole in the celestial reference system, also known as the precession-nutation matrix. This depends on the observed corrections of nutation $(D\psi, D\varepsilon \text{ or } DX, DY)$ (depending on the selected form)

The observed corrections to the rotation model x_p , y_p , dUT1, $D\psi$, $D\varepsilon$ or DX, DY or EOP (Earth Orientation Parameters) are included in the pole file in the environment block of the **Director** file (*pole* line).



GINS can compute Earth rotation in a number of ways (see the User Manual):

- either using the conventional representation based on the IAU 1996 nutation model and using the correction to the nutation model $D\psi D\varepsilon$ delivered by the IERS (file EOP97C04_ITRF2008). The amplitude of these corrections is around 80 mas (arc milliseconds);
- or by using the conventional representation $(D\psi D\varepsilon)$, or NRO (X Y) based on the IAU2000/IAU2006 nutation models and the observed corrections in relation to the DX/DY nutation model delivered by the IERS (file EOP97C04_NRO_ITRF2008). The amplitude of the latter corrections is less than one mas.

The IERS has recommended the second formulation since the early 2000s. The advantages in terms of the prediction of the nutation model are quite obvious. In the absence of any corrections, the IAU2006 nutation model reduces the nutation errors to about 1 mas, which corresponds to an angle of \sim 3 cm on the surface of the Earth. Model IAU2006 of the Earth rotation matrix M(t) and its derivatives with regard to the rotation parameters is calculated using the Standards of Fundamental Astronomy library (SOFA, see www.iausofa.org, Code fortran f77 [Released 2010-12-01]). The movement of the pole (x_p, y_p) includes the diurnal IERS corrections read in the pole file, the diurnal and sub-diurnal correction caused by ocean tides and the prograde corrections of the diurnal and sub-diurnal librations. The EOPs read in the pole file, the tabulated values of the nutation models and the rapid corrections of the pole movement are interpolated on the requested date.

<u>3.2.9.</u> Barycentric reference systems

This coordinate system is the reference system used to express the coordinates of extragalactic objects. In particular, it is used in the modeling of VLBI measurements in order to identify the coordinates of quasars as a function of their right ascension and their declination (see below). The Lorentz transformation is used to switch from the J2000 reference system to the barycentric reference system.

<u>3.2.10.</u> *Planetary reference systems*

The planetary rotation vector is defined by two angles α (right ascension) and δ (declination) relative to the J2000 terrestrial equator (inertial). The plane that is orthogonal to the vector of rotation defines the mean planetary equator of the body, which may feature geometric nutations (trigonometric terms). Therefore, unlike the definition for the Earth, the term "mean" shall stand for "no motion of the pole" of the body in question, i.e. the previously defined celestial equator always coincides with the physical equator that is fixed relative to the body.



Figure 5: Diagram representing orientation angles α and δ

3.3. Planetary ephemerides

The ephemerides of the bodies in the solar system (required for the transformation of Earth-planet references or to calculate the gravitational forces induced by these bodies) are provided in the J2000 reference by the DE403/405 models (Standish, 1998) or the INPOP models from the Institut de Mécanique Céleste et de Calcul des Ephémérides (Fienga et al, 2011). The file of planetary ephemerides assigned in the environment block of the **Director** file (*planet* line).

4. CALCULATION OF TRAJECTORIES

4.1. Numerical integration

The fundamental equation of the dynamics must be integrated in order to calculate the trajectory of the satellites:

$$\frac{d^2\bar{r}}{dt^2} = \sum_n \overline{A}(\bar{r}, \dot{\bar{r}}, \alpha_i)$$

where:

 $\vec{r}, \dot{r}, \frac{d^2 \vec{r}}{dt^2}$: the position, speed and acceleration vectors of the satellite, \vec{A} the sum of the forces acting on the satellite and α_i , the adjustable parameters on which the n forces depend.

We also want to adjust the position and speed of the satellite at the starting point of the orbit calculation and the dynamic parameters, which depend on the forces. This involves deriving the above expression in order to obtain the so-called variations equation. This produces:

$$\frac{d^{2}}{dt^{2}}\left(\frac{\partial \bar{r}}{\partial \bar{r}_{0}}\right) = \frac{\partial \ddot{r}}{\partial \bar{r}_{0}} = \sum_{n} \frac{\partial \overline{A}}{\partial \bar{r}} \frac{\partial \bar{r}}{\partial \bar{r}_{0}} + \sum_{n} \frac{\partial \overline{A}}{\partial \dot{\bar{r}}} \frac{d}{dt} \left(\frac{\partial \bar{r}}{\partial \bar{r}_{0}}\right)$$
$$\frac{d^{2}}{dt^{2}}\left(\frac{\partial \bar{r}}{\partial \alpha_{i}}\right) = \frac{\partial \ddot{\bar{r}}}{\partial \alpha_{i}} = \sum_{n} \frac{\partial \overline{A}}{\partial \bar{r}} \frac{\partial \bar{r}}{\partial \alpha_{i}} + \sum_{n} \frac{\partial \overline{A}}{\partial \dot{\bar{r}}} \frac{d}{dt} \left(\frac{\partial \bar{r}}{\partial \alpha_{i}}\right) + \sum_{n} \frac{\partial \overline{A}}{\partial \alpha_{i}}$$

The Cowell method is used to integrate these differential equations. This multistep numerical integrator is built on m constant intervals (Barriot, 1988). The order of the Cowell integrator is variable (usually 8 or 10) and the duration of the integration interval depends on the minimum detectable period of the perturbations (typical values range from 10 seconds for the satellites closest to Earth to 300 seconds for GNSS satellites). Note that this technique can be used to integrate circular or eccentric orbits, but in the later case, a regularization is required to integrate in terms of anomaly angle rather than time. The amplitudes of the various accelerations vary according to the satellites in question (see Figure 6). The various forces taken into consideration are presented in the following sections.



Figure 6: Some examples of the amplitude of the acceleration taken into consideration for the numerical integration of the movement (the min. and max. values in the course of the arc are entered for each satellite).

4.2. Gravitational force models / Free parameters

The description of the force models, or more precisely of the accelerations, is taken from the documentation in the "Obelix" numerical library that brings together and describes in greater detail all the force models in the GINS software.

The calculation of the potential of the central body (of the associated force and its derivatives) forms the basis of the calculation of all the accelerations deriving from a potential and that can be expressed in spherical harmonic functions.

4.2.1. The potential of the central body

The gravitational acceleration of the central body is derived from a potential that is conventionally expressed in a system of spherical coordinates.

If is written using spherical harmonic functions for $r \ge a_e$, in the following form:

$$U = \frac{\mu}{r} \sum_{l=0}^{l \max} \sum_{m=0}^{l} \left(\frac{a_e}{r} \right)^l \overline{P}_{lm} (\sin \varphi) (\overline{C}_{lm} \cos m\lambda + \overline{S}_{lm} \sin m\lambda)$$

= GM : from the model of potential
: normalized Legendre function

where $\mu = GM$ $\overline{P}_{lm}(\sin \varphi)$ $\overline{C}_{lm}, \overline{S}_{lm}$

: normalized Stokes coefficients taken from the model of potential

 n_e : semi-major axis of the central body

 (r, φ, λ) : spherical coordinates of the satellite

l and m are respectively referred to as the degree and the order of the Legendre functions or of the Stokes harmonic coefficients.

The acceleration is calculated in the rotating frame linked to the central body: $\vec{a} = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right)$. In practice, the derivation is initially calculated in a system of spherical coordinates, then restored in the system of Cartesian coordinates (x, y, z).

The gradient of the acceleration $\left(\frac{\partial^2 U}{\partial x^2}, \frac{\partial^2 U}{\partial y^2}, \frac{\partial^2 U}{\partial z^2}\right)$ and the temporal derivatives of the potential are also calculated if necessary, if these coefficients are variable over time.

Free parameters of the model

The gravity field of a body is defined for a given μ and a_e . All the other coefficients of the field $(C_{l,m} / S_{l,m})$ are free. Partial derivatives are calculated using the following expressions:

$$\frac{\partial U}{\partial \overline{C}_{l,m}} = \frac{\mu}{r} \left(\frac{a_e}{r}\right)^l \cos^m \varphi \ \overline{H}_l^m(\sin\varphi) \cos m\lambda$$
$$\frac{\partial U}{\partial \overline{S}_{l,m}} = \frac{\mu}{r} \left(\frac{a_e}{r}\right)^l \cos^m \varphi \ \overline{H}_l^m(\sin\varphi) \sin m\lambda$$

 \overline{H}_l^m : Helmoltz polynomials

4.2.2. The potential of perturbing bodies

A perturbing body is any celestial body, other than the central body, that exercises a gravitational influence on the satellite. For satellites in Earth orbit, the Moon, the Sun, Mars, Jupiter, etc. are perturbing bodies. For a satellite orbiting Mars, the Sun, Jupiter, the Earth, etc. are perturbing bodies. Since the movement of the satellite is studied in a reference system linked to the central body, it is in fact the differential attraction



exercised by the central body, or bodies, between the satellite and the central body that must be taken into consideration. In practice, the accelerations of all the central bodies in question are added together.

Central term of the acceleration of a perturbing body



C is the central body, *S* is the satellite, *P* is a perturbing body and *O* is the origin of an inertial reference. If bodies *C* and *P* are considered as isolated, then the differential acceleration of the satellite perturbed by body *P* is:

$$\vec{a}_{s}^{\Re c} = -GM_{p} \left(\frac{\overrightarrow{PS}}{PS^{3}} + \frac{\overrightarrow{CP}}{CP^{3}} \right)$$

This quantity is referred to as the central term of the acceleration.

Coupling term of the acceleration of a perturbing body

The hypothesis of the point-mass for both body is too reductive to take account of the observed perturbing acceleration. A coupling term of the acceleration is added (limited to the C_{20} coefficient of the harmonic development of the potential of the central body) in the following form:

$$\vec{a}_{p \to s}^{couplage} = -\frac{3}{2}\sqrt{5} \frac{GM_c}{CP^5} a_e^2 \overline{C}_{20} \left[(5\sin^2\varphi_p - 1) \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} - 2 \begin{pmatrix} 0 \\ 0 \\ z_p \end{pmatrix} \right]$$

Note: no parameters are free for this model.

The acceleration of Earth tides is derived from the deformation potential of the central body, of degree 2 and 3, under the gravitational effect of the perturbing bodies. Deformation potential is made up of four terms:

$U = U_k$: Earth tide potential
+ $\Delta U_{\delta k}$: frequency dependent correction of the Love numbers
+ ΔU_{ell}	: correction of ellipticity
+ ΔU_{pole}	: correction of Earth polar tide (described in section 3.2.5.)

For a point $S(r,\varphi,\lambda)$ around the central body, the perturbing gravitational potentials written in the form of a development in spherical harmonics (of the order of 3):

$$U_{A}(r,\lambda,\varphi) = \sum_{l=2}^{\infty} U_{l}(r,\lambda,\varphi) = \frac{\mu}{r} \sum_{l=2}^{\infty} \left(\frac{a_{e}}{r}\right)^{l} \left[\sum_{m=0}^{l} (C_{lm}(t)\cos m\lambda + S_{lm}(t)\sin m\lambda)P_{lm}(\sin \varphi)\right]$$

Where μ is the gravitational constant of the central body and a_e is the mean equatorial radius of the central body. The harmonic coefficients of the potential $C_{lm}(t)$, $S_{lm}(t)$ (of degree *l* and order *m*) depend on the position of the perturbing body $(r_p, \lambda_p, \varphi_p)$, the Love numbers (complex) of the deformation k_{lm} and the gravitational constant μ_p of the perturbing body, as per:

$$\overline{C}_{lm} - i\overline{S}_{lm} = \frac{k_{lm}}{2l+1} \frac{\mu_p}{\mu} \left(\frac{a_e}{r_p}\right)^{l+1} \overline{P}_{lm} \left(\sin\varphi_p\right) e^{-im\lambda_p}$$

The frequency dependent **correction** is added to the preceding model in order to take account of the frequency of excitation of the perturbation. This model requires the use of a tides formalism, where $C_{2m}(t) / S_{2m}(t)$ are expressed by Doodson formulae (Petit and Luzum, 2010) as a function of the argument θ_s of the wave of the tide in question and the amplitude H_s of the balancing tide:

$$\Delta \overline{C}_{20} = \sum_{s} \frac{H_{s}}{R\sqrt{4\pi}} \left(\delta k_{s}^{R} \sin \theta_{s} - \delta k_{s}^{I} \sin \theta_{s} \right)$$
$$\begin{cases} \Delta \overline{C}_{21} \\ \Delta \overline{S}_{21} \end{cases} = \sum_{s} \frac{H_{s}}{R\sqrt{8\pi}} \delta k_{s}^{R} \begin{cases} \sin \theta_{s} \\ \cos \theta_{s} \end{cases}$$
$$\begin{cases} \Delta \overline{C}_{22} \\ \Delta \overline{S}_{22} \end{cases} = \sum_{s} \frac{H_{s}}{R\sqrt{8\pi}} \delta k_{s}^{I} \begin{cases} \cos \theta_{s} \\ -\sin \theta_{s} \end{cases}$$

The number of tide waves taken into consideration depends on the modeling process (IERS 1996, IERS 2003).

The ellipticity effect of the terrestrial potential introduces corrections into C_{4m} and S_{4m} (m=0, 1, 2) that depend on the corrections of ellipticity of the Love numbers k_{2m}^+ :

$$\overline{C}_{4m} - i\overline{S}_{4m} = \frac{k_{2m}^{+}}{5} \frac{\mu_p}{\mu} \left(\frac{a_e}{r_p}\right)^3 \overline{P}_{2m} \left(\sin\varphi_p\right) e^{-im\lambda_p}$$



Free parameters of the mode

The degree 2 Love numbers (real and imaginary parts) used in the calculation of the degree 2 Earth tide potential can be freed.

4.2.4. The potential of fluid tides

The fluid tide potential is induced by the movement of masses of water under the effect of the potential of the perturbing bodies. This movement is not only radial and the horizontal displacement is highly perturbed by the presence of the continents. The potential itself is calculated at one point $S(r,\varphi,\lambda)$ on the basis of the surface density $q(\varphi,\lambda)$ developed in spherical harmonics (of coefficients q_{lm}^c and q_{lm}^s) as per:

$$U_{p}(r,\varphi,\lambda) = 4\pi GR \sum_{l=1}^{\infty} \frac{1+k_{l}'}{2l+1} \left(\frac{R}{r}\right)^{l+1} \sum_{m=0}^{l} P_{lm}(\sin\varphi) \left(q_{lm}^{c}\cos m\lambda + q_{lm}^{s}\sin m\lambda\right)$$

The coefficients of surface density q are obtained by the product of the density of the water ρ by the depth of water, which is the result of an addition on all the tide waves:

$$q_{lm}^{c} = \rho \sum_{n} \left[\cos(\theta_{n}(t) + \chi_{n}) (S_{n,lm}^{+} + S_{n,lm}^{-}) + \sin(\theta_{n}(t) + \chi_{n}) (C_{n,lm}^{+} + C_{n,lm}^{-}) \right]$$
$$q_{lm}^{s} = \rho \sum_{n} \left[\cos(\theta_{n}(t) + \chi_{n}) (C_{n,lm}^{-} - C_{n,lm}^{+}) + \sin(\theta_{n}(t) + \chi_{n}) (S_{n,lm}^{-} - S_{n,lm}^{+}) \right]$$

The various waves are characterized by their astronomical argument $\theta_n(t)$ and their Doodson-Warburg phase χ_n and their coefficients $C_{n,lm}^+, C_{n,lm}^-, S_{n,lm}^+, S_{n,lm}^-$ are provided by the selected tide model. A distinction is made between long-period waves, diurnal waves and semi-diurnal waves. They are split between main waves (whose amplitudes are provided by the model) and secondary waves whose amplitudes are calculated by admittance.

Load of the atmospheric tides

The physical considerations are equivalent to those defined for ocean tides.

Free parameters of the model

The tide coefficients of the waves used as input for the model can be freed.

<u>4.2.5.</u> The potential of solid and ocean polar tides

The correction of the solid polar tide models the solid Earth's response to the rotational movement. It is calculated like the other forces of gravity using spherical harmonic coefficients and depends on the position of the rotation pole relative to the mean pole x_p , y_p as per:



$$\begin{cases} \Delta \overline{C}_{21} \\ \Delta \overline{S}_{21} \end{cases} = w_1 \widetilde{k}_2 \begin{cases} -x_p + w_2 y_p \\ y_p + w_2 x_p \end{cases}$$

where \tilde{k}_2 is the Love number of the solid polar tide and w_1 , w_2 are the response coefficients, which depend on the selected model.

As is the case for solid polar tides, the ocean polar tide models the deformation of the oceans under the influence of the variations in the rotation movement. The general form, used by Desai's model, is given by the harmonic coefficients:

$$\begin{cases} \Delta \overline{C}_{nm} \\ \Delta \overline{S}_{nm} \end{cases} = R_n \left\{ \begin{bmatrix} \overline{A}_{nm} \\ \overline{B}_{nm}^R \end{bmatrix} (x_p \gamma_2^R + y_p \gamma_2^I) + \begin{bmatrix} \overline{A}_{nm} \\ \overline{B}_{nm}^I \end{bmatrix} (y_p \gamma_2^R - x_p \gamma_2^I) \right\}$$

where $R_n = \frac{\Omega^2 a_E^4}{GM} \frac{4\pi G \rho_{\omega}}{g_e} \left(\frac{1+k'_n}{2n+1}\right)$

The coefficients (Ar/Ai and Br/Bi) are provided up to degree 360 and degree 2 represents 90% of the effects (Desai, 2002).

