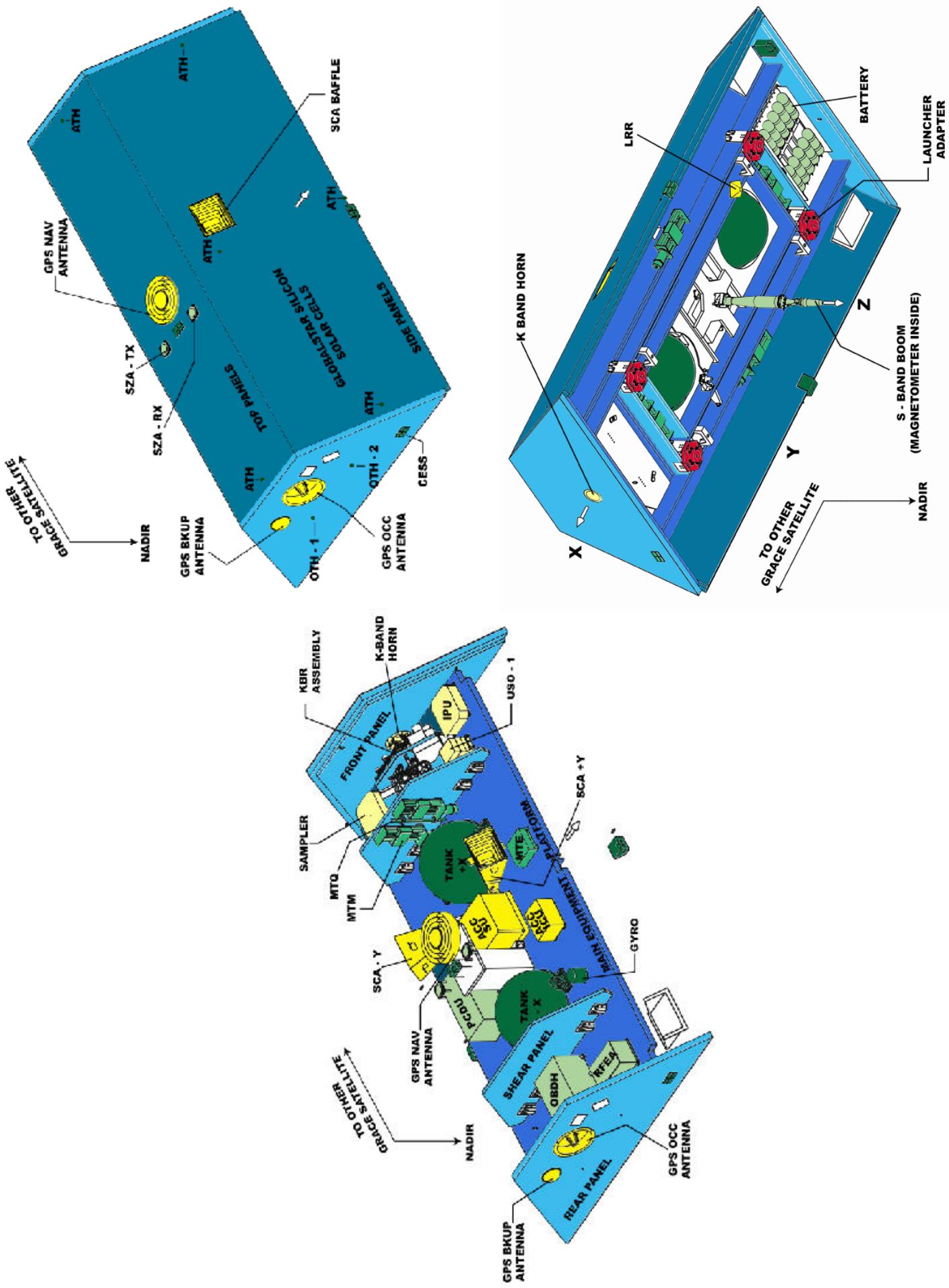
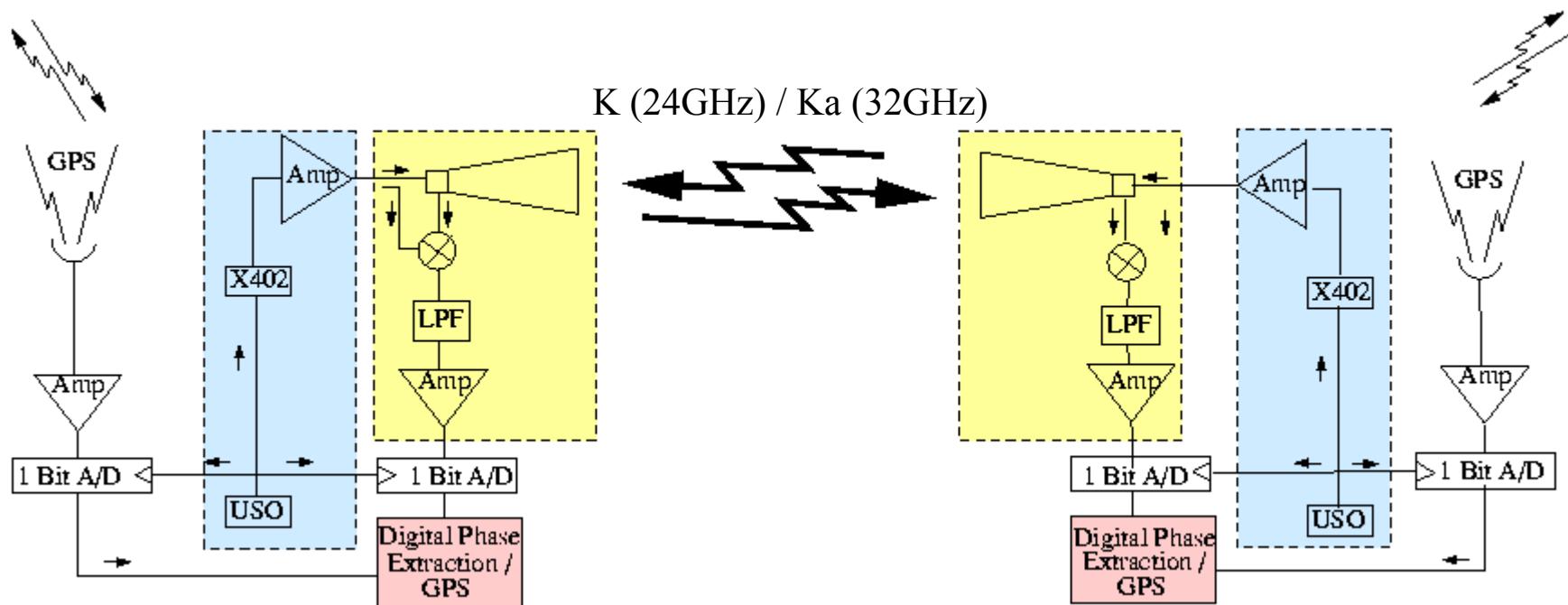


