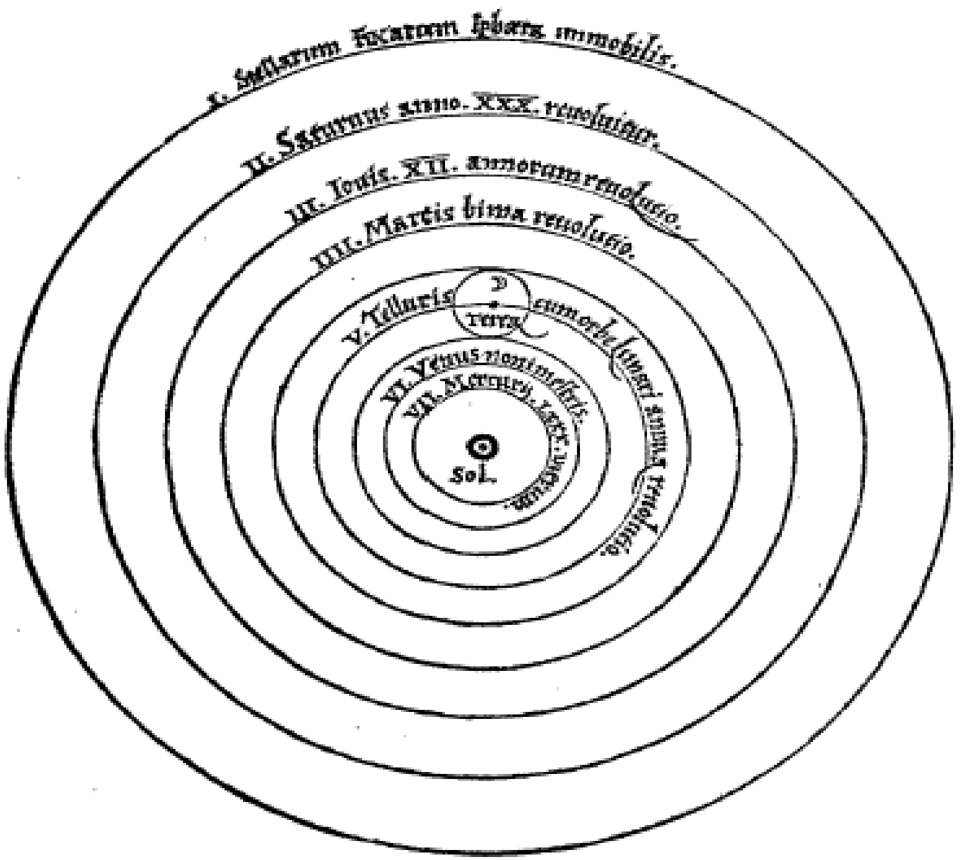
8ième Ecole d'été du GRGS

Généralités sur les trajectoires interplanétaires

Thierry MARTIN

CNES DCT/SB/MS

thierry.martin@cnes.fr



CONTENTS

- Keplerian motion and orbital elements
- interplanetary missions, definition, background history
- spacecraft motion within the solar system
- patched conics method
- ballistic trajectories and direct transfers
- Tisserand graphs
- complex trajectories: gravity assist, low thrust…
- Lagrange points

KEPLER’S PRECURSORS

Nicolas Copernicus (1473 1543)

De Revolutionibus Orbium Coelestium (1530)

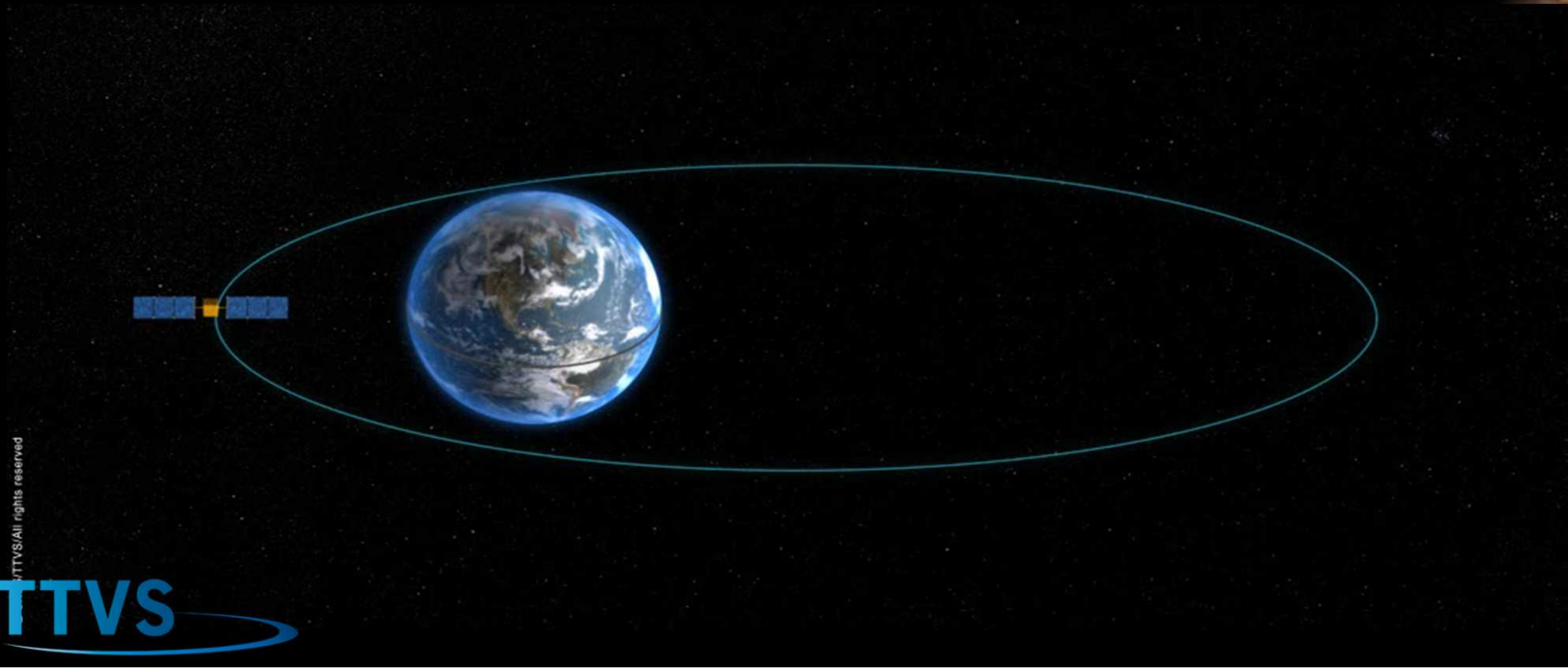
Tycho Brahe (1546 1601)