Free parameters of the model

The Loveumber of the Earth polar tide can be freed.

<u>4.2.6.</u> The potential of atmospheric pressure variations

Displacement of atmospheric masses induces a potential which can be written as a single layer potential including the deformation due to the loading (see previous chapters for notations).

$$U_{p}(r,\varphi,\lambda) = 4\pi GR \sum_{l=1}^{\infty} \frac{1+k_{l}'}{2l+1} \left(\frac{R}{r}\right)^{l+1} \sum_{m=0}^{l} P_{lm}(\sin\varphi) \left(q_{lm}^{c}\cos m\lambda + q_{lm}^{s}\sin m\lambda\right)$$

Mass distribution is represented by the loading coefficients q_{lm}^c , q_{lm}^s .

If ΔPa_{lm}^c , ΔPa_{lm}^s are the coefficients of the spherical harmonic development of atmospheric pressure variations the loading coefficients can be written as follows:

$$q_{lm}^{c} = \frac{\Delta P a_{lm}^{c}}{g} \quad ; \quad q_{lm}^{s} = \frac{\Delta P a_{lm}^{s}}{g}$$

with g the g-force at the surface the body ($\sim 9.81 ms^{-2}$ for the Earth).

Using the following formulas:

$$C_{lm} = \frac{4\pi R^2}{M} \frac{1 + k_l'}{2l + 1} \frac{\Delta P a_{lm}^c}{g}$$
$$S_{lm} = \frac{4\pi R^2}{M} \frac{1 + k_l'}{2l + 1} \frac{\Delta P a_{lm}^s}{g}$$

We can use the expression of the potential of the central body (see chapter 4.2.1).

<u>4.2.7.</u> Relativistic forces

The movement equations of the satellite are integrated in terms of non-relativistic mechanics. In this formulation, and as long as the conditions remain "reasonable", the relativistic effects intervene as perturbations of the second order. The relativistic force model is introduced to take these perturbations into consideration. The model used (Huang et al., 1990) comprises three terms: the Schwarzschild term (the most important), the relativistic Coriolis term (or geodetic precession) and the Lense-Thirring term (relativistic effects due to the rotation of the central body). These terms are expressed as follows:

$\vec{a}_{sch} = \frac{1}{c^2}$	$\frac{\mu}{r^3} \bigg[\bigg[2(\beta + \gamma) \bigg] \bigg] \bigg] = 0$	$(r)\frac{\mu}{r} - \gamma v^2 \bigg] \vec{r} + 2(1 + r)^2 \vec{r}$	γ) $(\vec{v}\cdot\vec{r})\vec{v}$]
$\vec{a}_{body} = 2$	$2\vec{\Omega}_{body} \wedge \vec{v}$		
v	where $\vec{\Omega}_{body}$	$=\frac{(1+2\gamma)}{2c^2}\sqrt{\frac{\mu_{Sun}^3}{r_{body/Sun}^5}}$	$\vec{u}_{zEcliptic}$ (standard model)
o	$\hat{\Omega}_{body}$	$= -\frac{(1+2\gamma)}{2} \frac{\mu_{Sun}}{c^2 r_{Sun}^3} (\bar{\eta})$	$\vec{v}_{Sun} \wedge \vec{r}_{Sun}$ (IERS 2003 standard)
$\vec{a}_{LT} = (1$	$+\gamma)\frac{\mu}{c^2r^3}\bigg[\frac{2}{r}\bigg]$	$\frac{3}{2}\left(\vec{r}\cdot\vec{J}\right)\left(\vec{r}\wedge\vec{v}\right)+\left(\vec{v}\wedge\vec{v}\right)$	$\left(\vec{J}\right) = (1+\gamma)\frac{\mu}{c^2 r^3} \left[\frac{3}{r^2} \left(\vec{r} \cdot \vec{J}\right) \vec{r} - \vec{J}\right] \wedge \vec{v}$
W	where $\vec{J} = \frac{2}{5}$	$\times R_{Earth}^{2} \times \Omega_{Earth} \times \vec{u}$	zEarth (standard model)
W	where $\vec{J} = 9$	$8 \times 10^8 \vec{u}_{zEarth}$	(IERS 2010 standard)
where:			
	ŕ	:	the position of the satellite in an inertial reference centred on the central body
	\vec{v}	:	the speed of the satellite in an inertial reference
			centred on the central body
	\vec{r}_{Sun}	:	the position of the Sun in an inertial reference centred on the central body
	\vec{v}_{Sun}	:	the speed of the Sun in an inertial reference centred on the central body
	\vec{J}	:	angular moment of the central body by unit of
			 →standard formulation: the central body is considered as a homogeneous sphere →IERS 2010 formulation: integration of the non-homogeneous character, for the Earth only.
	$\mathbf{\Omega}_{\textit{Earth}}$:	represents the rotation speed of the central body on itself
	\vec{u}_{zEarth}	:	unit vector according to the rotation axis of the central body in the positive direction relative to the rotation.
	4271111	βν 4271112 ·	PPN parameters equaling 1 in the theory of general relativity
		P?/	



The macro model brings together all the input used to physically describe the satellite and to model its interactions with space (see the example in Figure 7). It contains the geometry (type of surfaces, size, positions and orientations), its physical characteristics (mass, thermo-optical properties of the surfaces), the characteristics of the onboard instruments (type, positions of the phase centres in the satellite reference, orientation of the antennas), the attitude model to be used (see section 3.4) and the numbers used in the measurement files (Cospar numbers, Prn, Prv, etc.).

In GINS, this data is read in a **macro model** file containing the description of all the satellites that the software knows.





Figure 7: Diagram representing the Topex satellite (credits: NASA). The macro model contains the geometrical description of the satellite.

4.3.2. Surfaces and optical properties

The macro model (or Box & Wings model) describes the satellite as a set of surfaces defined by their type (see Table 2), their orientation in the satellite reference and their thermo-optical properties. These characteristics can be used to calculate the non-gravitational forces (see section 3.5). Three optical coefficients without units are used to calculate the reflectivity of the surface to the incident radiation: the absorption coefficient K_A , the coefficient of diffuse reflection K_D and the coefficient of specular reflection K_S . They are linked by the relation $K_A + K_D + K_S = 1$. (only the coefficients is provided to calculate the infrared reflectivity. A final coefficient of emissivity ε and the temperature law are used to calculate the thermal emission associated with each of the surfaces.

Type of surface	Normal	Other geometric characteristics	
Plate	Normal external to the plane	Surface	
Sphere	N/A	Radius of the sphere	
Cylinder	Axis of the cylinder	Surface	
Parabola ⁽¹⁾	Axis of the parabola	Radius and focus	
⁽¹⁾ In practice, the parabola is automatically broken down into n plate-type triangular			
facets.			

Table 2: Types of known elementary surfaces in GINS for the geometric description of satellites.



4.4. Attitude of the satellites

<u>4.4.1.</u> General

The **attitude law** of a satellite or an orbiter is the law used to switch from the integration reference (usually J2000) to the reference linked to the satellite, in which the various specific characteristics of the satellite are expressed (see 3.3). The actual attitude can be measured (e.g., by star trackers) and given in the form of attitude quaternions or, more generally, modeled by a **nominal law**. Most Earth observation satellites have more or less complicated specific attitude laws that depend on their assigned mission. These various attitude laws aim, for example, to maximize the exposure time of the solar panels, to minimize the number of manoeuvres or to respect certain constraints resulting from the onboard instruments (e.g. directing an instrument towards the Earth). The nominal laws are usually calculated on the basis of all or part of the following factors: the date, the instantaneous position and speed of the satellite (or keplerian elements of the osculating orbit), the satellite-Earth and Satellite-sun direction, etc.

The known attitude laws in GINS are listed in Table 3. Numerical values of the attitude used can be obtained in the form of quaternions in the GINS tabulated orbit output file.

Type of attitude law = the name appearing in the macro model file	Type of attitude law	Satellite(s) / Orbiter(s) concerned	References		
4.4.1.1.1.1.1 ty pe_cryos2	Geodetic point positioning with a 6° offset in the satellite's X axis relative to the normal to the reference ellipsoid. Satellite reference X,Y,Z \Leftrightarrow T,N,R	CRYOSAT2	(Cerri 2012)	and	Ferrage
type_topex	Since the orbit is not heliosynchronous, a complex attitude law to obtain geodetic point positioning, with solar panels that are oriented as much as possible in the normal direction to the direction of the sun. Satellite reference $X,Y,Z \Leftrightarrow T,-N,R$	TOPEX, JASON1, JASON2	(Cerri 2012)	and	Ferrage
type_spot	Z axis of the satellite oriented in the radial direction. Solar panels oriented towards the sun as much as possible (heliosynchronous orbit). Satellite reference X,Y,Z \Leftrightarrow N,-T,R	Satellites SPOTs (SPOT2, SPOT3, SPOT4, SPOT5)	(Cerri 2012)	and	Ferrage
type_goce	Attitude given by the quaternions from the onboard measurements of the attitude sensors.	GOCE			
type_champ	Attitude given by the quaternions from the onboard measurements of the attitude sensors.	CHAMP, GRACE			
type_cosmic	Specific.	COSMIC satellites			
type_ers	Specific.	ERS1, ERS2			
type_envisat	Geodetic point positioning, Z axis normal to the reference ellipsoid. Solar	ENVISAT	(Cerri 2012)	and	Ferrage



	panels oriented towards the sun as much		
	as possible (heliosynchronous orbit).		
	Satellite reference X,Y,Z \Leftrightarrow N,-T,R		
type_diadème	Specific.	DIADEME_C,	
		DIADEME_D	
type_geos3	Specific.	GEOSAT3	
type_mgs	Specific.	Mars Global	
		Surveyor	
type_mro	Specific.		
type_mex	Specific.	Mars Express	
type_mo1	Specific.		
type_vex	Specific.	Venus Express	
type_gnss	Nominal GNSS		(Kouba 2008, 2009)
type_koubaIIR	Specific to GPS IIR in eclipses.	GPS IIR	(Kouba 2008, 2009)
type_koubaIIA	Specific to GPS IIA in eclipses.	GPS IIA	(Kouba 2008, 2009)
type_koubaIIF	Specific to GPS IIF in eclipses.	GPS IIF	(Kouba 2008, 2009)
type_koubaGL	Specific to GLONASS in eclipses.	GLONASS	(Kouba 2008, 2009)
0			
sphérique	Attitude law for spherical satellites only.	LAGEOS1,	No attitude
		LAGEOS2,	
		STARLETTE,	
		STELLA,	
		AJISAI	
rtn	General RTN attitude law: the main axes	COURIER 1B	(see section 2.2)
	of the satellite are assumed to be		
	oriented according to the local orbital		
	RTN reference		
	(Radial/Tangential/Normal)		

Table 3: List of	f known attitude	laws in (GINS	(January	2012)
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4.4.2. Attitude quaternions

When the attitude is measured onboard, e.g. by star trackers, it may be accessible in the form of time series of attitude quaternions (four unit module values). In this case, the attitude matrix $M_{att}(t)$ is rebuilt on the basis of the quaternion $q(t) = (q_1, q_2, q_3, q_4)$, according to (Hamilton, 1830):

$$\boldsymbol{M_{att}} = \begin{pmatrix} q_1^2 + q_2^2 - q_3^2 - q_4^2 & 2(q_2q_3 - q_4q_1) & 2(q_2q_4 + q_3q_1) \\ 2(q_2q_3 + q_4q_1) & q_1^2 - q_2^2 + q_3^2 - q_4^2 & 2(q_3q_4 - q_2q_1) \\ 2(q_2q_4 - q_3q_1) & 2(q_3q_4 + q_2q_1) & q_1^2 - q_2^2 - q_3^2 + q_4^2 \end{pmatrix}$$

The time series of quaternions may possibly present all the specific characteristics of the measured data: noise, aberrant measurements, gap(s) in the temporal series. GINS is capable of handling some of these anomalies. In particular, it can validate the measurements, by comparing the attitude with a nominal law, and interpolate quaternions when there are gaps in the observations (provided that the gap is of a reasonable size in relation to the temporal variations in attitude). In some cases, the data should be pre-processed externally before using it in GINS. In addition to the quaternions that define the orientation of the satellite relative to the integration reference, quaternions are also used to direct high-gain antennas for planetary satellites.

4.5. Non gravitational force models / Free parameters

4.5.1. Atmospheric layers

The atmosphere (see Figure 8) is made up of several layers characterized by their main physical properties (constituents, pressure, temperature).

The thermosphere: the temperature can reach 1,200°C in a rarefied atmosphere. Conductivity is low because the heating energy is weak.

The mesosphere: from 50 to 80 km, the temperature drops as far as -90°C.

The stratosphere: from 11 km to 50 km on average. Contains about 19.9% of the mass of the atmosphere. The temperature rises due to the absorption of ultra-violet rays by the ozone layer.

The troposphere: from the ground to 16 km (in the tropics). Contains about 80% of the mass of the atmosphere. The gradient is $-6^{\circ}C/km$.



Figure 8: Diagram representing the layers of the Earth's atmosphere up to 120 km



DTM (Berger, 1997) or MSIS (Hedin, 1986) type models are semi-empirical. They are based on certain physical hypotheses that allow for the application of the following laws:

- the equilibriums of the different constituents are independent of one another,
- the vertical columns of atmosphere are independent of one another and in a state of statically balanced diffusion,
- the parameters of the state of the atmosphere are known at the lower boundary of the thermosphere.

The simplified integration of the differential equation of the static equilibrium of diffusion produces the partial density of each constituent of the thermosphere (H, He, N2, O, O2, etc.) according to an altitude law that depends on the temperature.

Moreover, these models are completed by an empirical G(L) function (Hedin, 1986) that takes account of different parameters, such as the indices of solar flux F10.7 and of geomagnetic activity K_p , the latitude, the local time and the season.

Solar flux is the main source that heats the thermosphere. Variations, which are linked to the number of Sun spots, are measured more efficiently at high or low frequencies, e.g. in the 10.7 cm radio band, or more representatively (but more recently) in the absorption ray of magnesium II at 280 nm. The solar wind carries charged particles that precipitate in the high latitudes, are trapped by the terrestrial magnetic field and heat the thermosphere by Joules effect. This is why the geomagnetic indices Kp or K_m are used to indirectly represent this heating by friction.



Figure 9: DTM-94 model: density by constituent (g/cm3)





4.5.3. Solar and geomagnetic indices

- The F_{10.7} (2800 MHz) index represents the intensity of the solar radio flux at 10.7 cm, with a temporal resolution of 1 day. It is measured in units of solar flux, in which each unit equals 10⁻²² Watts m⁻² Hz⁻¹, or 104 Janskys, which is the standard measuring unit in radioastronomy.
- The AE (Auroral Electrojet) index, which characterizes the intensity of the electrojets flowing in the auroral ionosphere. This index is calculated on the basis of the variations in the horizontal components of the magnetic field, measured by 12 stations located in the auroral zone in the Northern hemisphere. This index is expressed in nT.
- The **Dst** (Disturbed storm time) index, which characterizes the activity of the ring current, located in the equatorial plane of the magnetosphere at a distance of ~ 3 to 5 RT. Currently, the Dst is calculated on the basis of the hourly averages of the horizontal component *H* of the field observed by a network of four observatories that are sufficiently distant from the auroral electrojet and the equatorial electrojet, and are regularly spaced in longitude.
- The **PC** (Polar Cap) index, which measures the DP2 magnetic perturbations in the central ice cap, is calculated and its relation to the solar wind parameters is studied by a linear correlation analysis. This index is derived from the Thule station near the North Pole and the Vostok station near the South Pole, and the values derived from these two stations are then compared. This index shows a proper correlation with the component directed towards the South of the interplanetary magnetic field.
- The K_p index is established on the basis of measurements taken by several stations, located at latitudes between 44° and 60°. Due to the geopolitical conditions that reigned during the Cold War when this index was created, the stations are unevenly spread over the surface of the Earth. There are 11 in the Northern hemisphere (four in North America and seven in Europe) and two in the Southern hemisphere, in Australia and New Zealand. The K indices of each of these stations are standardized using tables drawn up by J. Bartels on the basis of a reference sample, in order to eliminate the effects of variations with universal time and the season of the magnetic activity. For each three-hourly interval, the value of the K_p index is the arithmetic mean of the standardized K indices. It is



expressed on a scale of 0 to 9, with a resolution of 1/3. The + and – symbols are used for non-integer values. For example, 5- corresponds to 4 2/3, and 5+ to 5 1/3.

- The ap index, expressed in 2 nT units, is deduced from the K_p index using a conversion table.
- The *am* index is calculated on the basis of the *K* indices measured by a network of 21 stations (12 in the Northern hemisphere and nine in the Southern hemisphere), with a corrected geomagnetic latitude as close to 50° as possible. These stations are grouped in sectors of longitude (five in the Northern hemisphere and four in the Southern hemisphere). For each three-hourly interval, the magnetic activity in each sector, expressed in nT, is estimated on the basis of the mean value of the *K* indices measured at the observatories located in the sector.