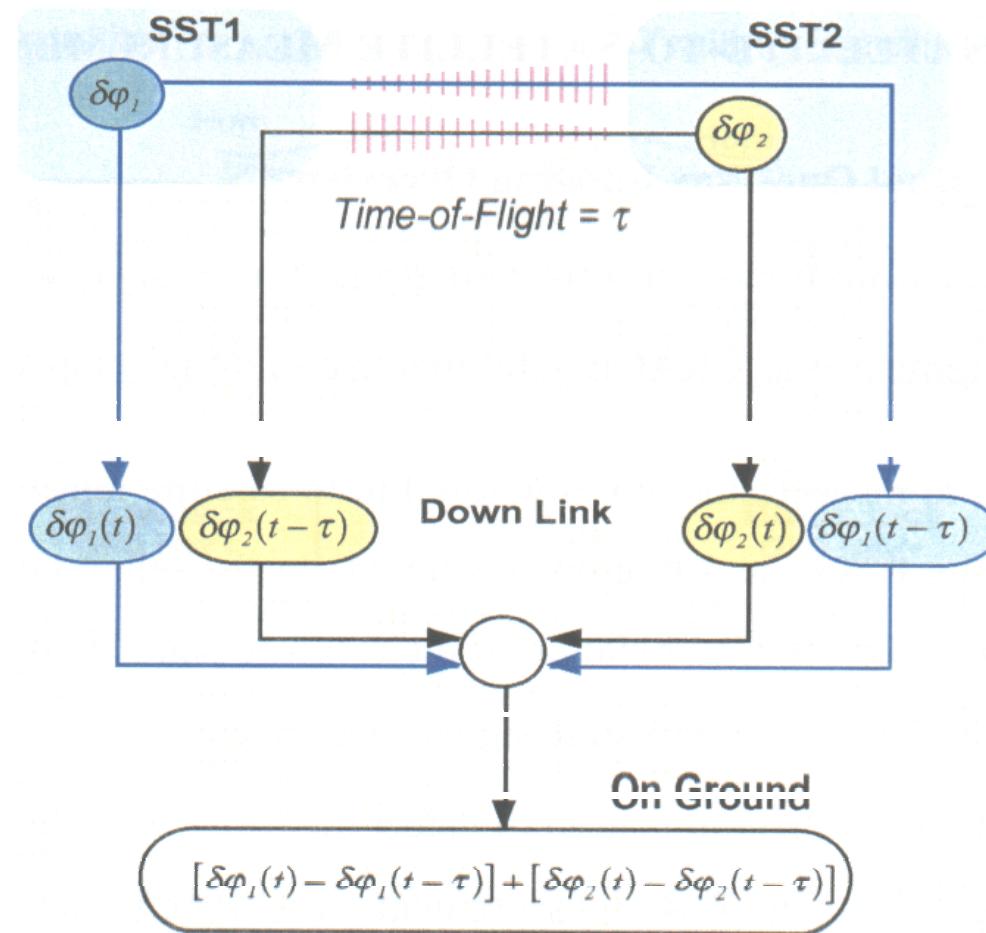
News: lancé le 17 mars 2002 à 9h21mn TUC depuis Plesetsk sur une orbite à 500 km d'altitude