Accurate and comprehensive astronomical and planetary observations

Accurate measurements of Mars orbit
➔ eccentricity

Kepler Laws (1605)

- The orbit of every planet is an ellipse with the sun at a focus.
- A line joining a planet and the sun sweeps out equal areas during equal intervals of time.
- The square of the orbital period of a planet is directly proportional to the cube of the semi major axis of its orbit.



$$T = 2\Pi\sqrt{\frac{a^3}{\mu}}$$

- Newton's second law (fundamental principle of dynamics)
  - Inertial frame
  - Evaluation of the forces applied at the center of mass of the vehicle

- Newton's law of universal gravitation **:** **(μ=GM$_{Earth}$)**

$$\vec{F} = -m \frac{\mu}{\| \vec{r} \|^3} \vec{r}$$

$$\Rightarrow$$

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3}$$

- Hypothesis for the two body problem :
- Force induced by the Earth supposed to be homogeneous and spherical

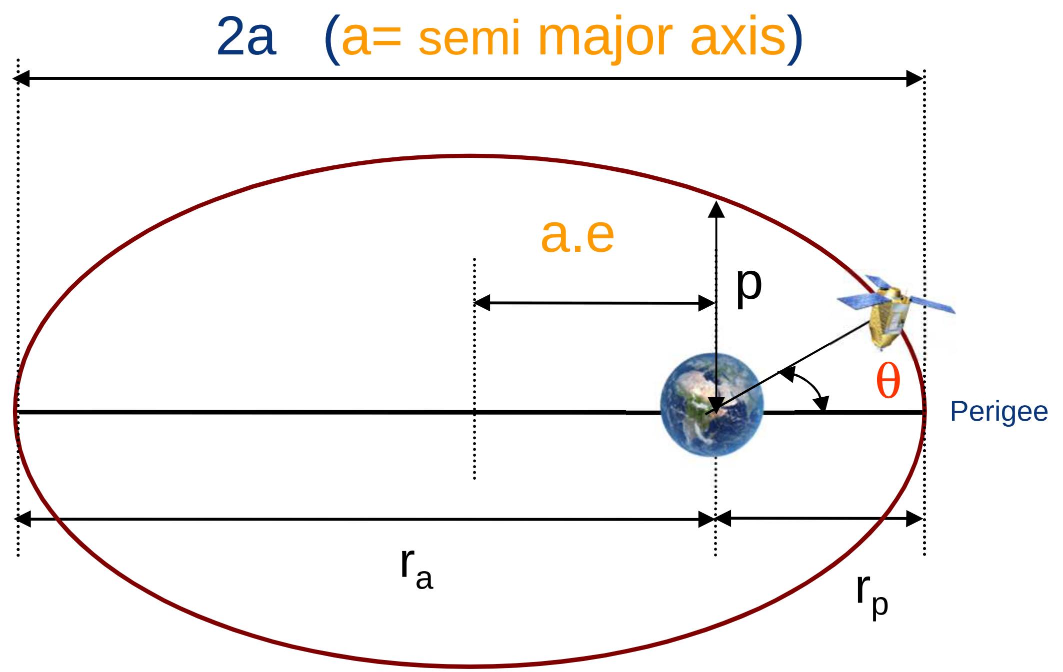
Elliptic motion (e<1)

- Kepler's 1st law: Ellipse of semi major axis a, eccentricity e and parameter with θ the anomaly.

$$r = \frac{p}{1+e\cos(\theta)}$$

$$with \quad p = a(1-e^2)$$

$$r_p = a(1-e) \quad \theta = 0$$
$$r_a = a(1+e) \quad \theta = \pi$$



GTO Orbit :
Alt apogee ~ 36000 km

GTO Orbit :
Alt aperigee ~ 500km

- Conservation of angular momentum C.

$$\frac{d\vec{C}}{dt} = \frac{d}{dt}(\vec{r} \wedge \dot{\vec{r}}) = \underbrace{\vec{r} \wedge \ddot{\vec{r}}}_{\vec{0}} + \underbrace{\dot{\vec{r}} \wedge \dot{\vec{r}}}_{\vec{0}} = \vec{0}$$

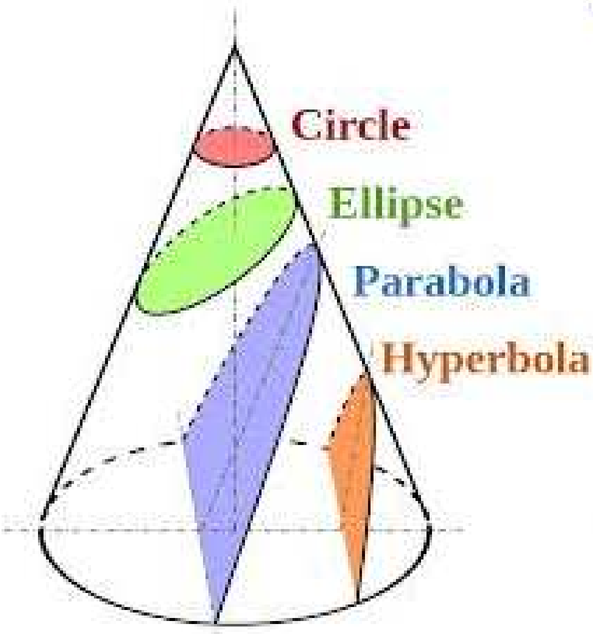
➔The trajectory lies in a single plane

- Conservation of specific mechanical energy

$$\frac{V^2}{2} - \frac{\mu}{r} = K$$

Specific mechanical energy

- Depending on the value of the eccentricity “e“ the orbit is:



e= 0	a circle	$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2r}$	K < 0 Closed orbit
0<e<1	an ellipse	$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$	
e=1	a parabola	$\frac{V^2}{2} - \frac{\mu}{r} = 0$	K ≥ 0 Liberation orbit
e>1	an hyperbola	$\frac{V^2}{2} - \frac{\mu}{r} = \frac{\mu}{2a}$	

Elliptic motion (e<1)