4.5.4. Acceleration of atmospheric friction

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The acceleration of the atmospheric friction applied to a satellite is calculated as the sum of the frictional forces applying to each elementary part (flat plates, spheres, cylinders, etc.). Each part produces an acceleration in drag and an acceleration in lift:

$$\vec{a} = -F_D \frac{1}{2} \rho \left(\sum_i C_{Di} \frac{S_i}{m} (\vec{v}_r \cdot \vec{n}_i) \cdot \vec{v}_r + \sum_i C_{Li} \frac{S_i}{m} (\vec{v}_r \wedge \vec{n}_i) \wedge \vec{v}_r \right),$$

where:

reference surface of the body i

\vec{n}_i	:	mass of the satellite unit vector normal to the reference surface of the body i
ρ	:	atmospheric density provided by the atmosphere model (depends on the position and the date)
\vec{v}_r	:	relative speed vector of the satellite in relation to the atmosphere
$egin{array}{ccc} C_{Di} \ C_{Li} \end{array}$:	aerodynamic drag coefficient of the body i aerodynamic lift coefficient of the body i
	4.5.4.1.1.1.3. :	
F_D		scale factor of the frictional force (ideally equals 1)
	4.5.4.1.1.1.4. :	

The aerodynamic coefficients are the sum of the coefficients of absorption (of gas molecules) and of supposedly diffuse re-emission. They are explained differently (see Obelix documentation), depending on the form and the temperature of the wall (Maxwell's hypothesis on the distribution of molecular speeds).

Free parameters of the model

A global multiplying coefficient of the force of atmospheric (or "Drag Factor", referred to as FD in GINS) is generally freed. The partial derivative of this coefficient is simply the acceleration itself.

<u>4.5.5.</u> Direct solar radiation pressure

The acceleration of direct solar radiation pressure is the acceleration to which the satellite is subjected due to the action of the received solar flux. It is calculated by adding up all the elementary radiation accelerations to which each of the exposed surface parts of the satellite is subjected (flat facets, cylinders, spheres or semi-spheres), according to:



$\vec{A} = F_s \frac{C_{Su}}{m}$	$\frac{n}{d_s} \left(\frac{\overline{d}_s}{d_s}\right)^2$	$\sum_{k}^{2} \sum_{k} S_{k} \overline{R}_{k}$
----------------------------------	---	--

where:

С

т F_s

- acceleration (m.s⁻²) \overline{A} : solar constant = $\frac{\Phi}{c \,\overline{d}_s^2} \approx 4.5 \text{ N.m}^{-2}$ C_{Sun} 4 solar flux at 1 au ≈ 1367 W.m⁻² : Φ : 4.5.5.1.1.1.2. speed of light (m.s⁻²) $rac{\overline{D}_{S}}{D_{S}}$: ratio between the mean/true solar distances S_k area of face k of the satellite (m^2) 4 : reflectivity vectorised coefficient on face k of the satellite (depends on the \overline{R}_k reflectivity coefficients of the face and one the angle of incidence lighting(without units)
 - : mass of the satellite (kg)
 - scale factor of the solar pressure force (ideally equals 1)
 - 4



The transmitted solar flux Φ_s is supplied as an input for the software. The flux that is actually received (Φ) may be attenuated by the presence of one or more bodies between the Sun and the satellite (eclipses). A shadow function is introduced (between 0 and 1) to represent the portion of the solar flux received by the satellite ($\phi = \phi_s \times$ shadow function). The shadow function depends on empirical coefficients and the relative positions and shapes of the various objects. The description of the obstructing bodies is limited to an elliptical geometric description.



The reflectivity of each of the surfaces is calculated on the basis of the thermo-optical coefficients (see 3.3). From an analytical perspective, it depends on the type of surface and on the incidental direction of the flux on the surface.

Free parameters of the model

A global multiplying coefficient of the force of solar pressure (or "Solar Factor", referred to as FS in GINS) is generally adjusted. The partial derivative of this coefficient is simply the acceleration itself.

The thermo-optical coefficients of the faces that make up the satellite can also be adjusted.

4.5.6. Rediffused and infra-red radiation pressures

The acceleration of rediffused radiation pressure is the acceleration to which the satellite is subjected due to the action of the solar flux that is re-transmitted by the central body.

The part of the central body that is in the shadow does not transmit any albedo flux. If the complete subsatellite cap is in the shadow zone, then we consider that the satellite does not receive any albedo flux.

The acceleration of infra-red radiation pressure is the acceleration to which the satellite is subjected due to the action of the infra-red flux that is transmitted by the central body.

Each surface element transmits an albedo flux and an infra-red flux in the same direction (see Figure 11).



Figure 11: Diagram of the geometry of the breakdown of the surface elements of the central body

Fluxes of re-diffused and infra-red radiations are calculated either on the basis of the emissivity grids derived from observations (see Figure 13 and Figure 12), that are input into GINS and interpolated for the date of the calculation, or on the basis of mean models that can also be given in the form of a grid:

$S_{el} = \frac{2\pi R_E^2}{n_{el}} \cdot \frac{r_{sat} - R_E}{r_{sat}}$
$\boldsymbol{\Phi}_{al} = \rho_{al} \cdot \frac{.C_{S}}{\pi d_{sat}^{2}} \cdot \left(\frac{\overline{D}_{S}}{D_{S}}\right)^{2} \cdot \cos \chi \cdot \cos \psi \cdot A_{surfel}$
$\Phi_{ir} = e_{ir} \cdot \frac{C_s}{4\pi d_{sat}^2} \cdot \cos \chi \cdot A_{surfel}$

where

S_{el}		area of the Earth surface element (m ²)
	4.5.6.1.1.1.1.:	
n_{el}		number of surface elements (without units)
	4.5.6.1.1.1.2. :	
$oldsymbol{\Phi}_{al}$:	albedo flux (N.m ⁻²)
$arPsi_{ir}$:	infra-red flux (N.m ⁻²)
R_E	:	mean radius of the Earth (m)
$ ho_{\scriptscriptstyle al}$		albedo coefficient of the surface element (without units)
	4.5.6.1.1.1.3. :	
e_{ir}		infra-red emissivity of the surface element (without units)
	4.5.6.1.1.1.4. :	
r _{sat}		geocentric satellite distance (m)
	4.5.6.1.1.1.5. :	
d_{sat}	:	
		4.5.6.1.1.1.6. surface element – satellite distance (m)
χ	:	angle between the normal to the surface element and the direction of the satellite from the surface element (rad)
Ψ	:	angle between the normal to the surface element and the direction of the Sun (rad)
S_{al}	:	area of the Earth surface element (m^2)
$\sim e_l$ n_{el}	-	number of surface elements (without units)
C1	4.5.6.1.1.1.7. :	

The acceleration of the pressure of re-diffused solar radiation on the satellite is calculated by adding up all the elementary accelerations (from each surface element of the central body) to which each of the exposed surface parts of the satellite is subjected (flat facets, cylinders, spheres or semi-spheres), according to:

$$\vec{A} = \sum_{el} \frac{\Phi_{el}}{m} \sum_{i} S_{i} \vec{R_{i}}$$

where:

\vec{A}	:	acceleration $(m.s^{-2})$
$oldsymbol{\Phi}_{el} \ S_i$:	albedo or infra-red flux from each surface $element(N.m^{-2})$ area of the satellite face i (m^2)
	4.5.6.1.1.1.8. :	
$\overrightarrow{R_i}$:	reflectivity vectorised coefficient on face k of the satellite (depends on the reflectivity coefficients of the face and on the angle of incidence lighting (without units) mass of the satellite (kg)
	4.5.6.1.1.1.9. :	

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It it to note that the reflectivity coefficients (for specular and diffuse reflections) are different in the visible spectrum (for direct or re-emitted fluxes) and in infra-red spectrum (depending on face emissivity).



Figure 13: Example of terrestrial emissivity values (March 2011)

The accelerometric measurements are made by an onboard accelerometer (Touboul, 2000) that is usually located in the centre of the mass of the satellite. These measurements provide access to the non-gravitational accelerations to which the satellite is subjected. If such measurements are available, as is the case for the CHAMP, GRACE and GOCE satellites, the sum of the non-gravitational forces can be replaced by

^{4.5.7.} Accelerometric measurements



accelerometric measurements (which may be interpolated) in the fundamental equation of the dynamics. Unlike the force models, these measurements are subject to noise and may include interruptions. These faults usually require suitable pre-processing containing elements that filter the signal and interpolation elements before use in the software. And like all the measurements, they must be calibrated, either at the time when the orbit is restored or afterwards. In an ideal case, in which the measurements are made at the centre of mass of the satellite, the acceleration is written in the instrumental reference (linked to the satellite) for each of the directions (i=1,2,3) according to:

$$A_{sat,i} = f_i A_{meas,i} + b_{meas,i} \quad (m/s^2)$$

where:

 $A_{meas,i}$ is the component *i* of the uncalibrated acceleration,

A_{sat,i} is the component *i* of the calibrated acceleration expressed in the instrumental reference,

 f_i is the scale factor associated with the component *i* (without units),

 $b_{meas,i}$ is the bias in (m/s^2) associated with component *i*.



Figure 14: Diagram representing the CHAMP satellite accelerometer

The calibration parameters f and b may vary over time, depending on temperature or instrumental ageing.

The acceleration is obtained in the integration reference by applying the attitude matrix to the calibrated accelerations:

$$\vec{A}_{inert} = M_{attitude}(t)\vec{A}_{sat}$$

The partial derivatives $\left(\frac{\partial \vec{A}_{inert}}{\partial f_i}, \frac{\partial \vec{A}_{inert}}{\partial b_{meas,i}}\right)_{i=1,3}$ of the calibrat

ion parameters are calculated at each step of the integration and are processed in the same way as the satellite's other dynamic parameters (see section 3.1).

The accelerometric data is input into the software by attaching a file of accelerometric measurements in ACC-CHAMP format (Föerste, 2002) in the Director file (lines acc and ac2).

<u>4.5.8.</u> Empirical accelerations (RTN/XYZ/stochastic/eclipses/manoeuvres)

A set of empirical acceleration adjusted on the basis of the measurements is used to take the modeling faults, the imprecision of the models, the poor knowledge of the physical or thermo-optical properties of the



satellite into consideration or to model manoeuvres. Several types of empirical accelerations can be modeled in GINS:

- accelerations that are constant in a given arc or period,
- accelerations that are variable to the period of the revolution (or to sub-multiples of the period of revolution),
- stochastic accelerations (or pulses) acting on an integration step.

Either the local orbital reference (see section 2.2.4) or the satellite reference (see section 2.2.5) is used, depending on the satellite and the physical problems to be modeled. In practice, the periodical terms are calculated according to the position in orbit relative to a reference direction defined in the orbital plane, rather than directly using the orbital period, which is difficult to access (see Figure 15).



Figure 15: Position **in** orbit

Stochastic empirical accelerations can be added in the three directions to each integration step. Their amplitudes are linked by time-based correlation law. It is also possible to add only pulses at predefined dates (manoeuvres or exits from an eclipse, for example). These accelerations are added to the sum total of the other accelerations in the selected dynamic model.

5. THE STATIONS

The measuring instruments on the ground are not fixed in the reference frame related to the body. They are subject to all the deformations of the terrestrial or planetary crust on which they are located. The reference point on the ground to which each instrument is attached is referred to as the *marker*. The movements affecting the position of the marker are caused by various factors. In addition to the tectonic motion, the IERS conventions (Petit and Luzum, 2010) define two main classes of displacements: the effects of tides that can be accurately predicted (Earth tides and oceanic load) and other load effects, due to variations in atmospheric pressure, the humidity in the soil, etc... Most of these are unpredictable and cover the entire temporal spectrum but they can be accounted for by using models based on observational data. In order to precisely model the position of the *phase centre* (the geometric point at which the measurement is done by the instrument), many corrections have to be taken into account: the eccentricity between the marker and the point of reference of the antenna or instrument, the phase centre vector and any corrections to the antennas that model the variation in this phase centre location or even the deformation of the instruments themselves (for example, due to the effect of temperature).



5.1. Tectonic displacements



Tectonic displacements include the drift of the various plates and the deformations due to earthquakes. Drift can reach several centimetres per year (see Figure 16). These motions are modeled as a bias + drift over a given period of time. The reference system of the body is made up of a coherent set of coordinates and velocities of geodetic markers. For example, the International Terrestrial Reference Frame (ITRF) is the practical representation of this kind of system (see Altamini et al, 2011 for the realization of ITRF2008). In GINS, the position and the velocity (\vec{p}_0, \vec{v}_0) of the marker are logged in the station file attached in the environment block of the **Director** file ("*stations*" line).





Figure 16: Horizontal tectonic ITRF2008 velocities observed using the DORIS technique, compared with the predictions of the Nuvel1A-NNR geological model (DeMets, 1994)

5.2. Deformations caused by tides and loads

The effects of tides and loads make up all the deformation effects of the crust. They cause the coordinates of the stations to vary. GINS features a number of deformation models that are consistent with the standards (Petit and Luzum, 2010).

5.2.1. Earth tides

Earth tide deformation caused by the attraction of the Moon and the Sun is proportional to the gravitational potential generated by these bodies at the point of the deformation, and takes account of the elastic character of the Earth, represented by the Love/Shida numbers of vertical/horizontal deformation (h_n/l_n , n=2.3). In vectorial terms, it is represented by:

$$\begin{split} \Delta \bar{r} &= \sum_{p=L,S} \frac{Gm_p}{GM} \frac{R_e^4}{r_p^3} \left\{ h_2 \bar{r} \left(\frac{3(\bar{r}_p, \bar{r})^2 - I}{2} \right) + 3l_2 (\bar{r}_p, \bar{r}) [\bar{r}_p - (\bar{r}_p, \bar{r})\bar{r}] \right\} \\ &\sum_{p=L,S} \frac{Gm_p}{GM} \frac{R_e^5}{r_p^4} \left\{ h_3 \bar{r} \left(\frac{5}{2} (\bar{r}_p, \bar{r})^3 - \frac{3}{2} (\bar{r}_p, \bar{r}) \right) + l_3 \left(\frac{15}{2} (\bar{r}_p, \bar{r})^2 - \frac{3}{2} \right) [\bar{r}_p - (\bar{r}_p, \bar{r})\bar{r}] \right\} \end{split}$$

The corrections for the ellipticity of the Earth, for the frequency and for the visco-elastic delay, as described in the IERS 2010 Conventions, are then added. The correction of the Earth's ellipticity is applied to the Love/Shida numbers h_2/l_2 by the introduction of an additional term that is proportional to the Legendre P_{20} polynomials. Taking the visco-elasticity into consideration requires complex Love numbers to be taken into account that express delays in deformation. Finally, the small oscillations of the Earth that cause the Love numbers to resonate, mainly at frequencies close to the diurnal and long periods (see Mathews et al.) are taken into consideration.

Note that the application of Earth tide deformation in this form also takes permanent zonal deformation into consideration and implies that the terrestrial reference is in a so-called "tide-free" system.

The polar tide is driven by small movements of the rotation axis of the Earth described by the polhode. The movements of polar Earth tides are calculated according to the resulting axifugal potential and the degree 2 Love/Shida numbers of elastic deformation:

$$\begin{cases} u_{r} = \frac{h_{2}}{g} \Delta V_{c} = -\frac{h_{2}}{g} \Omega_{o}^{2} R^{2} \sin \varphi \cos \varphi (m_{1} \cos \lambda + m_{2} \sin \lambda) = -0.033 \sin 2\varphi (\bar{x}_{p} \cos \lambda - \bar{y}_{p} \sin \lambda) \\ u_{\varphi} = \frac{\ell_{2}}{g} \frac{\partial \Delta V_{c}}{\partial \varphi} = -\frac{\ell_{2}}{g} \Omega_{o}^{2} R^{2} \cos 2\varphi (m_{1} \cos \lambda + m_{2} \sin \lambda) = -0.009 \cos 2\varphi (\bar{x}_{p} \cos \lambda - \bar{y}_{p} \sin \lambda) \\ u_{\lambda} = \frac{\ell_{2}}{g \cos \varphi} \frac{\partial \Delta V_{c}}{\partial \lambda} = \frac{\ell_{2}}{g} \Omega_{o}^{2} R^{2} \sin \varphi (m_{1} \sin \lambda - m_{2} \cos \lambda) = 0.009 \sin \varphi (\bar{x}_{p} \sin \lambda + \bar{y}_{p} \cos \lambda) \end{cases}$$

The variables \overline{x}_p , \overline{y}_p represent the difference between the coordinates of the pole and the centre of the polhode, which is more or less aligned with the axis of inertia.

5.2.3. Surface loads

The displacements of water masses due to oceanic tides or variations in oceanic currents, or even continental waters, result in a change in the superficial load per unit of surface to which the Earth responds elastically (in an opposing movement). The same applies to the variations in atmospheric pressure. The deformation for each tracked site is calculated outside GINS by the Green operator, a convolution integral between a Green function (including the Love/Shida load numbers) and a load model, or by a spherical harmonic development when the load is developed in this base.