Microwave ranging system

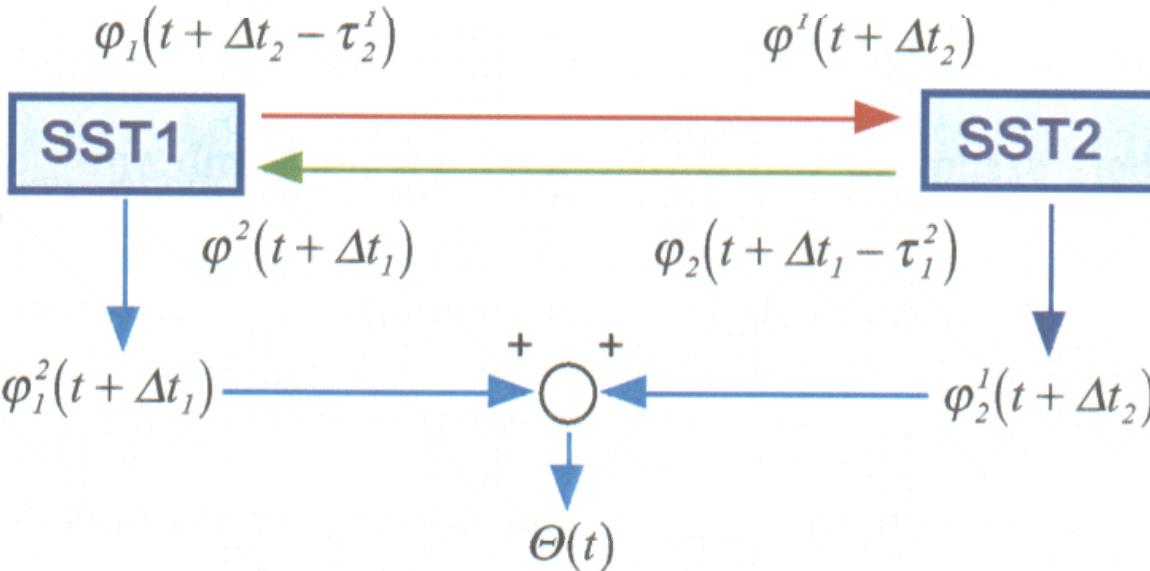


Dual one-way ranging principle Illustration of the oscillator noise reduction



Dual one-way ranging formulation

Transmit phase:



Dual one-way ranging phase measurement :

$$\begin{aligned}
 \Theta(t) &= \varphi_1^2(t + \Delta t_1) + \varphi_2^1(t + \Delta t_2) \quad t : \text{nominal reception time}, \Delta t : \text{time tag error} \\
 &= \bar{\varphi}_1(t + \Delta t_1) - \bar{\varphi}_2(t + \Delta t_1 - \tau_1^2) + \bar{\varphi}_2(t + \Delta t_2) - \bar{\varphi}_1(t + \Delta t_2 - \tau_2^1) \quad \text{reference frequency} \\
 &\quad + \delta\varphi_1(t + \Delta t_1) - \delta\varphi_2(t + \Delta t_1 - \tau_1^2) + \delta\varphi_2(t + \Delta t_2) - \delta\varphi_1(t + \Delta t_2 - \tau_2^1) \quad \text{phase error} \\
 &\quad + (N_1^2 + N_2^1) + (I_1^2 + I_2^1) + (d_1^2 + d_2^1) + (\varepsilon_1^2 + \varepsilon_2^1) \quad \text{integer ambiguities, iono phase shift,} \\
 &\quad \text{instrument offset, multipath, noise}
 \end{aligned}$$