Conservation of mechanical energy

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

Orbital Velocity

- Circular
- Apogee
- Perigee

Circular orbit

$$V_{circ} = \sqrt{\frac{\mu}{r}}$$

V_{circ}=7.6 km/s at 500 km

Elliptical orbit

Ex : GTO V_{ap}=1.6km/s

$$V_a = \sqrt{\frac{\mu}{a}\frac{1-e}{1+e}}$$

V_{per}=10.2km/s

$$V_p = \sqrt{\frac{\mu}{a}\frac{1+e}{1-e}}$$

Example : GTO orbit

Orbital elements

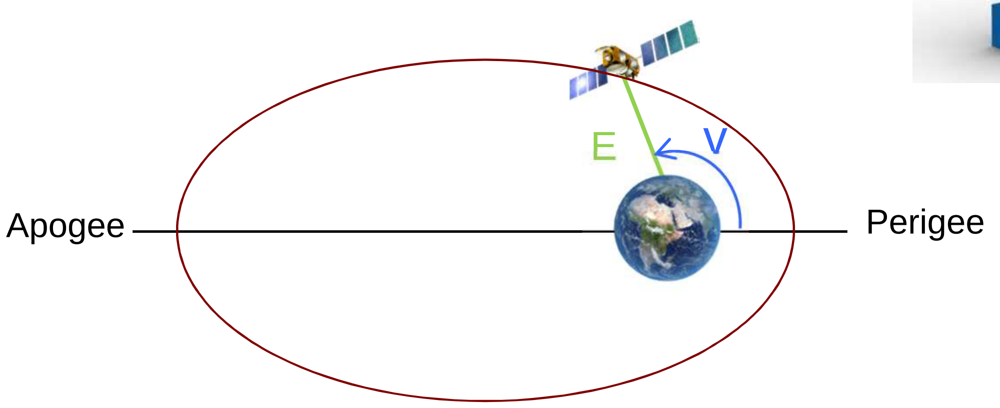
- The orbital elements define:
  - Shape of the orbit: a and e
  - Position of the orbit in the plane: ω
  - Position of the orbital plane: i and Ω
    - Inclination angle
    - Right ascension of ascending node
  - Position of the satellite on the orbit: the anomaly (M,E,v)
- Keplerian elements = Classical orbit elements.
- Also used :
  - Orbital element adapted to near circular orbit
  - Orbital element adapted to near circular orbit near equatorial
  - Others …

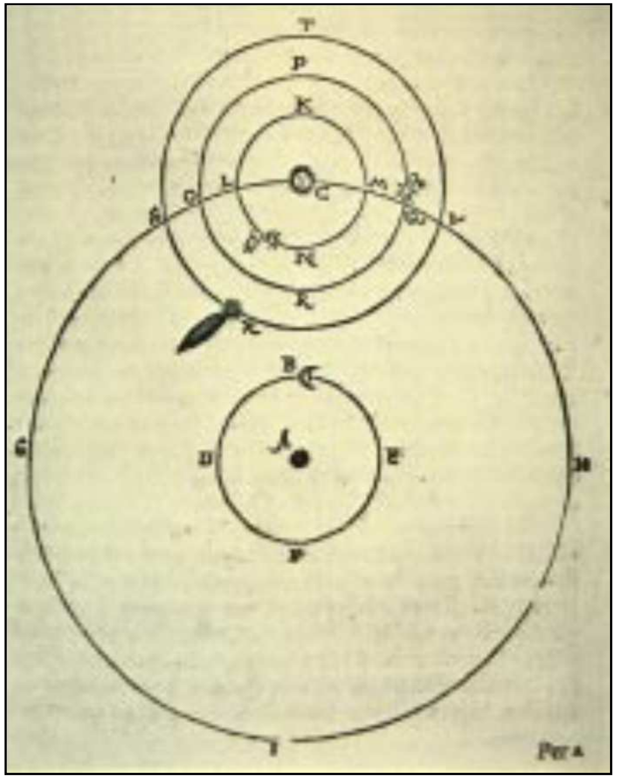
Relating orbital position and time

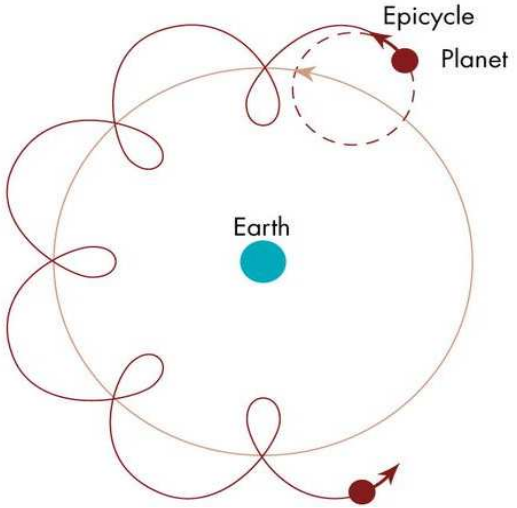
- The velocity of the satellite is not constant on its orbit

- How to relate the on orbit position
  (defined by the true anomaly v)
  of the satellite and time.

PROBLEM OF KEPLER







Relating orbital position and time

- The elliptic motion is a periodic motion as per Kepler's third law:

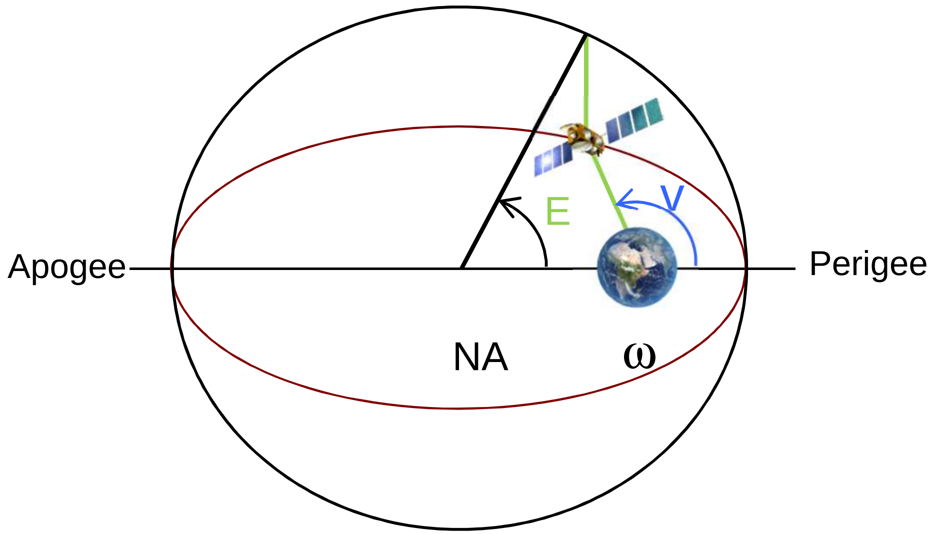
$$T = 2\Pi\sqrt{\frac{a^3}{\mu}}$$

- The motion can be described using :
  - mean motion
  - mean anomaly
- The position of a fictitious point given by:

$$n^2 a^3 = \mu$$

$$M = n(t - t_p)$$

- The 3 anomalies:
  - True anomalies (v)
  - The eccentric anomaly (E)
  - The mean anomaly (M)



Relating orbital position and time

- M is related to E by the Kepler's equation :

$$M = n(t - t_p) = E - e\sin E$$

The position of the satellite can be known by using M ➔E➔v.