The following load models are used:

- The oceanic tides from a "FES" model. In this case, the displacement is given for each site by amplitude and phase for each main wave of the model (semi-diurnal, diurnal and long period).
- The continental atmospheric pressure calculated on the basis of the ECMWF pressure fields or models.
- The inverse non-barometer response on the basis of the MOG2D model.
- And possibly, a hydrological load model.

When models other than tide models are used, the displacements of each site are given in a temporal series according to the resolution of the model.

5.3. Calculation of the phase centre of the instruments on the ground

The coordinates of the phase centre for a ground station are calculated in the reference frame of the central body (Earth, Mars, etc.) by adding up all the corrections at the signal reception date t (tectonic displacements, Earth tides, deformation due to oceanic load, deformation due to atmospheric load, eccentricity vector and phase centre vector between the marker and the phase centre of the measurement), according to:

$$\vec{p}_{station} = \vec{p}_0 + \vec{v}_0(t - t_0) + \overrightarrow{\Delta p}_{solid\ tide} + \overrightarrow{\Delta p}_{oceanic\ load} + \overrightarrow{\Delta p}_{atm.load} + \overrightarrow{\Delta p}_{ecc} + \overrightarrow{\Delta p}_{cdp}$$
The effects of Earth tides are always taken into consideration. The application (or not) of corrections in oceanic and atmospheric load are controlled by the **Director** file.

6. MEASUREMENTS / FREE PARAMETERS

6.1. The theoretical quantity of the measurement – general

6.1.1. Travel time/ Geometric distance

The travel time of an optical or radio signal is incorporated in most measurements. It is modeled in GINS in the same way for all range (Laser, DORIS, etc.) or range differences (GNSS) measurements. It includes the geometric travel time, to which relativistic corrections of signaletic element propagation and the delays that occur when crossing the atmosphere (for ground receivers) are added. For interplanetary measurements (DSN), corrections of the propagation in the interplanetary medium (plasma) and in the vicinity of the solar corona are also applied. The tropospheric, ionospheric and relativistic delays are described in detail below (see 5.3, 5.4 and 5.8). The travel time is broken down into four terms, according to:

$\tau = \tau_{geom} + \tau_{relat} + \tau_{tropo} + \tau_{iono}$

The geometric travel time is obtained directly from the geometric distance $(d_{geom} = c\tau_{geom})$, which is calculated by the *teodist* function that is common to most measurements. This function returns the distance between any two objects (orbiters, stations, quasars) according to their position at the instants of emission and reception t_1 and t_2 :

$$d_{geom} = \left| \vec{P}_{object2}(t_2) - \vec{P}_{object1}(t_1) \right|$$

The date on which the geometric distance is calculated depends on the measurement (for example, date of emission, of reception or of the laser echo). This function also calculates the elementary partial derivatives of this distance in relation to the required parameters $(X_i) \left(\frac{\partial d_{geom}}{\partial X_i}\right)$ on the date of the measurement.

The precise positions are the coordinates of the phase centres of the measuring instruments (for laser, DORIS, GNSS, VLBI, GRACE-KBR, etc. measurements) or the centre of mass (for ephemeris measurements). They, and the partial derivatives that can be calculated, depend on the type of object in question (see Table 4).



Type of object	Date of the event	Point used to calculate	List of calculated
		the distance	partial derivatives
receiver/transmitter	Date of emission or	Phase centre of the	Dynamic parameters
onboard a satellite	reception	instrument for GPS, Laser,	(bulletins, forces)
	Estimated date of the laser	DORIS, SST –GRACE	Derivatives of the phase
	echo on the retroreflector	measurements	centre of the instrument
	Date of the ephemeris	Mass centre (for	
	measurement	ephemeris measurements)	
ground	Date of emission or	Phase centre of the	Coordinates and
receiver/transmitter	reception	instrument	velocities of the station
(station)		The position of the station	marker
		is given by the sum total	Rotation parameters of
		of the displacements in	the body (EOP in
		question (see 4.3)	terrestrial cases)
			Tropospheric parameters
geocentre	Date of the ephemeris	Centre of the Earth	-
	measurement		
ocean surface	Estimated date of the	Point of reflection of the	Sea height
	moment when the signal is	signal on the ocean	
	reflected on the ocean	surface (altimetry /	
	surface	crossing points)	
quasar	Date of the measurement	No calculation of distance	Partial derivatives of the
	(in J2000 quasar	in this case	quasar coordinates
	coordinates are considered		
	as fixed)		
	4 4 4 1 4 6 4	• • • • • • • •	

Table 4: Output of the geometric distance function common to all objects that can be observed by GINS

<u>6.1.2.</u> Partial derivatives of the station coordinates

For measurements taken by instruments bound to the rotating central body, the reference change matrix M(t) appears in the expression of the geometric distance by writing $d_{geom} = \|\vec{P}_{satellite} - M(t)\vec{p}_{station}\|$, where $\vec{p}_{station}$ is expressed in the frame relating to the body.

If requested, (see common block of the **Director**), the program calculates the derivatives of the geometric distance in relation to the X_i coordinates of the stations for each measurement according to:

$$\frac{\partial d_{geom}}{\partial X_i}(t) = -\vec{u}.M(t)\frac{\partial \vec{p}_{station}}{\partial X_i}$$

where \vec{u} is the normed vector tangent to the line of sight.

The partial derivatives of the station coordinates can be expressed either as Cartesian coordinates (**XYZ**) or ellipsoidal coordinates (φ , λ , h).

6.1.3. Partial derivatives of the rotation parameters of the body

When one of the objects is a station linked to the central body, the partial derivative of the geometric distance for each rotation parameter is calculated according to:

$$\frac{\partial d_{geom}}{\partial X_i}(t) = -\vec{u} \cdot \left[\frac{\partial M}{\partial X_i} M^{-1}(t)\right] \vec{p}_{station}(t)$$

The list of X_i orientation parameters depends on the body in question and the form of the orientation matrix. Usually, for the Earth, these are the coordinates of the pole x_p and y_p , UT1 and the coordinates of the celestial pole, dX and dY.

6.2. Specifics of the types of measurement

6.2.1. Notion of pass

As a general rule, the term **pass** is used to refer to all the measurements between a ground instrument and a satellite that are made between the moments when the satellite rises and sets, as seen from the station. In this case, the term **visibility pass** is used. Clearly, the length of the pass depends on the orbital parameters of the satellite and the position of the station. It can range from just a few minutes to several hours. A number of parameters are sometimes defined (usually measurement biases) for the duration of these passes, in which case they are considered as constant for this duration. The notion of pass is different in the case of GNSS observable objects, since it refers to all the measurements with the same ambiguity (see 5.2 Specifics of the types of measurement).

6.2.2. Laser telemetry measurements

The laser telemetry measurement is the measurement of the time-of-flight of a laser pulse transmitted by a ground station and reflected on a retroreflector on an artificial satellite or the Moon. Laser stations are fitted with precision clocks that are used to date the instants of transmission and reception of the transmitted laser pulses. The pulses are powerful (in order to obtain a meaningful reflected signal) and their wavelength varies between technologies.

Theoretical quantity

The theoretical quantity associated with laser measurements is expressed in terms of the distance (in metres) between the point of emission and the point of reflection and is linked to the outward τ_1 and return τ_2 travel times, according to:

$$Q_{theo} (t + B_{datation}) = c \frac{\tau_1 + \tau_2}{2} + B_{distance}$$

The date of the measurement t is the mean date between the date of emission and the date of reception of the laser pulse by the station. This expression includes corrective terms B that take account of the calibration bias (range and time bias) associated with the instruments. The software can adjust range and/or time bias terms (by pass or by station) according to the options in the Director.





Figure 17: Fixed laser station of the Côte d'Azur Observatory (Calern plateau, France)



Figure 18: Laser retroreflectors of the Lageos satellite

The travel time terms are calculated in a conventional manner and take account of the phase centre corrections of the onboard instruments, the emission and reflection dates of the signal and the echo date on the retroreflector. As a general rule, the position of the phase centre of the retroreflector in orbit is obtained on the basis of the coordinates of the retroreflector in the satellite reference and the attitude matrix on the date of the echo. In the special case of spherical satellites that are fully equipped with laser retroreflectors (Lageos, Starlette, Stella, etc.), the phase centre is corrected by a distance correction that is independent of the orientation of the satellite, which is usually unknown.

The software can be used to make the distinction between the different types of laser station according to the transmitted wavelength. This data is entered in the station file using the codes shown in Table 5. These wavelengths are used in the calculation of the tropospheric correction of the Marini-Murray (Marini, 1972) model.

Laser station code that appears in the station file	Wavelength	Colour
Lts	423 nm	Purple
Lyg1	532 nm	Yag1 (green)
Lyg2	539 nm	Yag2
		(Green)
Lrb	694 nm	Ruby
Lir	1064 nm	Infra-red

Table 5: Codes and wavelengths of the different types of laser station known in GINS

6.2.3. Doppler measurements

Theoretical quantity

Definition of the four events:

- ① Emission of the first cycle by the transmitter
- ② Emission of the N_e -th cycle by the transmitter
- ①' Reception of the first cycle by the transmitter
- ②' Reception of the N_e -th cycle by the receiver



The events are dated τ_{e_1} , τ_{e_2} , in transmitter specific time, τ_{r_1} , τ_{r_2} in receiver specific time and t_1 , t_1 , t_2 , t_2 in coordinated time in the reference (S).

In the interval $\Delta \tau_r = \tau_{r_2} - \tau_{r_1}$, the receiver received the N_e cycles sent by the transmitter, where $N_e = f_e \Delta \tau_e$, f_e is the <u>natural</u> frequency of the transmitter. The receiver also has an oscillator and, in the same interval $\Delta \tau_r$, it also generated a number $N_r = f_r \Delta \tau_r$ of cycles, f_r being the <u>natural</u> frequency of the receiver. The Doppler measurement is represented by the difference between the number of cycles N_e and N_r generated at the receiver:

$$N_{DOP} = N_e - N_r \implies N_{DOP} = f_e \Delta \tau_e - f_r \Delta \tau_r$$

If we express $\Delta \tau_e$ in $\Delta \tau_r$, using coordinated time:

 $\Delta \tau_e = \tau_{e_2} - \tau_{e_1} \approx \left(1 - \frac{U_e}{c^2} - \frac{V_e^2}{2c^2}\right) (t_2 - t_1) \text{ assuming that the gravitational potential } U_e \text{ and the speed } V_e \text{ of } V$

the transmitter are constant for the interval $(t_2 - t_1)$ considering,

$$t_{2} - t_{1} = (t_{2} - t_{2'}) + (t_{2'} - t_{1'}) + (t_{1'} - t_{1})$$

= $-(t_{2'} - t_{2}) + (t_{2'} - t_{1'}) + (t_{1'} - t_{1})$

where:

 $t_{2'} - t_2 = \frac{\rho_2}{c} + \frac{2GM}{c^3} \ln\left(\frac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)$ $t_{2'} - t_{1'} = \Delta t_r \approx \left(1 + \frac{U_r}{c^2} + \frac{V_r^2}{2c^2}\right) \Delta \tau_r \text{ supposing that } U_r \text{ and } V_r \text{ are constants for } (\tau_2 - \tau_1), \text{ i.e. in the case of a}$

slightly eccentric orbit.

$$t_{1'} - t_1 = \frac{\rho_1}{c} + \frac{2GM}{c^3} \ln\left(\frac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right)$$

and neglecting the terms that produce an effect inferior to 7.10^{-5} cycles¹, the expression of N_{DOF} (GM is noted μ) becomes:

¹ The value of 7.10⁻⁵ cycles corresponds, once converted into speed by the conversion factor $\frac{c}{f_o\Delta t}$

where $f_e \approx 2.10^9$ Hz and $\Delta t \approx 10$ s to a relative transmitter-receiver speed of 10^{-6} m/s.



$$N_{DOP} = (f_e - f_r)\Delta\tau_r - f_e \left(1 - \frac{\mu}{R_e c^2} - \frac{V_e^2}{2c^2}\right) \frac{\rho_2 - \rho_1}{c} - \frac{f_e}{c^2} \left(\frac{\mu}{R_e} - \frac{\mu}{R_r} + \frac{V_e^2}{2} - \frac{V_r^2}{2}\right) \Delta\tau_r + 2\frac{GM}{c^3} f_e \left[\ln\left(\frac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right) - \ln\left(\frac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)\right]$$

Therefore, the theoretical Doppler count can be written as the sum of four terms, to which the corrections of tropospheric and ionospheric propagation must be added:

$$N_{DOP THE} = \underbrace{\Delta N_{BATT}}_{\text{Beat frequency}} + \underbrace{\Delta N_{DOP}}_{\text{Doppler}} + \underbrace{\Delta N_{REL_{H}}}_{\text{Clock}} + \underbrace{\Delta N_{REL_{T}}}_{\text{Trajectory}}$$
Where:

$$\Delta N_{BATT} = \left(f_e - f_r\right) \Delta \tau_r$$

$$\Delta N_{DOP} = -f_e \left(1 - \frac{\mu}{R_e c^2} - \frac{V_e^2}{2c^2}\right) \frac{\rho_2 - \rho_1}{c}$$

$$\Delta N_{REL_{H}} = \frac{-f_e \Delta \tau_r}{c^2} \left[\mu \left(\frac{1}{R_e} - \frac{1}{R_r}\right) + \frac{V_e^2 - V_r^2}{2}\right]$$

$$\Delta N_{REL_{T}} = 2 - \frac{\mu}{c^3} \left[\ln \left(\frac{R_1 + R_{1'} + \rho_1}{R_1 + R_{1'} - \rho_1}\right) - \ln \left(\frac{R_2 + R_{2'} + \rho_2}{R_2 + R_{2'} - \rho_2}\right)\right]$$

These equations are valid in upgoing and downgoing Doppler. This theoretical formulation is then used to constitute the quantity $\Delta Q = N_{OBS} - N_{THE}$ and to calculate the corrections of the parameters to be determined.

Important remarks:

 $\Delta \tau_r$ is a receiver proper time interval. ρ_2 and ρ_1 are distances calculated on the basis of the <u>coordinated</u> times $(t_2, t_{2'})$ and $(t_1, t_{1'})$. This has three consequences:

It is essential to know the time scale used to date the measurements and how the scale was converted. If the observation equation is to be rewritten to express it in terms of relative transmitter-receiver speed in the reference (S), or if the available time interval is a IAT² time interval (Δt_{TAI} instead of $\Delta \tau_r$), then the

relativistic corrections must be applied to $\Delta \tau_r$ in the equation : $\begin{vmatrix} \Delta \tau_r \approx \left(1 - \frac{U_r}{c^2} - \frac{V_r^2}{2c^2}\right) \Delta t \\ \Delta t_{TAI} \approx \left(1 - \frac{U_{GEO}}{c^2}\right) \Delta t \end{vmatrix}$

 U_{GEO} being the gravitational potential of the geoid.

$$\Rightarrow \Delta \tau_r \approx \left(1 - \frac{U_r}{c^2} - \frac{V_r^2}{2c^2} + \frac{U_{GEO}}{c^2}\right) \Delta t_{TAI}$$

(in ΔN_{REL_H} , this correction of $\Delta \tau_r$, will introduce totally negligible effects of the second order). Regarding the calculation of ρ : ρ is calculated on the basis of the position of the transmitter at the instant t_{α} and of the receiver at the instant $t_{\alpha'}$. Usually, the known quantity is the coordinated time of reception $t_{\alpha'}$.

² IAT (International Atomic Time) is the *former* TT time. It can be considered as the realization, at the geoid level, of the coordinated time t of the reference (S). Therefore, it differs from t at a constant rate: $L_G = \frac{U_{GEO}}{c^2} = 6.96929013010^{-10}$ (see IERS 2000 Conventions, Chapters 1 and 10).



$$t_{\alpha} = t_{\alpha'} - \frac{\rho_{app}}{c}$$

In the equations (4) the weakest corrective terms are ΔN_{REL_T} and the term in $\left(-\frac{\mu}{R_ec^2}-\frac{V_e^2}{2c^2}\right)$ in ΔN_{DOP} .

Their maximum values are respectively 8. and 4. 10^{-4} cycles, i.e. 11. and 6. 10^{-6} m/s. They can therefore be neglected, depending on the required precision.

Free parameters

The free parameters specific to Doppler measurements are a frequency bias and, possibly, a drift in the frequency by station. These parameters are constant for the duration of the entire arc or by pass.

References:

Test on relativistic corrections applied to the LASSO experiment - Richard Biancale. Relativistic correction to satellite Doppler Observation – Claude Boucher Relativity and cosmology – Robertson & Noonan. IERS Convention 2000: http://maia.usno.navy.mil/conv2000.html

6.2.4. VLBI measurements

Very Long Base Interferometry (VLBI) measurements are the measurement of the difference in reception time of radio radiation from an extra-galactic source (quasar) between two terrestrial radio telescopes. The S (2.3 GHz) and X (8.4 GHz) frequency bands used in astrometry and geodesy correspond respectively to wavelengths of 13 cm and 3.6 cm. The diameter of the antenna ranges from 9 to 60 metres (see Figure 19).