Phases and phase errors can be linearized as follows:

$$\bar{\varphi}_i(t + \Delta t_i) \approx \bar{\varphi}_i(t) + \dot{\bar{\varphi}}_i(t) \Delta t_i = \bar{\varphi}_i(t) + f_i \Delta t_i$$

$$\bar{\varphi}_i(t + \Delta t_j - \tau_j^i) \approx \bar{\varphi}_i(t) + \dot{\bar{\varphi}}_i(t) \Delta t_j - \dot{\bar{\varphi}}_i(t) \tau_j^i = \bar{\varphi}_i(t) + f_i \Delta t_j - f_i \tau_j^i$$

$$\delta\varphi_i(t + \Delta t_i) \approx \delta\varphi_i(t) + \delta f_i \Delta t_i$$

$$\delta\varphi_i(t + \Delta t_j - \tau_j^i) \approx \delta\varphi_i(t) + \delta f_i \Delta t_j - \delta f_i \tau_j^i$$

They cancel after combining the two phase signals and the dual one-way ranging phase measurement becomes a function of the time-of-flight and other error terms:

$$\begin{aligned} \Theta(t) &= (f_1 \tau_2^1 + f_2 \tau_1^2) + (\delta f_1 \tau_2^1 + \delta f_2 \tau_1^2) \\ &\quad + (f_1 - f_2)(\Delta t_1 - \Delta t_2) + (\delta f_1 - \delta f_2)(\Delta t_1 - \Delta t_2) \\ &\quad + (N_1^2 + N_2^1) + (I_1^2 + I_2^1) + (d_1^2 + d_2^1) + (\varepsilon_1^2 + \varepsilon_2^1) \end{aligned}$$

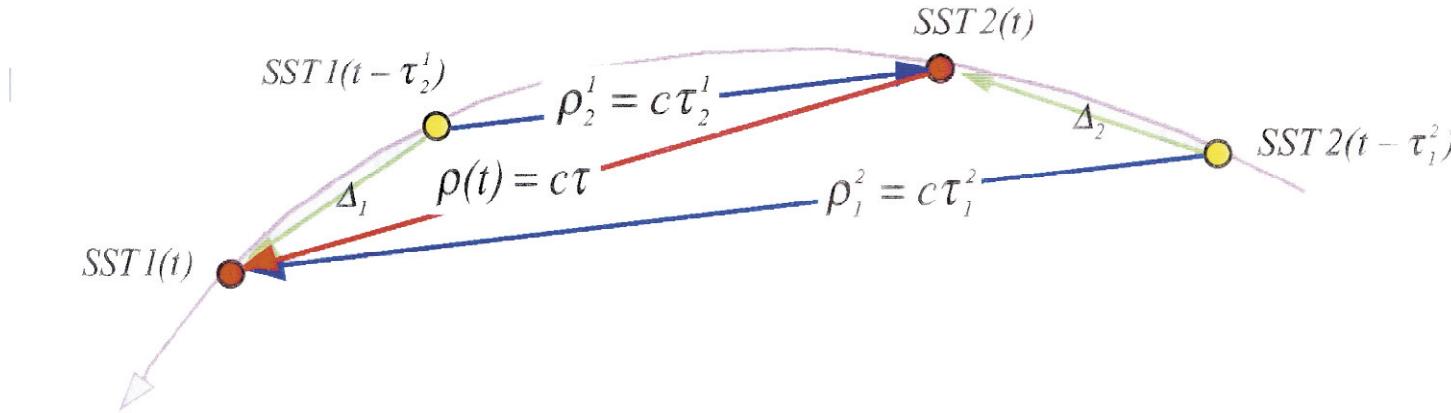
The time-of-flight τ (<1ms) corresponding to the instantaneous inter-satellite range at nominal time t is then computed by algorithm introducing an instantaneous range correction :