- v is related to E thanks to an analytical expression :

$$v = 2\arctan\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right]$$

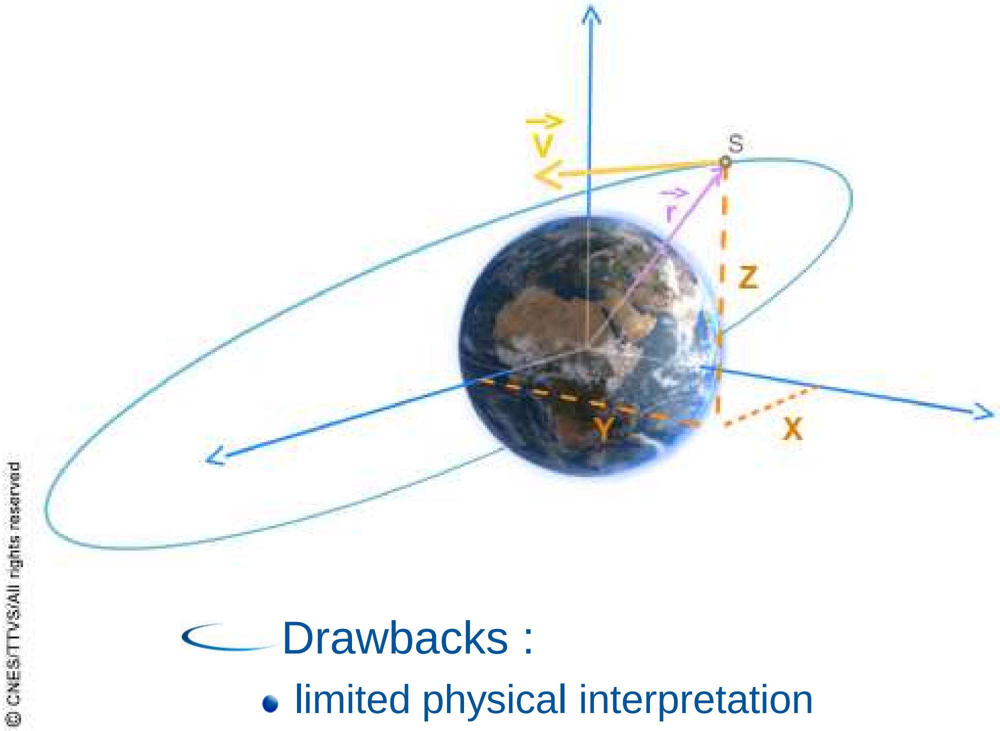
NB.The Kepler equation is a transcendental equation that requires iterative methods or approximations by infinite series.

Orbit parameters

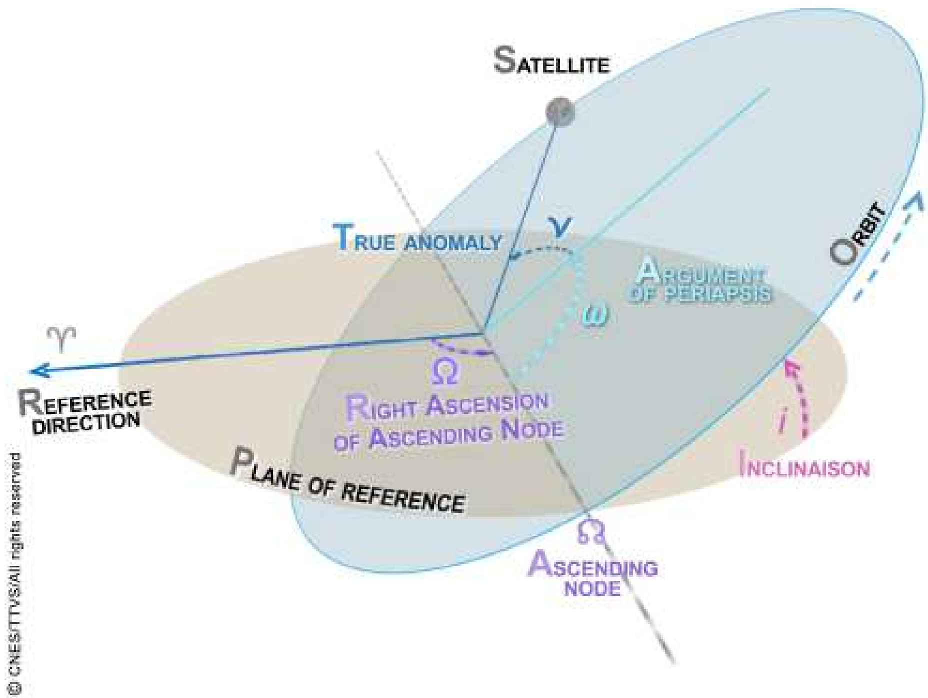
Mathematical transformation

$\tau(\mu)$

- Cartesians elements
  - 3 position coordinates
  - 3 velocity coordinates
  - ➔ a state vector



- Orbital elements
  (a,e,i,ω,Ω,v)

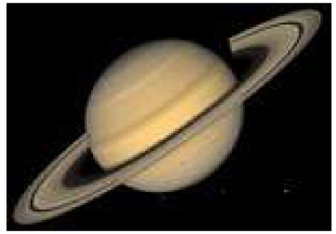


- Drawbacks :
  - limited physical interpretation
  - mathematical simplification impossible

Background history of interplanetary missions

Towards the inner planets (Mercury, Venus)

- MARINER 10, MESSENGER
- MAGELLAN, VENUS-EXPRESS…

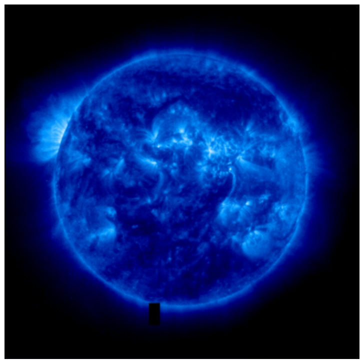


Towards the outer planets (Mars, Jupiter, Saturn…)