Figure 19: The radio telescope antenna in Fairbanks, Alaska. Diameter: 26 m

The measurement is obtained by correlating the signals recorded on each site and dated by a local clock. For each observed object, this correlation gives the delay and the drift $(\tau, \dot{\tau})$ between the times of arrival on the two sites on the date of the measurement.





Figure 20: VLBI measurements (Source: NASA/GSFC)

Theoretical quantity

If there is no perturbation (atmospheric, gravitational), the difference between the arrival times of the signals at the two antennas is made up of the geometric delay, which is written:

$$\tau_{geom} = \frac{\vec{k}.\vec{b}}{c}$$

where,

 $\vec{k} = -(\cos d \cos \alpha, \cos d \sin \alpha, \sin d)$, a unit vector in the direction of the radiosource with a right ascension α and declination d (in the barycentric frame). These coordinates are entered in the file of quasar sources.

 $\vec{b} = \vec{p}_{station1} - \vec{p}_{station2}$ base vector between the two stations (see Figure 20).

The precise theoretical quantity of this measurement is calculated by adding the corrections related to the various observed sources, the corrections due to the deflection of the signals by the gravitating masses in the solar system, the various delays caused when crossing the atmosphere and instrumental delays (synchronization faults of the clocks, antenna corrections) to the geometric delay τ_{g} .

The total theoretical quantity is therefore calculated according to the sum:

$$Q_{theo}(t) = \tau_{source} + \tau_{gravity} + \tau_{geom} + \tau_{tropo} + \tau_{iono} + \tau_{instruments}$$

 τ_{geom} is calculated in the barycentric frame by successively applying the transformation between the terrestrial reference and the geocentric reference, then Lorentz's transformation (see section 2.2) between the celestial frame and the barycentric frame to the base line vector \vec{B} . The relativistic delays ($\tau_{gravity}$) due to the curve of path are calculated in this frame by adding up the contributions on all of the bodies in question (Sun, Moon and Earth), which are obtained by the expression (Petit and Luzum, 2010):

$$\tau_{gravity,body} = (1+\gamma) \frac{GM_{body}}{c^3} \ln\left(\frac{\left|\vec{R}_{1,body}\right| + \vec{R}_{1,body}.\vec{k}}{\left|\vec{R}_{2,body}\right| + \vec{R}_{2,body}.\vec{k}}\right)$$



where,

 $\vec{R}_{n,body}$: the vector between the centre of mass of the body in question and the station *n* GM_{body} : product of the constant of gravitation by the mass of the body

$$\gamma = \frac{1}{\sqrt{\left(1 - \left|\frac{V_{Earth}}{c}\right|\right)}}$$
: Lorentz factor

The resulting geometric delay is restored to the celestial frame according to the reverse Lorentz transformation.

The instrumental corrections include the geometric corrections of the antennas (which depend on the type of assembly used) and the clock corrections.

The geometric corrections are applied according to the code of each radio telescope in the station file. The different types of assemblies known in GINS are listed in Table 6. The corrections linked to the thermal deformation of the assemblies (Nothnagel, 2009) are not applied for the time being.

Type of radio telescope	Code in the	Formula / reference
mounting	station file	
Azimuth	Vaze	No correction applied
Equatorial	Vequ	?
Richmond	Veqr	?
XY North horizontal	Vx-y / Vxyn	?
XY East horizontal	Vxye	?
Other	V ????	No correction applied

 Table 6: The different types of radio telescope assemblies known in GINS



Figure 21: Example of a polar mounting (credits: Elena Skurikhina)

Ionospheric corrections are obtained either by the iono-free combination on two observed frequencies, or on the basis of models.



The tropospheric corrections along the line of sight are calculated by making the difference between the contributions of the two stations. The observed zenith bias can be freed for both stations.

Free parameters

The clock parameters and derivatives.

In addition to the Earth orientation parameters (EOP), the coordinates of the radio sources (right ascension and declination) can also be freed.

6.2.5. Optical measurements

Optical measurements consist of two angular parameters: the right ascension (α) and the declination (δ) of satellites observed by camera (mostly Baker-Nunn). The observations made in the 1960s and 1970s were generally reduced in relation to the FK4/FK5 catalogues in the γ 50 system. The theoretical quantity is simply calculated in the J2000 inertial system. The right ascension is calculated by:

$$\alpha = \arctan\left(\frac{x_{sat} - X_{STA}}{y_{sat} - Y_{STA}}\right)$$

The declination is calculated by:

$$\delta = \frac{(z_{sat} - Z_{sta})}{\left[(x_{sat} - X_{sta})^2 + (y_{sat} - Y_{sta})^2 + (z_{sat} - Z_{sta})^2 \right]}$$

The declination measurement is corrected by an old Veis refraction model.

<u>6.2.6.</u> Altimetric measurements

The altimetric measurement is the measurement of the pseudo-distance (based on the travel time) between the phase centre of an onboard altimeter that transmits a radar signal and the point of reflection of this signal on the ocean surface (see Figure 22).

$$Q_{theo_{alt}}(t) = c \frac{\tau}{2}$$

Theoretical quantity

The travel time is conventionally modeled by replacing the coordinate of the station by the point of reflection of the signal on the ocean surface in the calculation of the geometric travel time. Itimetric measurements are radial measurements that are used to define the ellipsoid coordinates (φ , λ) of this point of reflection as the intersection between the position vector of the satellite and the ocean surface (at the nadir of the satellite). The majority of the reflection from the satellite is made on a surface area measuring a few kilometres in diameter, and these coordinates can be obtained on the basis of an approximate position of the satellite. The measurements are affected by the various delays that occur when passing through the atmosphere. These delays are usually corrected when creating the measurement file for GINS using, on the one hand, the combination of the bi-frequency altimetric measurements (for the ionospheric delay) and, on the other hand, the simultaneous measurements from an onboard radiometer that provides access to the steam content along the line of sight (for the tropospheric correction).





Figure 22: Illustration of the altimetric range measurement (according to www.aviso.oceanobs.com)

The height of the ocean surface on the date of the measurement is obtained by adding up the variations in the height of the surface above the ellipsoid (with a given semi major axis and oblateness) according to:

$$\begin{aligned} h_{surf}(t) &= h_{geoid}(\varphi, \lambda) + h_{topo}(t, \varphi, \lambda) + h_{ocean\ tide}(t, \varphi, \lambda) + h_{atm.tide}\left(t, \varphi, \lambda\right) \\ &\quad h_{sol\ tide}(t, \varphi, \lambda) + h_{load}\left(t, \varphi, \lambda\right) + h_{baro}(t, \varphi, \lambda) \end{aligned}$$

where,

 $h_{geoid}(\phi, \lambda)$: the height of the geoid

 $h_{topo}(t, \varphi, \lambda)$: dynamic topography

 $h_{ocean\ tide}(t,\varphi,\lambda) + h_{atm.tide}(t,\varphi,\lambda) h_{sol\ tide}(t,\varphi,\lambda)$: the respective contributions of the Earth, oceanic and atmospheric tides

 h_{load} (t, ϕ , λ): the displacement of the ocean floor by the ocean tide

 $h_{baro}(t, \varphi, \lambda)$: atmospheric pressure and inverse barometer

For more details on the modeling of the altimetric measurements, refer to (Chelton, 2001).

Note: the sum total of the geoid height and the mean dynamic topography is called the mean sea surface or MSS. This is defined for a given period.

These model-based corrections are either integrated directly in the altimetric measurements file or can be recalculated, in certain cases, in GINS. Table 7 gives the list of possibilities offered by the software according to the various corrections.



Correction	Can be calculated in GINS, if	Present in the measurement
	requested	file
	(see description of the	
	Director)	
Geoid and dynamic	Yes (on the basis of harmonic	Yes
topography	models or MSS matrices	
	provided in the header of the	
	Director)	
Ocean tide	Yes (on the basis of the tide	Yes
	model provided in the header of	
	the Director)	
Earth tide	Yes (permanent tides can be	Yes
	included)	
Atmospheric tide	No	Yes
Load effect	No	Yes
Reverse barometer effect	Yes	No

Table 7: Altimetric corrections for the calculation of the height of the ocean surface

Free parameters

Like for any satellite, it is possible to calculate the partial derivatives of the dynamic parameters of an altimetric satellite.

It is also possible to ask for the partial derivatives of the geoid to be calculated through the contribution of $h_{geoid}(\phi, \lambda)$ to the measurement.

6.2.7. Measurement of the crosspoint

A crossover measurement is the difference between two altimetric measurements at two different times, one of which is made along the ascending track and the other along the descending track.

Theoretical quantity

The theoretical quantity is obtained by making the difference of the theoretical quantity of two altimetric measurements. In this case, all the corrections are taken from the measurement file and are not recalculated in GINS.

$$Q_{theo_{cro}} = Q_{theo_{alt}}(t_2) - Q_{theo_{alt}}(t_1) = c \frac{\tau(t_1) - \tau(t_2)}{2}$$

Free parameters

Like for any satellite, it is possible to calculate the partial derivatives of the dynamic and geometric parameters (the phase centre of the altimeter) of the satellite that are simply obtained on the basis of the differences in the partial derivatives of the two elementary measurements:

$$\frac{\partial Q_{theo_{cro}}}{\partial X_{i}} = \frac{\partial Q_{theo_{alt}}}{\partial X_{i}}(t_{2}) - \frac{\partial Q_{theo_{alt}}}{\partial X_{i}}(t_{1})$$

The German PRARE system was used on two satellites in the 1990s: METEOR3 in trials and ERS-2. The associated distance and Doppler functions are encoded in the software.

The measurement method of the PRARE instrument consists of two messages sent by the satellite's onboard sensor: one signal in the S band at 2.2 GHz and another in the X band at 8.5 GHz. The two signals are modulated with a PN code (pseudo-random noise). The time interval separating the reception of the two signals, which are sent at the same time, is measured at the ground station (see Figure 23) with very high accuracy (< 1 ns). This value is then sent back onboard for the ionospheric correction of the data. Around 20 ground stations are used to collect the meteorological measurements to correct the tropospheric refraction.



Figure 23: PRARE tracking station

The precision of this two-channel system is of the order of the cm in range (in normal points) and 0.1 mm/s in Doppler for an integration time of 30 s.

6.2.9. GNSS measurements

GNSS measurements are made on radio signals sent by GNSS satellites to GNSS receivers. The measurement is obtained by correlating the signal received with a signal of the same form generated in the receiver. The distinction is made between the pseudo-distance measurements, which are obtained by the correlation of the code, and the phase measurements, obtained by correlating the phase of the signal carrier. A GNSS station is made up of a calibrated antenna linked to a receiver and containing a high-precision clock. Modern GNSS receivers can make measurements on different channels. In this way, at each measurement step, all the observations made on several frequencies and for different satellites and constellations are available. The receivers are synchronized automatically and have the same sampling rate. Consequently, the measurements are simultaneous, allowing for the formation of numerous combinations. Hybrid processes consist in using observations from different constellations in the same process.

GNSS observations are pre-processed by a set of tools outside the GINS software on the basis of raw observations supplied in RINEX format. They can be used in GINS in a number of ways:

- Zero-difference or double-difference measurements
- Known or estimated ephemeris (orbits/clocks)
- Fixed, or mobile ground receivers
- Receivers, which may be onboard LEO satellites (Low Earth Orbiter)



For example, GINS can be used for the following applications:

- **PPP mode:** single-receiver processing of non-differentiated measurements: the clocks and the orbits of the GNSS satellites are fixed and are specified in the GINS Director. The software restitutes the clock parameters, the ambiguities, the positions (**static or high-frequency**) and the tropospheric parameters associated with the receiver.
- **Double-differences:** processing of the double-difference data of a network of stations. The software restitutes the positions, the differences in ambiguity and the tropospheric parameters associated with the receivers.
- **LEO orbit restitution:** the clocks and the orbits of the GNSS satellites are fixed and are specified in the GINS Director. The software restitutes the low orbit of one or two LEO satellites and the associated clock parameters and ambiguities.
- **Restitution of GNSS orbits and the network coordinates:** GINS processes the data from a global network of receivers and restitutes the orbits and the clocks of the GNSS satellites, the orientation parameters of the Earth, plus the coordinates of the stations, ambiguities, the clock parameters and the tropospheric parameters associated with the receivers.

The calculation of the elementary quantity is the same, according to the mode in question. The clock corrections are freed or applied, depending on the case.

Theoretical quantity

The non-differentiated elementary observation equations between a GNSS receiver and transmitter are modeled by the following equations (Laurichesse et al., 2009) for the phase (L) and pseudo-distance (P) observables on the two frequencies (a and b):

$$P_a = D_a + e + \Delta h_p + \Delta \tau_p$$

 $P_b = D_b + \gamma_e + \Delta h_p + \gamma \Delta \tau_p$

 $\lambda_a L_a = D_{L_a} + \lambda_a d_{windup} - e + \Delta h_L + \Delta \tau_L - \lambda_a N_a$

 $\lambda_b L_b = D_{L_b} + \lambda_b d_{windup} - \gamma e + \Delta h_L + \gamma \Delta \tau_L - \lambda_b N_b$

The travel time D_a , D_b , D_{L_a} and D_{L_b} of the various observations includes the geometric term and the tropospheric delay. The geometric term contains the antenna corrections that are usually different for the phase measurements and the pseudo-distance measurements, as well as for the frequency in question. The antenna and wind-up phase d_{windup} corrections are described in detail in section 5.5. GINS is currently limited to the signals and the frequencies in Table 8. The Δh_p , Δh_L terms are the clock differences between the receiver and the transmitter. These terms include possible bias between the different types of observables and the inter-system bias. The clock terms are eliminated when double differences are formed between the observations. The terms of ionospheric delay, which are written in the first order as a function of $\gamma = \frac{\lambda_B^2}{\lambda_a^2}$, and of the delay e on the first frequency (a), are eliminated by forming iono-free combinations in the input into GINS. This combination also makes the possible $\Delta \tau_p$, $\Delta \tau_L$ bias unobservable.

	L1	L2	L5b	L5a
GPS	1,575.42 MHz	1,227.60 MHz	-	1,176.45 MHz
GLONASS (index k,	(1602 +	(1246 +	-	-
between -7 and +6	k *0.5625 MHz)	k *0.4375 MHz)		
depending on the				
satellite)				
Galileo	1,575.42 MHz		1,207.14 MHz	1,176.45 MHz



Table 8: Known GNSS satellite frequencies in GINS

The GNSS observations processed in GINS are the non-differentiated, iono-free observables (Loyer, 2012) that are obtained on the basis of elementary non-differentiated observations according to:

$$P_{I} = \frac{\gamma P_{a} - P_{b}}{\gamma - 1} = \frac{\gamma D_{a} - D_{b}}{\gamma - 1} + \Delta h_{p}$$

$$L_{I} = \frac{\gamma \lambda_{a} L_{a} - \lambda_{b} L_{b}}{\gamma - 1} = \frac{\gamma D_{L_{a}} - D_{L_{b}}}{\gamma - 1} + \lambda_{NL} d_{windup} + \Delta h_{L} - R$$

$$L_{Icorr} = \frac{\gamma \lambda_{a} L_{a} - \lambda_{b} L_{b} - \lambda_{b} N_{WL}}{\gamma - 1} = \frac{\gamma D_{L_{a}} - D_{L_{b}}}{\gamma - 1} + \lambda_{NL} d_{windup} + \Delta h_{L} - \lambda_{NL} N_{a}$$

$$L_{Ibloquées} = \frac{\gamma \lambda_{a} L_{a} - \lambda_{b} L_{b} - \lambda_{b} N_{WL}}{\gamma - 1} - \lambda_{NL} N_{a} + \lambda_{NL} d_{windup} = \frac{\gamma D_{L_{a}} - D_{L_{b}}}{\gamma - 1} + \Delta h_{L}$$

Measurement of pseudo-distance

Phase measurement (*R* is the real ambiguity)

Phase measurement (correction of the wide-lane ambiguity) Blocked phase measurement (correction of the wide-lane and narrowlane ambiguities)

The iono-free, double-difference observations between two receivers (i, j) and two transmitters (k, l) are obtained on the basis of the iono-free, non-differentiated observables according to:

$$\Delta_{ij}^{kl} P_I = \begin{bmatrix} P_I \end{pmatrix}_i^k - P_I \end{bmatrix}_i^l \begin{bmatrix} P_I \end{pmatrix}_j^k - P_I \end{bmatrix}_j^l \qquad \begin{array}{c} \text{Measurement of pseudo-distance} \\ \Delta_{ij}^{kl} L_I = \begin{bmatrix} L_I \end{pmatrix}_i^k - L_I \end{bmatrix}_i^l \begin{bmatrix} -\begin{bmatrix} L_I \end{pmatrix}_j^k - L_I \end{bmatrix}_j^l \qquad \begin{array}{c} \text{Measurement of pseudo-distance} \\ \end{array}$$

The geometric distance between the transmitter antenna (onboard a GNSS satellite) and the receiver antenna is calculated in the same way as all the distance measurements (see section 5.1).

Free parameters

The free parameters that are specific to GNSS measurements are the receiver clock (MNS) and transmitter clock (MNG) parameters, the GPS (MNA), GLONASS (MNR) and Galileo (MNE) ambiguity parameters, and the inter-system bias in the case of hybrid processes (MBI). They are named according to the date, the satellite and the constellation, as shown in Table 9. The partial derivatives of these parameters are equal to 1, except for the satellite clocks, for which -1 is conventionally applied.