$$(f_1 \tau_2^1 + f_2 \tau_1^2) = (f_1 + f_2) \tau - \Delta \Theta_{TOF}(t)$$

The inter-satellite biased range is the computed from the dual one-way phase by multiplying the speed of light and by dividing the sum of the two carrier frequencies:

$$\begin{aligned} R(t) &= \frac{c \Theta(t)}{f_1 + f_2} = \rho(t) - \Delta \rho_{TOF}(t) + c \frac{\delta f_1 + \delta f_2}{f_1 + f_2} \tau \\ &\quad + c \frac{N_1^2 + N_2^1}{f_1 + f_2} + c \frac{I_1^2 + I_2^1}{f_1 + f_2} + c \frac{d_1^2 + d_2^1}{f_1 + f_2} + c \frac{\varepsilon_1^2 + \varepsilon_2^1}{f_1 + f_2} \end{aligned}$$

Instantaneous range correction



Since the estimation equations utilize the instantaneous range, it is necessary to convert the phase-derived range ρ_1^2 and ρ_2^1 into the instantaneous range:

$$(f_1\tau_2^1 + f_2\tau_1^2) = (f_1 + f_2)\tau - \Delta\Theta_{TOF}(t)$$

$$\Delta\rho_{TOF}(t) = c \frac{\delta f}{f_1 + f_2} (\Delta_1 - \Delta_2)^T \cdot \hat{e}_{12} - \frac{f_1 - f_2}{f_1 + f_2} (\Delta_2^T \cdot \hat{e}_{12})$$

$$\text{with : } r_1 - r_2 = \rho \cdot \hat{e}_{12} \quad , \quad \rho_2^1 = \rho - \Delta_1^T \cdot \hat{e}_{12} \quad , \quad \rho_1^2 = \rho - \Delta_2^T \cdot \hat{e}_{12}$$

Measurement errors

Oscillator noise :

it depends on the oscillator characteristics (quartz crystal oscillator with an Allan variance close to 10^{-13}) and on the dual one-way filter (function of frequency offset, carrier frequency and separation distance).

System noise :

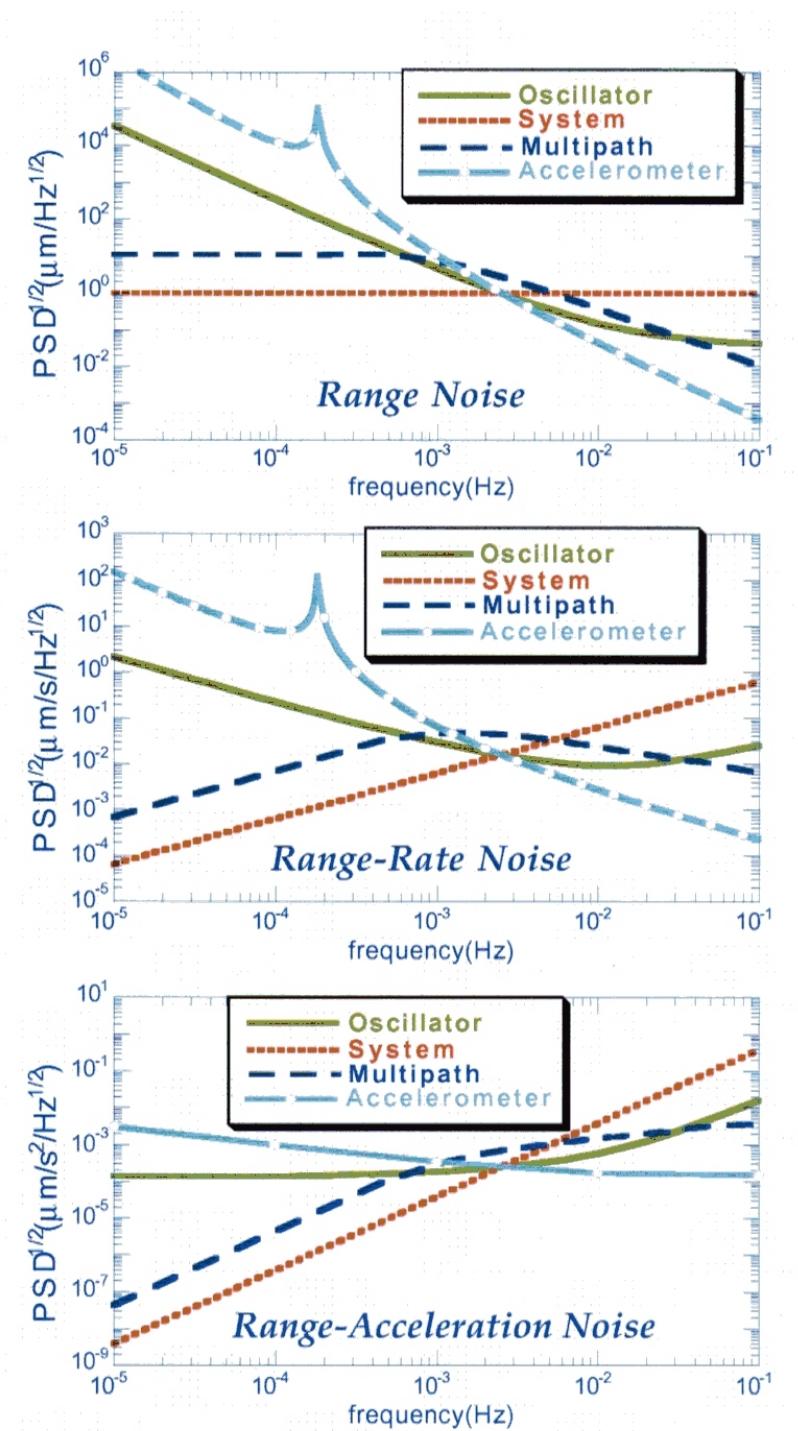
it comes from the receiver subsystem (white noise for the range constant to $1 \mu\text{m}$) and from the time-tag error (less than 70 ps when using GPS time-tag corrections).