- MARS GLOBAL SURVEYOR, MARS ODYSSEY, MARS EXPRESS, MSL…
- VOYAGER, PIONEER, CASSINI-HUYGENS, NEW HORIZONS…

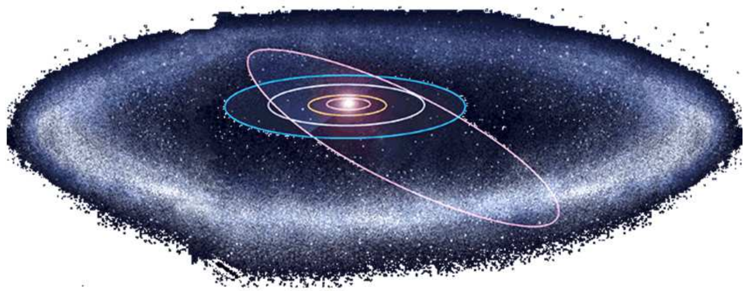
Towards small bodies (Moon, comets, asteroids, dwarf

- APOLLO, LUNA, SMART1…
- NEAR, DEEP IMPACT, HAYABUSA…
- GIOTTO, ROSETTA, STARDUST…



Towards the Sun

- ULYSSES, SOHO…

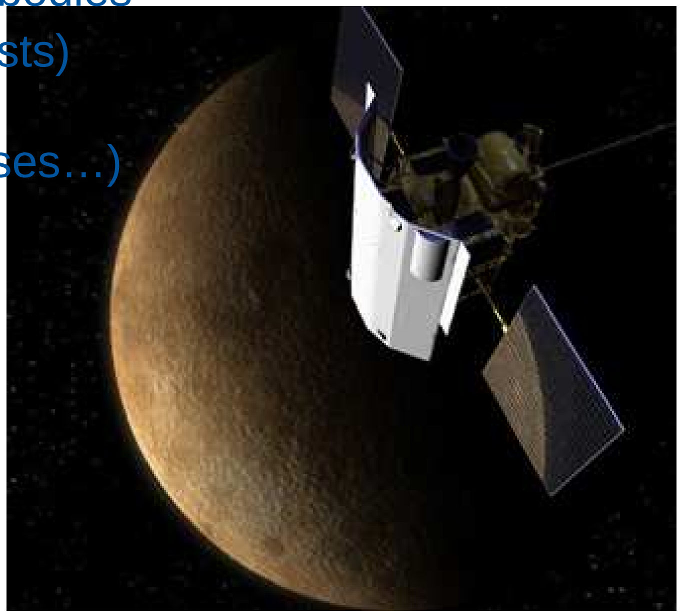


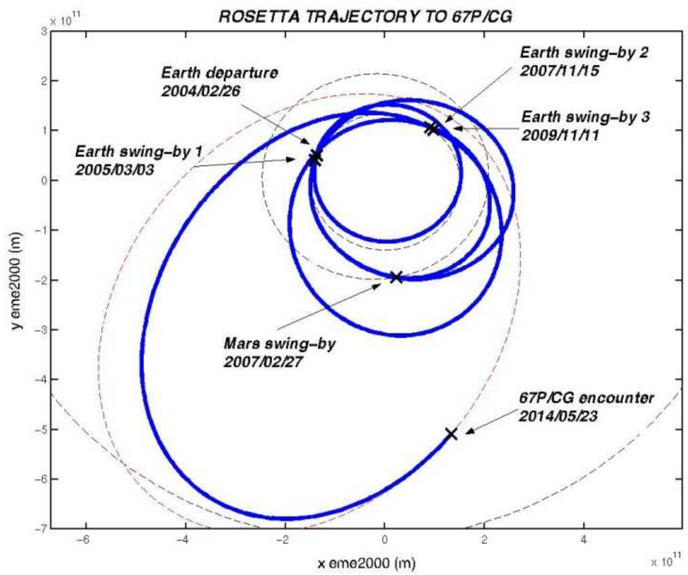
Beyond the solar system

- VOYAGER, NEW HORIZONS…

Scenario of an interplanetary mission

- Escape phase
  - direct injection by the LV
  - injection onto a parking orbit th
    escape
- Cruise phase
  - direct ballistic transfer
  - fly-by of intermediate bodies
  - swing-by (gravity assists)
  - powered phases
    (DSM, low-thrust phases…)
- Arrival phase
  - fly-by
  - insertion and capture
- Return phase
  - sample return
  - manned mission





Spacecraft motion within the solar system

Complete model

- all the forces are taken into account
- numerical integration is required
- a simpler method is required for mission design

Restricted three-body problem

- **definition of Lagrange's points**

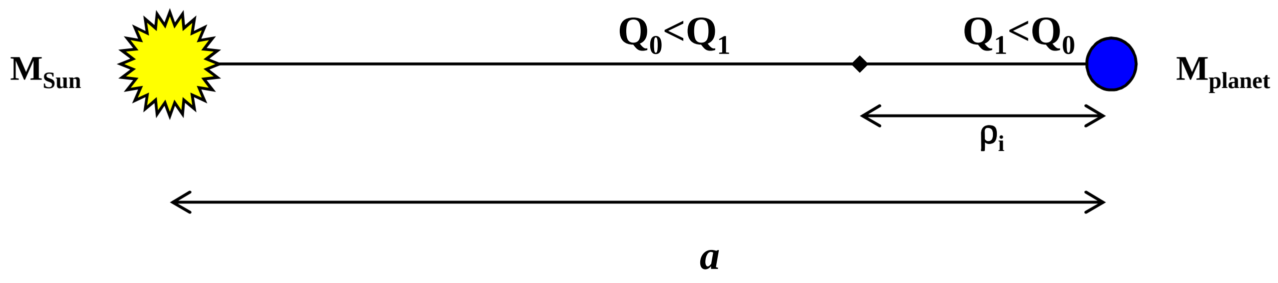
Two-body problem

- **only the central body and the probe**
- **Keplerian motion**
- **patched conics method**

Patched-conics: decomposition of an interplanetary trajectory

- Patched planetocentric and heliocentric arcs
- For each phase the keplerian trajectory is defined relatively to the central body
- The link between planetocentric and heliocentric arcs is ensured at the sphere of influence boundary.