Constellation	Rinex code	GINS number of the satellites	Code of the phase ambiguity unknown	Short code (clocks)	3-letter code
GPS	G	666nn (777/888/999)	MNA	GPnn	GPS
Glonass	R	500nn	MNR	GLnn	GLO
Galileo	Е	400nn	MNE	GAnn	GAL

Table 9: codes uses for GNSS observables and unknowns

GNSS data pre-processing program: PRAIRIE



The *PRAIRIE* data pre-processing software prepares the GNSS data for the GINS software. *PRAIRIE* processes the data from one or more files (of any sample rate) from the same receiver in Rinex2 or Rinex3 format and performs the steps described in Table 10.

	Step	Conditions/Details
1.	Reading and selecting the observations.	Default: GPS observations
	Elimination of observations that are unusable at	Option: GLONASS observations, which
	this point (e.g., single-frequency or no code)	require the GLONASS frequency history
		file to be present (historik_glonass)
		Option: Galileo (and/or Giove)
		observations
2.	Correction of millisecond discrepancies between	If any such discrepancies are detected
	the code and phase measurements	
3.	Breakdown of the observations into passages of	The thresholds applied at this point can
	constant ambiguity. The passages and the	be modified in the options file.
	ambiguities are named in this step.	
4.	Sets the WL ambiguities (GPS only)	Requires the MuSatRef.res.dat containing
		the Wide-Lane Satellite Biases (WSB
		file) to be present.
5.	Elimination of observations that are unusable at	The thresholds applied at this point can
	this point (extremities of the passes, overly noisy	be modified in the options file.
	measurements or passs).	
6.	Conversion to GINS input format.	Only the observations that have not been
		eliminated are copied.
		The name of the output file is always
		"sortie_PDGR90".

 Table 10: Key steps in the PRAIRIE GNSS data pre-processing software

The useful parameters can be defined in an options file. The output files of the various receivers can be concatenated to produce a multi-receiver file for the GINS software or for the "double90" program that forms the double-differences, which is described below.

GNSS double-difference formation program: double90

The program uses the files produced by concatenating several files from *PRAIRIE*. The operations to form double-difference measurement files are performed on three-dimensional logic tables (stations x satellites x epochs) that encode the presence or the absence of a non-differentiated measurement.

The double-differences are formed by using these tables, without really forming the observables. The differences are made when writing the data at the end of the program. The first step consists of counting all the double-difference measurements that can be formed on the basis of the non-differentiated data of the \mathbf{n} stations present in the input file for each of the possible baselines. The baselines are sorted in increasing order of potential measurements. The following algorithm is then applied to select the bases that will be formed:

- the first and second base in the list are retained.
- if these two bases share a common station, an identical family number is assigned to them. Otherwise, two different numbers are assigned.
- for each base, a test is run to see whether the two stations belong to families that are already linked and the bases selected according to the various cases are formed: the two stations belong to the same family: no base is formed. One of the two stations belongs to a family, but not the other one, or neither of them belong to a family: the base is kept and the two stations receive the number of the corresponding family. The stations belong to two different families: the base is kept and all the stations in the two families receive a common family number (the families are linked by the new baseline).



• proceed to the next base, and so on to the end of the list.

In this way, a total of (n-1) baselines are created out of the possible n(n-1)/2. This algorithm is partially inspired by the Bernese software manual (version 4.2, chapter 10). The above process guarantees that all the stations belong to at least one of the baselines that are retained. Moreover, all the stations are linked to the same network (see Figure 24). Once the station bases have been chosen, the couples of satellites to be formed are identified in each baseline. The algorithm used to select the couples is similar to the station algorithm described above. The couples of satellites that form the most double-differences are sorted, then the couples (n_{sat} -1) of independent satellites are added to the set of double-differences to be formed.



Figure 24: Example of a network formed by the double90 program. At this point, the formation algorithm prevents the formation of double-differences between stations E and F, which would result in double-differences that are not independent and would make the system of the normal equations of the measurements singular.

After these two steps, the logic tables are used to identify the non-differentiated measurements that have not yet been used. Since the verification of the non-redundancy of the measurements is costly, the measurements of one station are simply re-introduced to the set of unused non-differentiated measurements. The measurement that forms the most duplicates with the remaining measurements is added. The first two steps are then repeated on the remaining set. The output file contains the double-differences that can be used by GINS.

6.2.10. GRACE measurements

Inter-satellite GRACE measurements (Gravity Recovery And Climate Experiment) correspond to the projection onto the chord of the difference in the coordinates of the two GRACE-A and GRACE-B satellites that co-orbit at an altitude of less than 500 km and at a distance of between 170 and 270 km. This measurement, which is made using two bi-directional carriers in frequency bands K (24 GHz) and Ka (32 GHz), is referred to as KBR (K-Band Ranging) and produces a biased distance measurement that must be corrected due to the effects of differential propagation time (of the mm), antenna alignment (of 10 μ m) and ionospheric delay (of the mm), which are supplied in the measurement files.

If (x_A, y_A, z_A) represents the coordinates of satellite A and (x_B, y_B, z_B) represents the coordinates of satellite B, then the distance equation is written as follows:

$$\rho = \sqrt{((x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2)}$$

The measurement files also contain the most widely used derived KBRR (K-Band Range-Rate) measurements:

$$\dot{\rho} = (\dot{x}_B - \dot{x}_A)(x_B - x_A) / \rho + (\dot{y}_B - \dot{y}_A)(y_B - y_A) / \rho + (\dot{z}_B - \dot{z}_A)(z_B - z_A) / \rho$$

and the KBA (K-Band Acceleration):

$$\ddot{\rho} = \sum_{1}^{3} (\dot{x}_{B} - \dot{x}_{A})^{2} / \rho + \sum_{1}^{3} (\ddot{x}_{B} - \ddot{x}_{A}) (x_{B} - x_{A}) / \rho - \dot{\rho}^{2} / \rho$$

The derivatives of the propagation time and antenna alignment corrections must also be applied to the latter two functions.

These various measurement functions can be used together. Bias by period, or even linear or quadratic corrections, can also be applied to them, as well as corrections to the orbital period and to sub-multiples down to one quarter of this period.

6.2.11. Gradiometric measurements (GOCE)

A gradiometer is an instrument made up of two or more accelerometers that are symmetrically distributed around the centre of mass of the satellite (see Figure 25). The sum total of the measured accelerations provides access to the common mode (surface accelerations), while their differences provide access to the components of the volume forces gradient.

The difference between the accelerations is written in the non-inertial reference frame linked to the satellite according to:

$$\vec{m} = \left[\nabla^2 U - \left(\dot{\Omega} + \Omega^2\right)\right] \cdot \vec{L}$$

where $\nabla^2 U$ is the Laplacian of the gravity potential (at the centre of the accelerometers), $\vec{L} = \vec{A_i A_j}$ is the vector between the two accelerometers in question and $\dot{\Omega}$ et Ω^2 are the matrices associated with the components of the rotation vector of the satellite ($\omega_1, \omega_2, \omega_3$), produced by:



Figure 25: Block diagram of a three-axis gradiometer made up of six accelerometers. The distinction is made between the references specific to each accelerometer (ARF = accelerometer reference frame) and the reference of the gradiometer (GRF = gradiometer reference frame).

$$\dot{\Omega} = \begin{pmatrix} 0 & \dot{\omega}_{3} & -\dot{\omega}_{2} \\ -\dot{\omega}_{3} & 0 & \dot{\omega}_{1} \\ \dot{\omega}_{2} & -\dot{\omega}_{1} & 0 \end{pmatrix} \text{ and } \Omega^{2} = \begin{pmatrix} -\omega_{2}^{2} - \omega_{3}^{2} & \omega_{1}\omega_{2} & \omega_{1}\omega_{3} \\ \omega_{1}\omega_{2} & -\omega_{3}^{2} - \omega_{1}^{2} & \omega_{2}\omega_{3} \\ \omega_{1}\omega_{3} & \omega_{2}\omega_{3} & -\omega_{1}^{2} - \omega_{2}^{2} \end{pmatrix}$$

These observations are used in conjunction with the attitude data to restrict the corrections to the components of the tensor of total gravity $\nabla^2 U(t)$.

The a priori gravity tensor is modeled on the basis of all the gravitational models described in section 3.2 (gravity field of the Earth and parts that are variable over time).

In practice, this tensor and its partial derivatives are calculated on each measurement date by the "Obelix" software library. The free parameters are the coefficients of the gravity potential and the calibration parameters of the gradiometer. The GINS software can be used to filter the measurements and the variational equations, so that only the useful spectral band of the instrument is processed (see Figure 26).





Figure 26: Predicted spectral density for onboard accelerometers in the GOCE mission

6.2.12. DSN measurements

There are three types of DSN measurement: two-way Doppler, one-way Doppler and range.

2/3-way Doppler measurements

The signal is transmitted by an Earth station at a frequency f_T (usually in the S or X band) during an interval $[t_{1,s}, t_{1,e}]$. This frequency can be controlled and, therefore, can vary in a linear manner in each time interval, in order to optimize the reception of the final signal on Earth. The signal sent by the station is received onboard the satellite by a transponder that re-transmits the signal for an interval $[t_{2,s}, t_{2,e}]$ after multiplying it by a frequency factor M_2 . The signal is then received by a station on Earth during an interval $[t_{3,s}, t_{3,e}]$, also known as the counting time T_c . If the transmitter and receiver stations are different, the measurement is a 3-way measurement. The frequency received is compared with a reference frequency fT, which is itself multiplied by the frequency factor M_{2R} . Therefore, the measured quantity is expressed as follows:





where corrections are all the propagation corrections in the Earth's atmosphere and in the interplanetary environnement (plasma). The propagation corrections in the solar corona are added to these corrections if the electromagnetic signal passes close to the Sun.

1-way Doppler measurements

In this case, the satellite transmits a frequency (f_{T0}) , which may drift over time with a linear part (f_{T1}) and a quadratic part (f_{T2}) .



Receiving station (R)

$$q_{obs} = C_2 f_{T_0} \frac{(\rho_{1e} - \rho_{1s})}{T_c} - C_2 \left\{ \Delta f_{T_0} + f_{T_1} (t_{2m} - t_0) + f_{T_2} \left[(t_{2m} - t_0)^2 + \frac{1}{12} (T_c^{'})^2 \right] \right\} \frac{T_c^{'}}{T_c} + corrections$$

With $t_{2_m} = \frac{(t_{2e} - t_{2s})}{2}$ and t_0 the reference time of the onboard frequency.

Range measurement

This is the difference between the phase of the transmitted ranging code and the phase of the received ranging code.

$$q_{obs} = \left[\int_{T_B}^{t_3(ST)_R} F(t_3)dt_3 - \int_{T_A}^{t_1(ST)_T} F(t_1)dt_1\right], modulo M + corrections$$

where ST is the station time, and TA and TB are the phase references of the station upon emission and reception. F is the conversion factor of seconds into units of distance, and M is the length of the ranging code.

6.3. Troposphere models

The travel time (section 5.1) is linked to the geometric distance by the speed c of propagation of the optical or radioelectric signals in a vacuum. In practice, the precise modeling of the signal's travel time must incorporate the propagation through the atmosphere, which is not a vacuum and whose refraction index depends on the temperature, pressure and water vapour content. These effects are variable in space and over time according to altitude and meteorological phenomena (Marini 1972, Niells, 1996).



On the Earth, the precise modeling of travel time through the lowest layers of the atmosphere is generally carried out for an altitude up to 80 km. The models include the delay due to the drop in the speed of propagation (relative to the speed of light) and the effects due to the curve in the optical path caused by the heterogeneous nature of the medium. These two effects result in an apparent increase in the travel time relative to the straight line in a vacuum, known as the **tropospheric delay**. The models used in GINS break down the total tropospheric delay (*STD*) by allocating a geodetic measurement seen from an angle of elevation *el* and an azimuth *az* into a sum of two or three terms, which is the general form proposed in the IERS 2010 conventions (Petit and Luzum, 2010).

Two-term model:

$$STD(el, az) = ZTD.mf_{T}(el) + (G_{N}.\cos(az) + G_{E}.\sin(Az)).mf_{G}$$

Three-term model:

$$STD(el, az) = ZHD.mf_{H}(el) + ZWD.mf_{W}(el) + (G_{N}.\cos(az) + G_{E}.\sin(Az))mf_{G}$$

where:

ZHD: zenith hydrostatic delayZWD: zenith wet delayZTD: zenith total delay mf_H : hydrostatic projection function mf_W : wet projection function mf_T : total projection function G_N : North component of the tropospheric gradient G_E : East component of the total tropospheric gradient mf_G : projection function of the total tropospheric gradient

A projection function is associated with each type of zenith delay (hydrostatic and wet). Each projection function model produces a couple of functions for the two zenith components. Models that do not separate the hydrostatic and wet contributions of the zenith delay, modeling the total zenith delay instead, must clearly be associated with a total projection function.

The term associated with the tropospheric gradients also has a specific projection function. It is totally independent of the projection function models associated with the zenith delays. Consequently, the modeling of a tropospheric gradient can be associated with any type of zenith delay model.

As a general rule, the hydrostatic contribution is deterministic in the calculation and is corrected in the observations a priori. Only the wet contribution ZWD is estimated at the same time as the other geodetic parameters. The corresponding partial derivative is mf_W . In modeling processes that do not make the distinction between the hydrostatic and wet components, the partial derivative is naturally mf_T . In three-term models, the absence of ZWD, calculated a priori by the model (e.g. GPT (Boehm, 2007)), imposes the use of an a priori ZWD that is zero.

The projection function associated with the tropospheric gradients is also deterministic. Only the G_N and G_E components of the gradients are estimated. Their a priori value is zero.

Depending on the geodetic technique, the parameters ZWD, G_N and G_E , are not estimated with the same density. For example, it is customary for GNSS measurements to estimate one ZWD per station every 2 hours and one gradient per day. For DORIS, one ZWD is estimated each time a satellite passes over a station (~20 min).



6.4. Ionospheric corrections

The ionosphere is a dispersive medium that is ionized by solar radiation. The electronic content depends on the received solar flux, the time of day, the place in question, the period of the year and the solar cycles. An example of a global TEC (Total Electron Content) map is shown in Figure 27. The electromagnetic signals are affected when crossing this medium by an advance in the phase of the carrier and a delay that depends on the frequency for the code measurements (range).

This effect, which can reach tens of metres in terms of distance for GNSS signals (and up to about 100 metres for periods of intense solar activity), must be corrected in order to make proper use of the geodetic measurements.

provides an illustration of examples of ionospheric bias observed on GPS code and phase measurements.



CODE'S GLOBAL IONOSPHERE MAPS FOR DAY 098, 2006 - 00:00 UT

Figure 27: Example of a global map of the ionosphere calculated on the basis of GPS observations of the Astronomisches Institut Universität Bern (AIUB), source http://www.cx.unibe.ch/aiub/ionosphere.html

Multi-frequency measurements (GNSS, DORIS, Altimetric, VLBI) can be used to correct the ionospheric propagation errors of the first order (bi-frequency) or of the second order (tri-frequency). The iono-free combination between two distance measurements M_a and M_b (on frequencies *a* and *b*) is written as follows:

$$M = (M_a - M_b) \left(\frac{1}{f_a^2} - \frac{1}{f_b^2}\right)^{-1}$$

Effects of the second order, which are not corrected in bi-frequency, are of the order of a few centimetres for the L band of GNSS signals (Petrie et al., 2010).





Seconds (PRN 01)

Figure 28: Example of ionospheric correction observed by GPS measurements of code and phase (inverted correction) for four passages over the POTS GPS station (April 8, 2004). The values from the IRI2001 model (International Reference Ionosphere 2001) are shown by way of comparison (ref = http://nssdc.gsfc.nasa.gov/space/model/ionos/iri.html). The highest noise of the code measurements can also be distinguished in relation to the phase, especially at the start and the end of the passage.

6.5. Correction of the phase centre of the instruments on the ground

6.5.1. Antenna correction

The coordinate of a station provided in the station file is the coordinate of the associated geodetic marker. But the maker usually differs from the **phase centre** of the instrument, which is the geometrical point where the measurement is physically made. The phase centre depends on the type of instrument and the frequency. The integration of data used to calculate the phase centres of ground instruments depends on the type of measurement in question. The phase centre and all the coordinate corrections are calculated in the reference frame linked to the body.



Measurement	Details	Sources for the correction
Laser		
Doppler	There are two types of DORIS antennas (Starec and Alcatel). Each is associated with a phase centre at 2 GHz and 400 MHz	Station file (xxxA /dorb = Alcatel; xxxB/dloc = Starec). The eccentricity in the station file gives the 2 GHz phase
		centre. The 400 MHz offset is hard coded in GINS.
GNSS	Specific corrections according to the azimuth and the elevation of the received signal are applied in addition to the phase centre corrections (see below.)	The model of the GNSS antenna used on the site (and its history) is entered in the station file. Corrections specific to each antenna are entered in the ANTEX file attached in the Director.
VLBI		

Table 11: Sources of station antenna corrections

6.6. Correction of the phase centre of the onboard instruments

<u>6.6.1.</u> Antenna correction

The antenna corrections relating to onboard antennas (transmitters and receivers) are similar to the corrections of ground station antennas. The phase centres of the instruments are entered in the macro-model file for all types: DORIS, altimeter, laser, accelerometer, rare and kbr. The phase centre correction of GNSS antennas is read in the ANTEX file (as for station antennas).