Multipath noise :

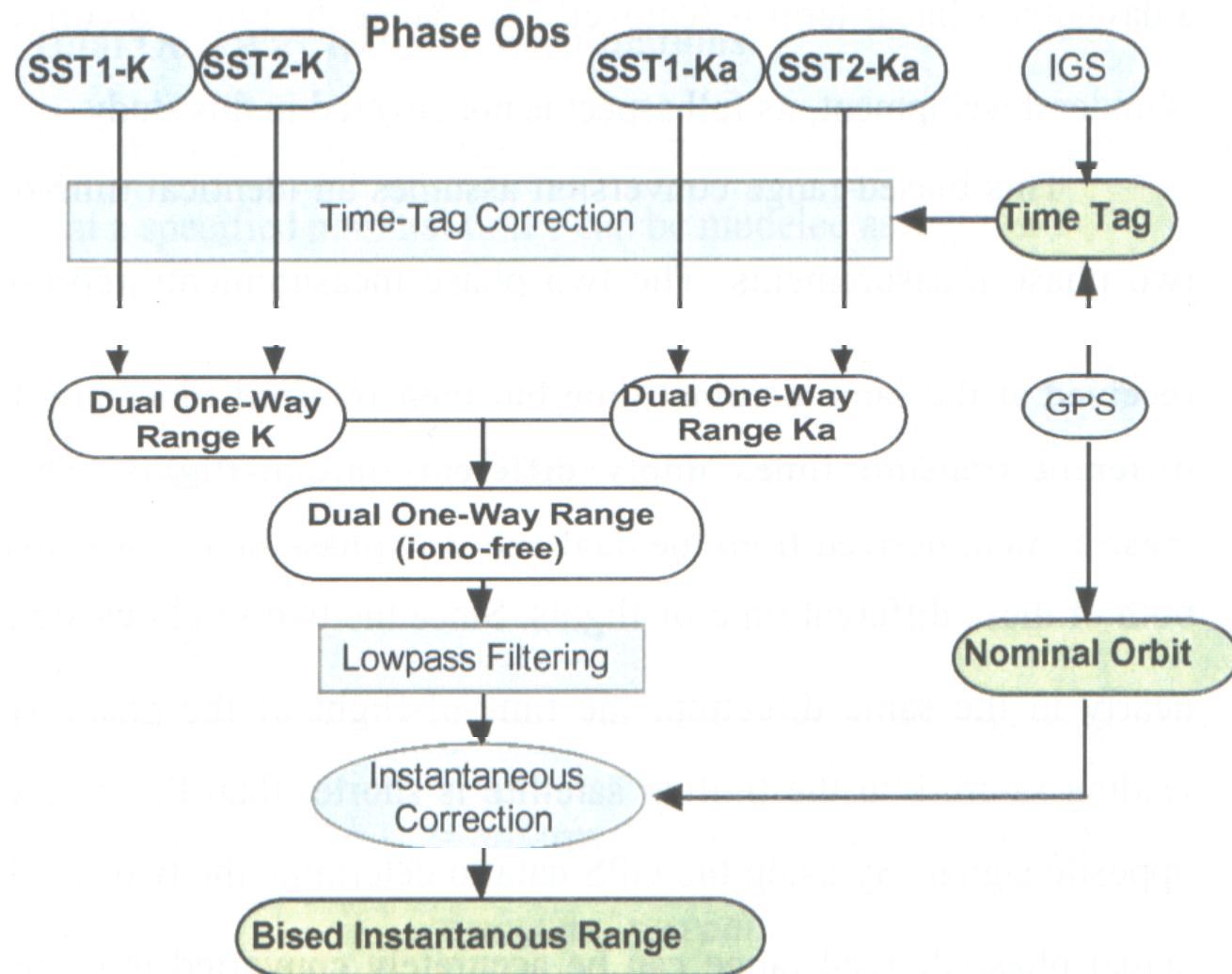
it is due to the indirect microwave signals which are reflected around the antenna horn. It depends on the reflectivity of the front surface and of the satellite attitude ($< 0.3 \text{ mrad}$).