The sphere of influence



- spacecraft motion around the Sun $Q_0 = \dfrac{\text{perturbing acceleration (planet)}}{\text{main acceleration (sun)}}$
- spacecraft motion around the planet $Q_1 = \dfrac{\text{perturbing acceleration (sun including contrifugal effect)}}{\text{main acceleration (planet)}}$
- a point located on the sphere of influence (radius *ρi*) satisfies *Q0=Q1*

$$\rho_i = 0.87055\left(\frac{\mu_p}{\mu_{sun}}\right)^{2/5} a$$

PLANET	Mercury	Venus	Earth	Mars	Jupiter	Saturn
ρ_i (10^6 km)	0.0978	0.536	0.805	0.502	41.9	47.5

The sphere of influence determines the validity space of the Keplerian model, planet centered, at the first order

The escape phase 1

- To 'tear away' the spacecraft out of the attraction of a planet (Earth)
- To reach the escape velocity -> "open trajectory" i.e hyperbole
  - spacecraft velocity at the periapsis > escape velocity L_0

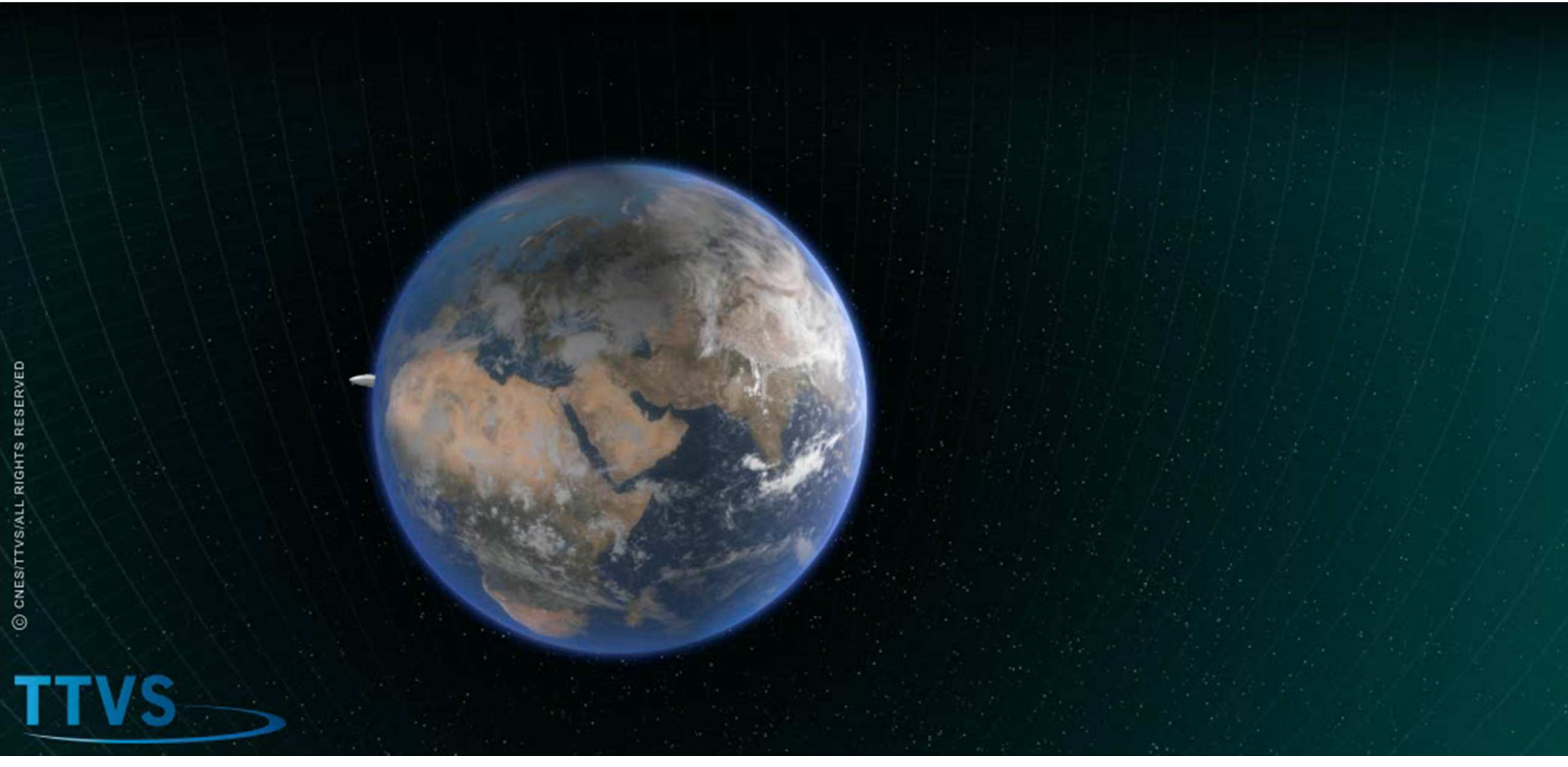
$$L_0 = \sqrt{\frac{2\mu}{r}}$$

PLANET	Mercure	Vénus	Earth	Mars	Jupiter	Saturn
L_0 [km/s] (r = Rpla)	4.3	10.36	11.2	5.03	59.5	35.5