Measurement	Details		
Laser	Phase centre entered in the macro-model file		
Doppler	Phase centre entered in the macro-model file		
GNSS	Corrections specific to each transmitter antenna are entered in the		
	ANTEX file attached in the Director.		
GRACE Kbr	Phase centre entered in the macro-model file		
DSN	Phase centre entered in the macro-model file		
Altimetric	Phase centre entered in the macro-model file		
T-11-12			

 Table 12: sources of onboard antenna corrections

<u>6.6.2.</u> Attitude correction

Like for the calculation of non-gravitational forces, the calculation of the phase centre of onboard instruments takes the attitude of the satellite into consideration (see section 3.4). The phase centre corrections expressed in the satellite reference frame are converted to the integration reference frame by applying the attitude of the satellite on the date when the signal is sent or received.



6.7. GNSS antenna correction

The GNSS antenna file, in ANTEX format, contains the information on the antenna phase corrections based on adjustments or calibration measurements made on the ground using robots (Schimdt, 2007). For each antenna model, it contains the vectors between the centre of reference of the antenna and the phase centre of the measurement for the different useful frequencies, plus the correction maps of the measured quantity according to the azimuth and the elevation of the incoming signal. This file is in ANTEX format, which is described at *igscb.jpl.nasa.gov/igscb/station/general/antex13.txt*. If combinations of measurements between several frequencies are processed, a phase centre correction is applied that depends on the individual phase centres by frequency read in the antenna file. In the case of the iono-free combination between two measurements on the frequencies **a** and **b**, the phase centre of the combined measurement $\overline{\Delta p}_{ab}$ is obtained on the basis of the individual phase centres, according to:

$$\Delta \vec{p}_{cdp}(a,b) = \Delta \vec{p}_a + (\Delta \vec{p}_a - \Delta \vec{p}_b) \frac{1}{\gamma - 1} \quad where: \quad \gamma = \frac{\lambda_2^2}{\lambda_1^2}$$

In addition to the vectorial correction of the phase centre, scalar corrections are also applied (from the antenna correction maps) that depend on the frequency, the type of measurement (code / phase) and the line of sight relative to the antenna (azimuth, elevation):

$$\Delta d_{geom} = \Delta d_{antenne} (azim., elev.)$$

The two above corrections (phase centre and scalar correction) are applied to transmitter antennas (GNSS satellites) and receiver antennas. The make of the antenna is included in the station file for receiver antennas, and the constellation file for transmitter antennas.

The ANTEX file is entered in the environment block of the GINS Director file.

6.7.1. Wind-up correction

For DORIS and GNSS measurements, a so-called phase wind-up correction is also applied, which depends on the relative orientation of the transmitter and receiver antennas (variable in the course of a pass). This correction can reach several cm for GNSS measurements. It is calculated according to the formalism described in (Kouba, 2009).

6.8. Relative curve correction

The relativistic distance correction relating to the curve of the travel due to the gravitational action of the central body is calculated for all the signals propagated in the vicinity of the central body according to (Petit & Luzum, 2010, Chapter 11):

$$d_{relat} = 2 \frac{GM}{c^2} log \left(\frac{r_{2+}r_1 + d_{geom}}{r_{2+}r_1 - d_{geom}} \right)$$

where r_1 and r_2 are the norms of the coordinates of the start and end points of the travel.

6.9. Partial derivatives linked to the satellite

The two main families of parameters associated with the satellite are the geometric parameters (coordinates of the phase centres of onboard instruments, geometric instrument bias) and the dynamic parameters



affecting the force model and, therefore, the position of the satellite over time. The numerical integration returns the derivative of the position (and of the speed) in relation to the dynamic parameters X_i . These integrated partial derivatives are interpolated on the date of the measurement by the onboard instrument (date of emission or reception, depending on the instrument) and linked to the partial derivatives of the measurement according to:

$$\frac{\partial d_{geom}}{\partial X_i}(t) = -\vec{u}.\frac{\overrightarrow{\partial P_{sat}}}{(\partial X_i)(t)}$$

where \vec{u} is the unit vector directed along the line of sight of the measurement.

7. ADJUSTMENT PROCEDURE

7.1. Observation equation (s)

At each iteration, GINS calculates all the *n* measurement residuals R_i obtained by deducting the theoretical quantity of the measurement Q_i from the value of the measurement M_i . A weight (π_i) specified in the measurement block(s) of the **Director** is allocated to each measurement. Therefore, the following is written for each measurement:

$$R_i = M_i - Q_i (X_1, X_2, \dots, X_p)$$
 (π_i) $i = l, n$

The theoretical quantity is a non-linear function of the parameters X_k (k = 1 à ρ) that are used in the calculation. The obtained residuals contain the instrumental noise and the contribution caused by errors modeling the theoretical quantity. The model is refined by adjusting all or part of the parameters entering the calculation of the theoretical quantity by writing the linearization in the first order of the measurement equations according to:

$$Q_i(\vec{X} + \Delta \vec{X}) = Q_i(\vec{X}) + \sum_k \frac{\partial Q_i}{\partial X_k} \Delta X_k \quad (\pi_i) \qquad i = l, n$$

In this case, variable X_k represents the a priori value (or current value) of the parameter, and variable ΔX_k represents the correction to this value. By considering the set of p parameters $\vec{X} = (X_1, X_2, ..., X_p)$ and the residuals of the set of the *n* weighted measurements $\vec{R} = (R_1, R_2, ..., R_n)$, the matrix system of the linear observation equations is built:

$$A_{n,p}\Delta \vec{X} = \vec{R} + \vec{\varepsilon} \qquad (\pi_{n,n})$$

 $\vec{\epsilon}$ is the residual error between $Q_I(\vec{X} + \Delta \vec{X})$ and M_i .

The partial derivatives are calculated with the theoretical quantities in the measurement functions (see section 5). Matrix $A_{n,p}$ is the matrix of partial derivatives (size nxp) and $\pi_{n,n}$ is the matrix of the weights of the measurements. The latter is purely diagonal in the case of independent measurements, but may contain non-diagonal elements in the case of measurements that are correlated with one another (e.g., GNSS double-differences).



7.2. Least square method

<u>7.2.1.</u> General

We look for the parameters correction vector $\Delta \vec{X}$ which minimizes the residual error $\vec{\epsilon}$. Litterature contains several methods to resolve this type of equation system. GINS uses the conventional least square method. With this method it can be shown that the amount $A^T \pi \vec{\epsilon}$ equals the null vector.

Multiplying the observation equations by $A^T \pi$ on both sides allows to build the normal equations:

$$A^t \pi A \ \Delta \vec{X} = A^t \pi \vec{R}$$

In "degraded" cases, in which the system is under-determined (due to a low number of measurements in relation to the parameters in question, or if certain parameters are correlated with one another, or in order to take account of gaps in the measurements), it may be useful to add a set of equations of n_{cont} equations of constraints applying to all or part of the parameters, that takes the following general form:

$$C_{P,P}\Delta \vec{X} = 0$$

The matrix of the constraints on the parameters $C_{P,P}$, which may not be diagonal, is added to the normal equations to form the normal constrained system:

$$(A^t \pi A + C) \Delta \vec{X} = A^t \pi \vec{R}$$

The term on the right $(\vec{D} = A^t \pi \vec{R})$ is called the **second member** of the normal equation and the matrix N = $(A^t \pi A + C)$ is called the **normal matrix**, which is said to be constrained if the matrix *C* is non-zero.

The a priori variance of the residuals (before resolution) is:

$$\sigma^2 = \frac{\vec{R}^T \pi \vec{R}}{n}$$

7.2.2. Practical aspects in GINS

Only the residuals, and therefore the second member, are recalculated between two successive iterations. On the contrary, the normal matrix, whose calculation is costly, remains unchanged between the iterations. The hypothesis is justified in most of the cases encountered.

For certain problems, the number of observations and parameters is such that the calculation times become excessive, as does the amount of memory required to store the partial derivatives. This is the reason why more efficient methods have been developed to store and calculate the matrix of the observation A and of the normal equation N.

7.3. Resolution

The normal (constraint) equations are resolved by inversion according to:

$$\Delta \vec{X} = (A^t \pi A + C)^{-1} A^t \pi \vec{R}$$

The residuals and their a posteriori variance are thus obtained:



$$\vec{R}' = \vec{R} - A\Delta \vec{X}$$
$$\sigma'^2 = \frac{\overrightarrow{R'}^T \pi \overrightarrow{R'}}{n - n + n_{\text{const}}}$$

the a posteriori variance can be computed from the a priori variance, D and N through the formula:

$$\sigma'^{2} = \frac{n \cdot \sigma^{2} - \Delta \vec{X}^{T} \cdot D}{(n - p + n_{cont})}$$

The formal uncertainties are given by the diagonal terms of the variance-covariance matrix:

$$C_{OV} = \sigma^{\prime 2} (A^t \pi A + C)^{-1}$$

7.4. Iteration (s)

In the first iteration the software calculates the residuals and the partial derivatives of each of the measurements. The parameters are initialized at their a priori value, which depends on the selected model. The a priori values of the parameters can be changed by the user.

Certain measurements are excluded according to the selected elimination criteria (minimum elevation, minimum number of measurements per pass, residuals above a given threshold). Then the system of normal equations is built for the first time and inverted.

In each iteration, the obtained correction $\Delta \vec{X}$ is added to the current value \vec{X} , which is taken into consideration in the calculation of the theoretical quantities and of the residuals that are recalculated in the following iterations. The normal system is not completely recalculated in each iteration (the linearization obtained in the first iteration is assumed to be sufficiently correct), but in the course of the iterations, the contribution of the eliminated measurements is deducted from the system of normal equations.

The iterations continue until convergence is reached when the variation of the global residuals drops below the convergence criterion ε_{conv} according to:

$$\frac{\left(\left[\sum_{n} R_{i}^{2}\right]_{iter} - \left[\sum_{n} R_{i}^{2}\right]_{iter-1}\right)}{\left[\sum_{n} R_{i}^{2}\right]_{iter-1} < \varepsilon_{conv}}$$

The maximum number of iterations and the convergence criterion are selected by the user.

The final value of the parameters, which is the sum of the a priori values and the successive corrections obtained in the iterations, is used for the additional iteration.

If an additional iteration is decided on, the residuals of the retained measurements and the partial derivatives are recalculated. The normal system **without constraints** is rebuilt and output in the "EQNA" file ready for use by the programs in the DYNAMO chain.

8. GINS OUTPUTS

The main output of the GINS software is the listing that contains the screen output of the software. The most useful information includes the global or detailed statistics on the measurement residuals, the adjusted values of the parameters, the characteristics of the input models and the Director used. In addition to this output, it is also possible to ask for specific output in the form of independent files. More details about these outputs are included below.

8.1. Parameters

The distinction is made between the **dynamic parameters** that affect the calculation of the forces acting on the satellite(s) are integrated with the movement equation (see Table 13), and the **geometric and measurement parameters** (see Table 14). The adjusted parameters from GINS and the DYNAMO chain or any other source can also be used as input for GINS in order to impose the a priori values. There are two formats of a priori values: the **a priori value** format that gives the values and the sigmas of the parameters directly, and the **constraints** format (more general) that is used to specify the equations of linear constraints between one or more parameters.

The first letter "G" shows the group to which the unknown belongs. The letters "g g" indicate its name in the group. The fourth character is always left blank. The last four characters usually show the name of the satellite. The other "?" characters are used to encode details about the parameter, such as the degree and order for harmonic coefficients, the date of dated coefficients and the numbers or codes of the stations for measurement parameters.

Figure 29: General principles applying to the 24-character naming signaletic element



	First letter of	Dynamic parameters
	the signaletic	
	element	
Orbital elements	Е	Position and initial velocity (angular), keplerian or other
		elements.
Multiplying factors of	F	Factor of the atmospheric drag force (FD), of the
non-gravitational		atmospheric lift, solar pressure force factor (FS), albedo,
accelerations		thermal thrust.
Empirical accelerations	В	Bias and periodical acceleration in the local orbital
		reference frame (RTN) or in the satellite reference frame
		(XYZ).
Thrust parameters	Y	Stochastic empirical acceleration parameters, constant per
		segment or at specific dates (manoeuvres, eclipses).
Accelerometers	Х	Bias, scale factor and eccentricity of the onboard
		accelerometers and their dependancy with temperature or
		temperature drift.
Optical parameters	R	Thermo-optical parameters characterizing the different
		surfaces of the satellite.
Atmosphere	А	Temperature and density parameters of the components of
		the atmosphere.
Gravity	G	Spherical harmonic coefficients of the development of the
		gravity potential (C_{lm}/S_{lm}) and their temporal drift.
Oceanic	0	Spherical harmonic coefficients of the development of the
		different tide waves. Inverse barometer response of the
		ocean at various frequencies.

 Table 13: The families of dynamic parameters

Family	First letter of the signaletic	Geometric parameters
Cantra of maga	c	Economicity and control of mass vectors of the
Centre of mass	C	Eccentricity and centre of mass vectors of the
		ondoard instruments.
EOP	P	Earth rotation parameters.
		Coordinates of the pole (PX/PY) and UT1 (PT).
Nutation	Ν	Nutation parameters (NP/NE or NX/NY).
Love number	L	Love numbers associated with the loading
		deformation of the Earth.
Stations	S	Coordinates and drift in the coordinates of the
		stations (XYZ or Phi/Lambda/h).
Dynamic topography	Т	Spherical harmonic coefficients of the dynamic
		topography of the oceans.
Measurements	М	Any measurement parameters (bias and
		instrumental drift, clocks, ambiguities, dating bias
		and tropospheric gradients).
Quasar	Q	Coordinates (right ascension and declination) of
		extragalactic radio sources.

 Table 14: The families of geometric parameters

The ephemerid file contains the tabulated orbits of all the satellites and their possible extrapolation (beyond the duration of the arc). Various formats are possible (see *orbite_info*), some of which are "historical". The conventional format contains the positions and speeds over time in both the integration reference frame and the reference frame linked to the central body, plus the accelerations in the integration reference frame. Depending on the output options, it is possible to ask for an ephemerid output containing the details of each of the forces or the gravitational potential and its derivatives.

The ephemerid file is a multi-satellite file containing all the ephemerids that are adjusted in the course of the process. It can be input into GINS as an "ephemerid" measurement or as a bulletin source. A number of conversion utilities can be used to convert the internal formats into international exchange formats, such as sp3. Two different realizations of the same ephemerid can be compared and graphically displayed using the **ov** (orbito_visu) software. An example of output is shown in Figure 30.



Figure 30: Example of ov software output (comparison and visualization of ephemerids). Overlap (12hrs) of GPS and Giove orbits (data from December 2008). According to G. Bracher (2011).

8.3. GNSS clocks

The clock file contains the value of the $c\Delta h$ clocks converted in metres for each date and any GNSS transmitter or receiver. When processing GNSS data, a file of this type must be attached in the **Director** header to associate the observations with the clocks of the GNSS satellites. These clocks are used to calculate the date and therefore must be absolute (i.e. referenced according to TAI). If they are not part of the estimated parameters, the precision of clock corrections directly impacts the solution. The *hocomp* software is used to graphically display and/or compare two sets of clocks. It is used to:



- View the clocks themselves (satellites or stations) and to compile statistics after removing a drift.
- Compare the two sets of clocks after alignment(*) and graphically display the differences of the station and satellite clocks according to several alignment methods.
- Correct from narrow lane integers: one set of day D+1 compared with D day.
- Compare two realizations from different clocks (e.g. time transfer).
- •

^(*) The clock alignment operation is used to calculate a "mean" clock between two different sets of clocks and, possibly, to subtract this "mean" clock. It is also used to switch from a set of relative clocks to a set of absolute clocks. In the GINS software, the GNSS clocks are defined relative to a reference clock, which is forced to zero, otherwise the inversion problem is singular:

 $H(t) - h_0(t)$, (satellites) $h(t) - h_0(t)$ (stations)

The alignment operation uses the least square method to calculate the clock (which may be linear) that minimizes the differences between the clocks produced by GINS and a set of reference clocks based on T_{GPS} (or TAI), such as the BRDC (Broadcast Ephemeris) clocks. This clock is then subtracted from all the clocks of the initial set.



Figure 31: Clocks of a number of GPS stations (1point / 30 seconds). Values in metres ($c\Delta h$).

8.4. Measurement residues / Statistics file

The **statistics** output is a file containing the individual measurement residuals (one record per measurement). It also includes the information used to produce particular statistics or graphical representations. For example, it can be used to track the residuals along the satellite track, as a function of time or elevation.

The format of the statistics file, which is the same for all the measurements, except KBR, is shown in Table 15.