Accelerometer noise :

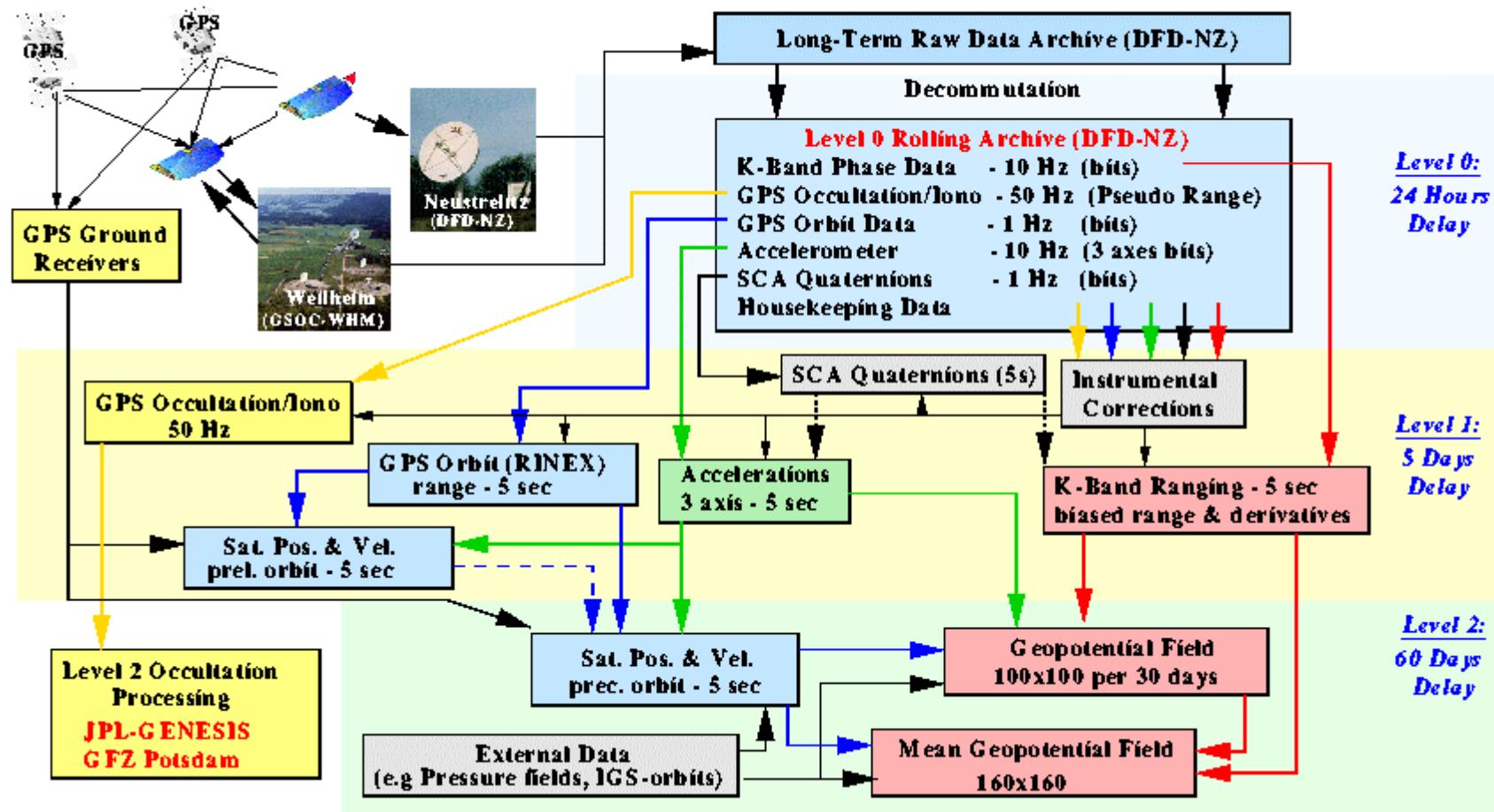
the SuperSTAR accelerometer (Onera) has a specification noise of $10^{-10} \text{ m/s}^2/\text{Hz}^{1/2}$ along R and T and of $10^{-9} \text{ m/s}^2/\text{Hz}^{1/2}$ along N. Attitude and misalignment errors may be as much as 0.3 mrad.