Escape strategies

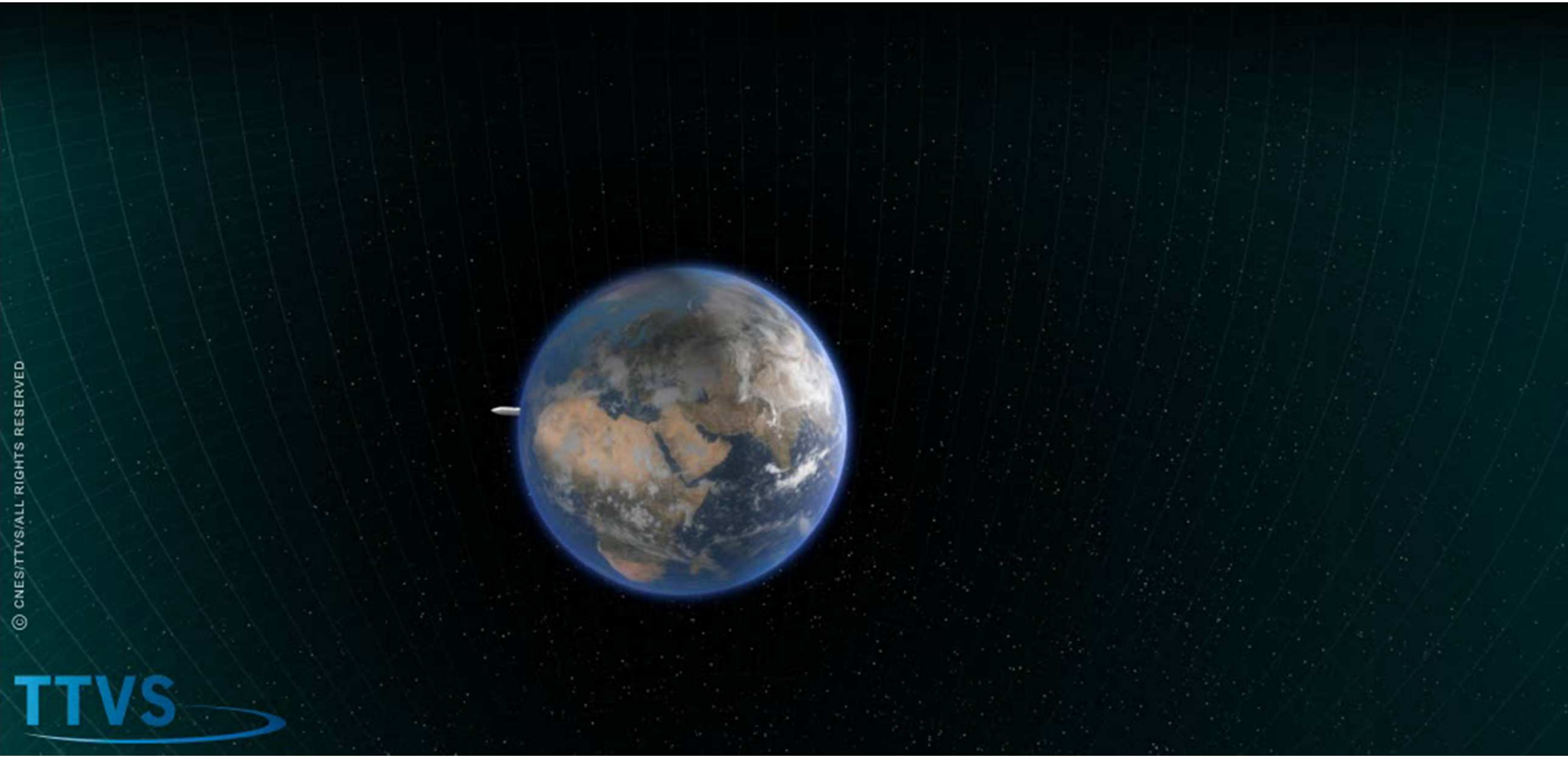
- **direct injection by the launch vehicle onto the escape hyperbola**
- **indirect strategy: first the spacecraft is put onto a parking orbit, then the transfer to the escape**
- **hyperbola is done by means of several maneuvers (upper stage, spacecraft engines…)**

The escape phase 2



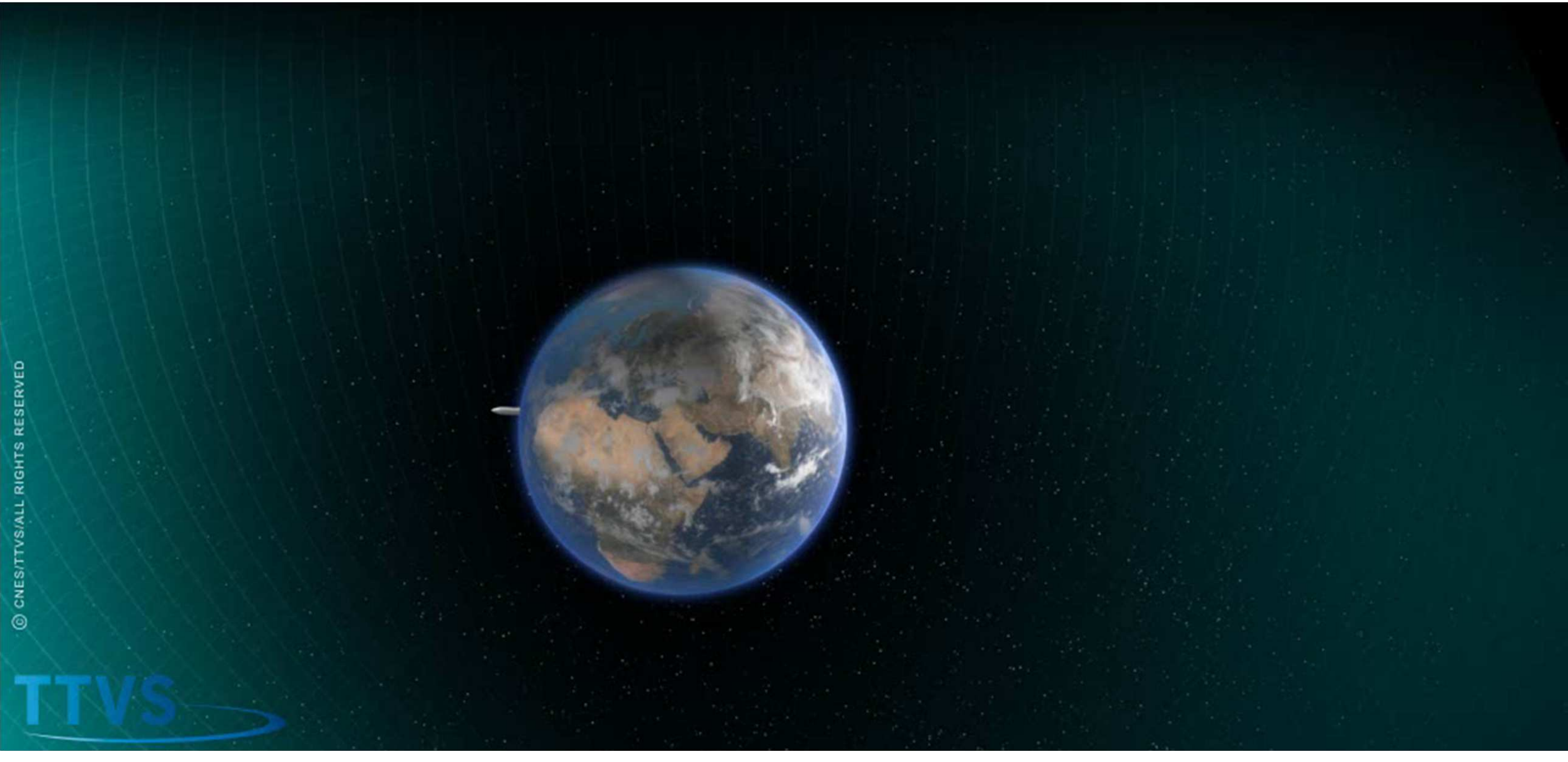
The escape phase 3

- Conservation of mechanical energy along the hyperbola $$\frac{V^2}{2} - \frac{\mu}{r} = \frac{V_\infty^2}{2}$$