Field (Fortran	Content	Unit
notation)		
I3	Type of measurement	Without units
1X,E16.10	Date	Julian 50 day
1X,E9.3	Weight	Unit according to the type of measurement
1X,F15.9	Residual	(m, mm/s, seconds, etc.)
1X,A8	Satellite code	Name of the satellite
1X, F8.3	Latitude of the satellite	Degrees
1X, F8.3	(trace on the ground) Longitude of the satellite	Degrees
I8	Station number	Without units
1X, F8.3	Measurement azimuth	Degrees
1X, F8.3	Measurement elevation	Degrees
1X, F12.3	Val1	Fields specific to the type of
1X, F9.3	Val2	measurement
1X, F9.3	Val3	

Fable 15: Format and	l content of	the statistics file
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8.5. Normal equation

The normal equation (see section 6.2) is an essential output of GINS. A normal equation (or rather a system of normal equations) is a set of p linear equations linking p unknowns that can be written in the form of a matrix. The normal matrix by construction is symmetrical definite positive.

<u>8.5.1.</u> Format

The normal equation can be saved on demand or in the additional iteration. In practice, it is written in **binary** format and contains:

- comments,
- statistical elements (for example, the number of measurements used per satellite),
- the number of parameters and the list of signaletic elements (see section 7.1),
- the a priori sigma2 (defined in section 6.2),
- the a priori values of the parameters: $\overrightarrow{X_o}$
- the second member of the normal equation: \vec{D}
- the normal matrix itself: \vec{N}

The whole represents the normal equation $N \overrightarrow{\Delta X} = \overrightarrow{D}$ associated with the a priori values \overrightarrow{X}_o .

These equations can be read and used by the various programs (or modules) in the **DYNAMO** package or converted into the SINEX international exchange format (Rothacher & Thaller, 2006) using the **Sinextool** utility.

<u>8.5.2.</u> DYNAMO

The **DYNAMO** chain is a serie of programs that use normal equations. They can be used to perform usual linear algebra operations. The main components of the chain are listed in Table 16 and described in detail below.



The general principle of these tools consists in performing operations on all or part of the parameters (or unknowns) of the normal equations. A powerful tool that is common to all the components of the chain has been developed in order to quickly select the parameters to which the operations are to be applied on the basis of their name. In addition to specific output, the set of modules also produces a listing to keep track of the proper execution of the operations.

The normal equations can be generated by GINS or produce otherwise.

One essential tool of the DYNAMO chain is used to generate so-called **constraint** equations that can be added to the normal equations in order to resolve them, to take account of a realistic physical constraint (for example, constraints of continuity between temporal parameters) or to add information from observations that are not processed by GINS (e.g., ground gravimetric data or measurements linking tracking stations obtained by a leveling process). The tool used to generate constraint equations (exe_genere_equation.ksh) builds the normal equations, on the basis of observations provided in a **constraints file**, in schematic form:

$$C_1[ELS \ 1] + C_2[ELS \ 1] + \dots = C_0 \pm \sigma$$

where [ELS i] are signaletic elements encoded on 24 characters (see section 7.1).

DYNAMO_D: resolution of a normal equation

This module is used to resolve normal equations using the following methods:

- inversion by the Cholesky method
- inversion by the conjugate gradients method
- inversion by the specific values and vectors method

Resolving the equation consists in inverting the normal matrix (with possible constraints) in order to obtain the solutions:

$$\vec{X} = \vec{X}_0 + \Delta X = \vec{X}_0 + N^{-1}\vec{D}$$

The a posteriori variance of the residuals is obtained from the a priori variance by :

$$\sigma_{post}^2 = \sigma_{pri}^2 - \frac{\vec{D}^T \cdot \Delta \vec{X}}{n-p}$$

The output consists of a file containing the solutions, possibly with the variances or the complete matrix of the covariances of the parameters (according to the selected inversion method). Before the inversion, it is possible to add a predefined constraint equation: Kaula's law for the coefficients of the gravity field, the minimum constraints for station network solutions or a set of constraint values specified by the user (see DYNAMO_C below).

DYNAMO_B: reduction of normal equations

By reducing a normal equation, only the useful parameters in the equation are retained. This operation is essential when working with equations containing a very high number of parameters to be determined by combining the observations over several months or years (for example, the coordinates of the stations or the gravity field). It consists in excluding from the equation those parameters that do not need to be solved (e.g., the measurement parameters). The parameters can either be completely eliminated (fixed to their initial values and ignored) or reduced (resolved and reinjected in the normal equation system). Reduction requires the sub-block of reduced parameters to be inverted and constraint equations can be added.

Once the parameters have been chosen, this module breaks down the initial normal equations (which may be combined with a constraint) $N\overrightarrow{\Delta X} = D$ into three blocks: reduced parameters, conserved or external parameters and eliminated parameters, according to:



$$\begin{pmatrix} N_{rr} & N_{rc}^t & N_{re}^t \\ N_{rc} & N_{cc} & N_{ce}^t \\ N_{re} & N_{ce} & N_{ee} \end{pmatrix} \begin{pmatrix} \Delta X_r \\ \Delta X_c \\ \Delta X_e \end{pmatrix} = \begin{pmatrix} D_r \\ D_c \\ D_e \end{pmatrix}$$

The blocks relating to the eliminated parameters are simply deleted:

$$\begin{pmatrix} N_{rr} & N_{rc}^t \\ N_{rc} & N_{cc} \end{pmatrix} \begin{pmatrix} \Delta X_r \\ \Delta X_c \end{pmatrix} = \begin{pmatrix} D_r \\ D_c \end{pmatrix}$$

This system is then reduced to only the "c" parameters, according to:

$$(N_{cc} - N_{rc}N_{rr}^{-1})\Delta X_{c} = D_{r} - N_{rr}^{-1}D_{c}$$

or $N^* \overrightarrow{\Delta X_c} = \overrightarrow{D}^*$

where $\sigma^{*2} = \sigma^2 - \frac{D_r^T N_{rr}^{-1} D_r}{n - p_r}$.

The program output is the normal equation restricted to only the conserved parameters.

DYNAMO_C: combination of normal equations

This module is used to combine several equations (which may be weighted) in a single equation by summing the various contributions on the common parameters. For two σ^2 equations σ_1 and σ_2 , weighted by p_1 and p_2 , this module performs the following operations:

$$\frac{p_1 \times (N_1 \Delta X = D_1)}{p_2 \times (N_2 \Delta X = D_2)}$$
$$\frac{p_1 N_1 + p_2 N_2 \Delta X = (p_1 D_1 + p_2 D_2)}{(p_1 N_1 + p_2 N_2) \Delta X = (p_1 D_1 + p_2 D_2)}$$

where $\sigma^2 = \sum_i p_i \sigma_i^2$

The order of the common parameters must be identical in the two equations. A permutation of the unknowns using DYNAMO_C may be needed.

DYNAMO_P: permutation of a normal equation

This module places the unknowns of a normal equation in a predefined order by permuting the order of the lines and columns of the matrix and of the second member. After the permutation, if two or more identical unknowns (i.e. identical signaletic elements) are detected, then they are compacted into a single unknown. Unknowns with different names can be renamed with identical names so that they are considered as a single unknown in the compaction stage.

DYNAMO_W: search for optimal weighting

It is often needed to combine normal equations derived from observations of different tracking systems (e.g. SLR, DORIS, GNSS, etc.) of different nature (altimeter, gradiometer, gravimeter) of different precision, etc... The search for the optimal weighting aims to estimate the weight of each set in order to obtain the optimal combination of the various measurements, thereby producing the most accurate solution of the parameters to be determined. Helmert's method (Sahin et al, 1992) is an iterative method used to search for the optimal weighting between different equations. Starting with an approximate initial weighting, the method achieves compatibility between the a priori weighting and the residuals *after* the global resolution of the parameters of the problem.



DYNAMO module	Role	Options for all or part	Program name (1)
name		of the parameters	
DYNAMO_B	Reduction of a normal equation	Conservation (EXT), elimination (ELI), or reduction (RED) (2)	exe_dynamo_b.ksh (3)
DYNAMO_C	Combination of normal equations	(2)	exe_dynamo_c.ksh (3)
DYNAMO_D	Resolution of a normal equation by Cholesky or conjugate gradients	Resolution (RES) or not (FIX) (2)	exe_dynamo_d.ksh (3)
DYNAMO_P	Permutation of a normal equation	Compaction of identical unknowns (2)	exe_dynamo_p.ksh (3)
VERIF	Verification of a normal equation	-	exe_verif.ksh
DYNAMO_W	Search for weighting by Helmert's method	Limitation of the test to the important parameters only	exe_dynamo_w.ksh (3)
GENERE_EQUATION	Generation of a normal equation on the basis of a "constraint" type file	Creation of parameters and the associated linear equations	exe_genere_equation.ksh

(1) All these commands have an online help function that can be accessed using the "-help" argument.

(2) It is also possible to rename the parameters and modify their a priori value for all these modules.

(3) A Director file specifying the operations to be performed is necessary for these modules.

Table 16: Main components of the DYNAMO program chain

9. SUPPLEMENTS TO THE ALGORITHMIC DOCUMENTION

Algorithmic documentation of the Obelix forces library, version L20, 2012 User documentation of the Obelix forces library (2012 version). Using external macro-models in GINS (2011). Processing parabolas in GINS (2011) GINS: description of the Director (upated with each new software version).

10. REFERENCES

Altamimi Z, Collilieux X, Métivier L (2011) ITRF2008: an improved solution of the International Terrestrial Reference Frame. Journal of Geodesy, doi: 10.1007/s00190-011-0444-4.

Barriot J.P (1988) Méthodes d'intégration numériques de Cowell (Systèmes du premier et deuxième ordre) : Théorie et implémentation. B.G.I, Techn. Rep. n°9.

Berger C, Biancale R, Ill M, Barlier F (1998) Improvement of the empirical thermo-spheric model dtm: DTM94 - a comparative review of various temporal variations and prospects in space geodesy applications, J of Geod 72:161-178.
Boehm J, Niell A, Tregoning P, Schuh H (2006) The Global Mapping Function (GMF): A new empirical mapping function based on numerical weather model data. Geophys Res Lett 33(L07304). doi:10.1029/2005GL025546.

Boehm J.R, Heinkelmann R, Schuh H (2007) Shrot Note: a global model of pressure and temperature for geodetic applications. J Geod doi:10.1007/s00190-007-0135-3.

Brouwer D., G. M. Clemence (1961) Methods of Celestial Mechanics, Academic Press, New York and London.

Cerri L, Ferrage P (2012) DORIS satellites models implemented in POE processing, Document SALP (<u>ftp.idsDORIS.org/pub/ids/satellites/DORISSatelliteModels.pdf</u>, (ref. SALP-NT-BORD-OP-16137-CN).

Chelton, D.B., J. Ries, B. Haines, L.-L. Fu, and P.Callahan (2001) Satellite altimetry. In "Satellite Altimetry and Earth Sciences: A Handbook forTechniques and Applications" edited by L.-L. Fu and A. Cazenave, pp. 1-131, Academic Press, San Diego, 423 pp.

Cheng G, Herring T.A (1997) Effects of atmospheric azimuthal asymmetry on the analysis of space geodetic data, J Geophys Res 102(B9):20489-20502.

DeMets Ch, Gordon R.G, Argus D.F, Stein S (1994) Effect of recent revisions to the geomagnetic reversal time scale on estimates of current plate motions, Geophys. Res. Lett., 21(20):2191-2194, doi:10.1029/94gl02118.

Desai S. D (2002) Observing the pole tide with satellite altimetry, J Geophys Res, 107, doi:10.1029/2001JC001224.

Fienga A, Laskar J, Kuchynka P, Manche H, Desvignes G, Gastineau M, Cognard I, Theureau G (2011) The INPOP10a planetary ephemeris and its applications in fundamental physics. Celestial Mechanics and Dynamical Astronomy, Volume 111, Issue 3, pp.363-385 doi:10.1007/s10569-011-9377-8.

Föerste Ch (2002) Format Description: The CHAMP Data Format. GFZ Postdam. http://op.gfz-potsdam.de/champ/docs_CHAMP/CH-GFZ-FD-001.pdf.

Guo et Langley (2003) A new tropospheric propoagation delay mapping for elevation angles down to 2deg. Proceedings of the ION 2003 meeting, Portland, Oregon Sept 9-12.

Hamilton (1830) Elements of quaternions (vol. I et II) Chelsea Publ. Comp., New York.

Hedin A (1987) MSIS-86 Thermospheric Model, Journal of Geophysical Research, Volume 92, No. A5, pp. 4649-4662, 1987, doi:10.1029/JA092iA05p04649.

Hopfield H.S (1969) Two quartic tropospheric refractivity profile for correcting satellite data. J Geophys Res,74:4487-4499.

Jackson J.D. (1975) Classical Electrodynamics. John Wiley & Sons, p 517, New York.

Kouba J (2008) A simplified yaw-attitude model for eclipsing GPS satellites, GPS solutions 13:1-12. doi:10.1007/s10291-008-0092-1.

Kouba J (2009) A Guide To using International GNSS Service (IGS) products. igscb.jpl.nasa.gov/igscb/resource/pubs/UsingIGSProductsVer21.pdf. Accessed 10 January 2011.

Laurichesse D, Mercier F, Berthias JP, Broca P, Cerri L (2009) Integer Ambiguity Resolution on Undifferenced GPS Phase Measurements and its Application to PPP and Satellite Precise Orbit Determination. Navig J Inst of Navig 56(2):135-149. Issn: 00281522.

Loyer S, Perosanz F, Mercier F, Capdeville H, Marty J.C (2012) Zero-difference GPS ambiguity resolution at CNES-CLS IGS Analysis Center, J of Geod, published on line (April 2012). doi: 10.1007/s00190-012-0559-2.

Marini J.W (1972) *Correction of satellite tracking data for an arbitray tropospheric profile. Radio Science* 7(2):223-231.

McCarthy DD, Petit G (2004) IERS Conventions 2003, IERS Technical Note 32, Verlag des Bundesamts für Kartographie und Geodäsie Frankfurt am Main.

Mendes V. Band Langley R.B (1998) Tropospheric Zenith Delay Prediction Accuracy for Airborne GPS High-Precision Positioning, Proc Inst Navig, 54th Annual Meeting, Denver (CO/USA) June 1-3 337-347.

<u>Nothnagel</u> A (2009) <u>Conventions on thermal expansion modelling of radio telescopes for geodetic and astrometric VLBI, Journal of Geodesy, Volume 83, Number 8, Pages 787-79. doi:10.1007/s00190-008-0284-z.</u>

Petit G, Luzum B (2010), IERS Conventions 2010, IERS Technical Note 36, Verlag des Bundesamts für Kartographie und Geodäsie Frankfurt am Main , 179 pp., ISBN 3-89888-989-6.

Petrie EJ, Hernández-Pajares M, Spalla P, Moore P, King MA (2010) A Review of Higher Order Ionospheric Refraction Effects on Dual Frequency GPS, Surv Geophys 32(3):197-253. doi:10.1007/s10712-010-9105-z.

Rothacher M, Thaller D (2006) SINEX - Solution (Software/technique) INdependent EXchange Format Version 2.02. Available at www.iers.org/IERS/EN/Organization/Analysis Coordinator/SinexFormat/sinex.html.

Saastamoinen J (1972) Atmospheric correction for the troposphere and stratosphere in radio ranging of satellites. The use of artificial satellites for geodesy, Geophy. Mono Ser 15:247-251, AGU.

Sahin M, Cross PA, Sellers PC Variance component estimation applied to satellite laser ranging. Bulletin Géodésique (1992) Volume: 66 Issue: 3 Pages: 284-295.

Schmid R, Steigenberger R, Gendt G, Ge M, Rothacher M (2007) Generation of a consistent absolute phase center correction model for GPS receiver and satellite antennas. J Geod 81(12):781-798. doi:10.1007/s00190-007-0148-y.

Standish E (1998) JPL planetary and lunar ephemerides DE405/LE405, Interoffice Memorandum IOM 312.F-98-048, Jet Propulsion Laboratory, Pasadena.



Touboul P, Foulon B, Willemenot E (2000) Electrostatic space accelerometers for present and future mission, Acta Astronautica, 45(10):605-617. doi: 10.1016/s0094-5765(99)00132-0.

11. ACRONYMS

CNES: Centre National d'Etudes Spatiales (French Space Agency) **CRS:** Celestial Reference System DORIS: Doppler Orbitography and Radiopositioning Integrated by Satellite DSN: Deep Space Network **EOP: Earth Orientation Parameters** EQNA: Equations normales (normal equations) ELS: Eléments signaletic elementalétiques (parameter names) ESA: European Space Agency GINS: Géodésie par Intégrations Numériques Simultanées (geodesy by simultaneous numerical integrations) **GNSS:** Global Navigation Satellite System GOCE: Gravity field and steady-state Ocean Circulation Explorer GPS: Global Positioning System **GRACE:** Gravity Recovery And Climate Experiment GRGS: Groupe de Recherche de Géodésie Spatiale (space geodesy research group) IAU: International Astronomical Union **ICRF:** International Celestial Reference Frame **IERS:** International Earth Rotation Service ITRF: International Terrestrial Reference Frame LEO: Low Earth Orbiter NRO: Non-Rotating Origin PRARE: Precise Range And Range-rate Equipment SLR: Satellite Laser Ranging SOFA: Standards Of Fundamental Astronomy IAT: International Atomic Time UTC: Coordinated Universal Time **BDT: Barycentric Dynamical Time TRS:** Terrestrial Reference System VLBI: Very Long Base Interferometry