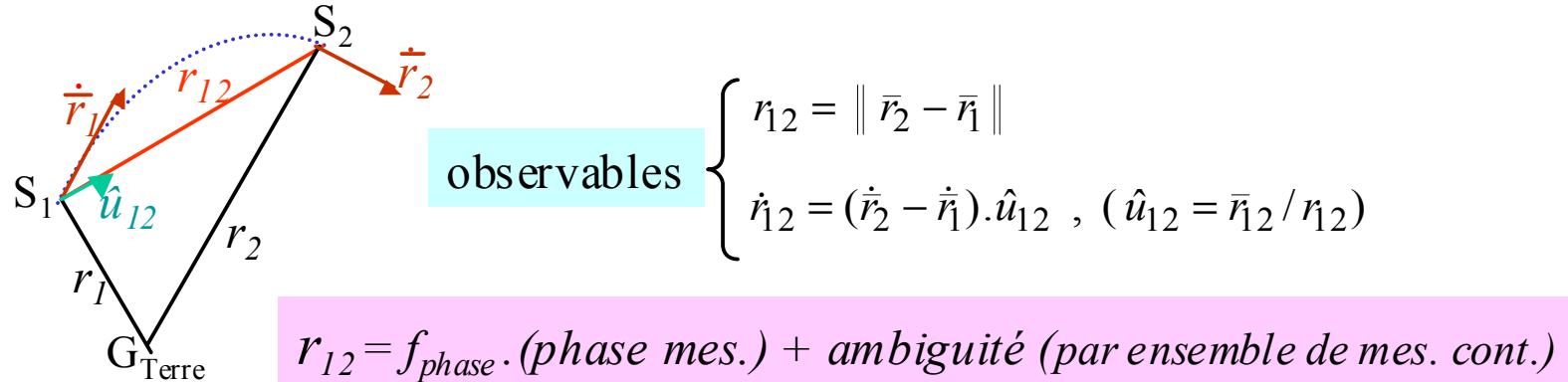
Pre-processing procedure (JPL)



GRACE products and data flow



Exemple d'équations d'observation pour GRACE



Distance

$$r_{12}^{obs} - r_{12}^{calc} \approx dr_{12} = \hat{u}_{12} \cdot (d\bar{r}_2 - d\bar{r}_1)$$

Vitesse
mesurées

$$\dot{r}_{12}^{obs} - \dot{r}_{12}^{calc} \approx d\dot{r}_{12} = \hat{u}_{12} \cdot (d\dot{\bar{r}}_2 - d\dot{\bar{r}}_1) + \frac{(\dot{\bar{r}}_2 - \dot{\bar{r}}_1) - \dot{r}_{12} \hat{u}_{12}}{r_{12}} \cdot (d\bar{r}_2 - d\bar{r}_1)$$

de l'intégr. num.

avec

$$d\{\bar{r}_i, \dot{\bar{r}}_i\} = \sum_j \frac{\partial\{\bar{r}_i, \dot{\bar{r}}_i\}}{\partial(PInt_i^j)} \cdot \Delta(PInt_i^j) + \sum_k \frac{\partial\{\bar{r}_i, \dot{\bar{r}}_i\}}{\partial(PExt^k)} \cdot \Delta(PExt^k)$$

pour l'arc { i }

Eq. aux variations
(intégr. num.)

Expected cumulative geoid error