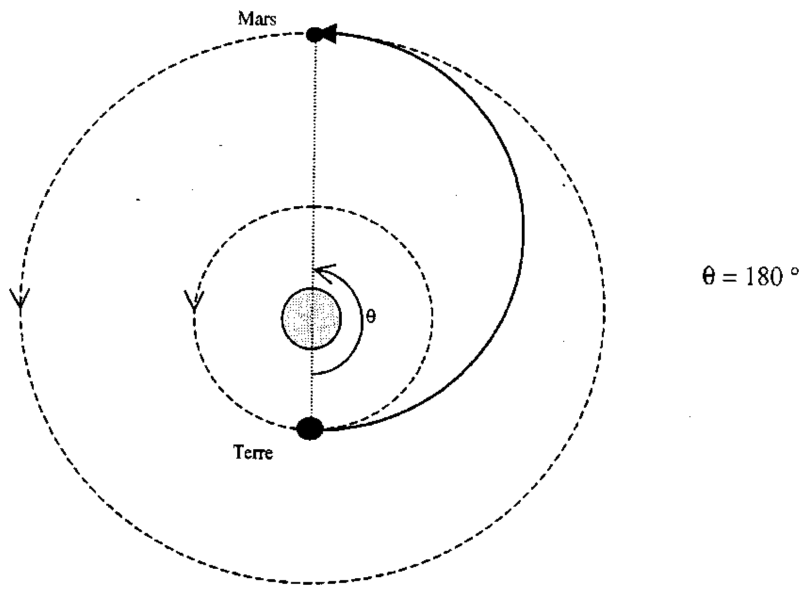
The escape phase

The hyperbolic excess velocity "V_∞" is equal to the relative velocity of the probe with respect to the planet



Computation of heliocentric ballistic trajectories 2/3

- How to find favorable configurations (in terms of energy required) ?



- The Hohmann's transfer
  - circular and coplanar orbits
  - transfer angle of 180deg
  - departure and arrival positions: periapsis and apoapsis of the transfer orbit

- The geometric planetary configuration defines the interplanetary windows

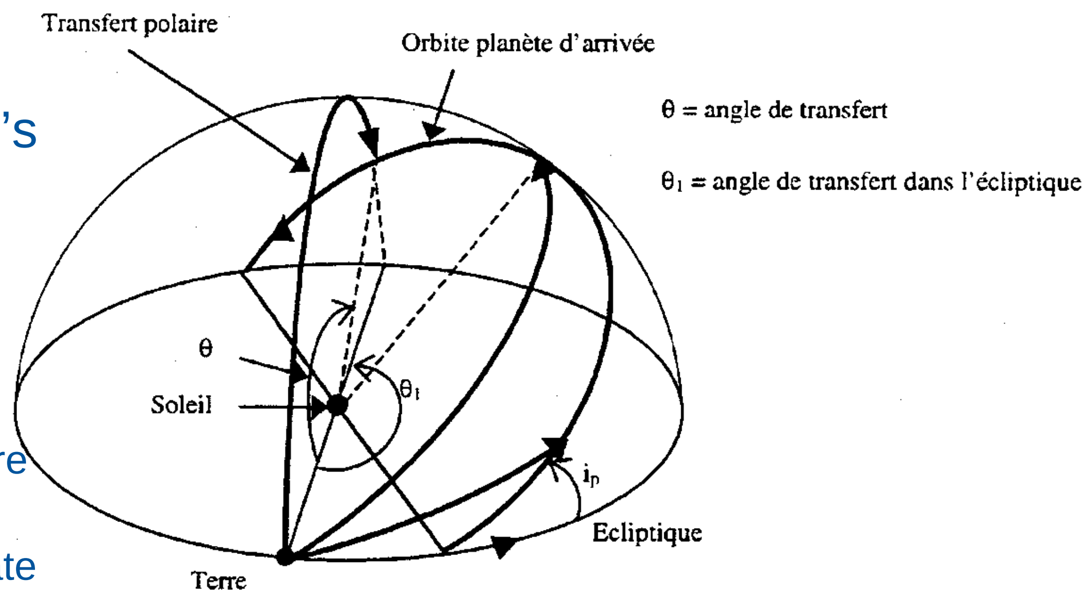
- Geometric repetitiveness: definition of the synodic period:

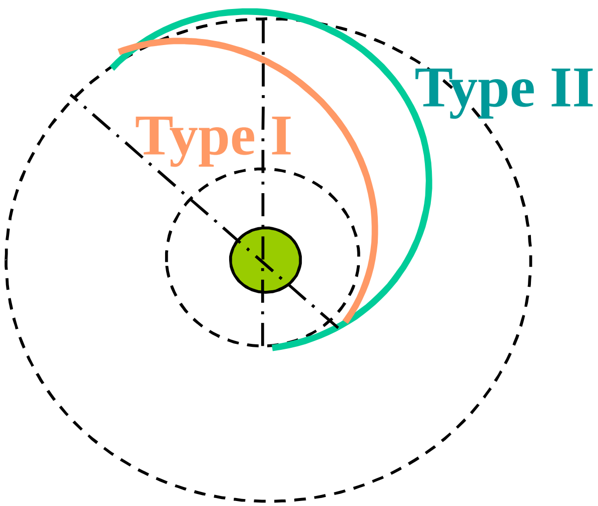
$$P = \frac{T_1 . T_2}{\left| T_1 - T_2 \right|}$$

PLANET	Mercure	Vénus	Mars	Jupiter	Saturn
PERIOD (d)	116	584	781	399	378

Computation of heliocentric ballistic trajectories 2/3

- In fact the assumptions of the Hohmann's transfer are not fulfilled, mainly because orbits are not coplanar
- Real transfer
  - The motion plane is defined by:
    - position the departure planet at the departure date
    - position of the arrival planet at the arrival date
    - position of the Sun





- **In terms of energy the180deg-transfer represents the worst case (-> motion plane far from the ecliptic)**
- **For a direct (less than 1 loop) ballistic transfer between two planets, two optimal trajectories are defined:**
  - **Type I : short trajectory**
  - **Type II : long trajectory**

Journey towards Mars:

Type I → 6-7 months **Type II → 10-12 months**

Computation of heliocentric ballistic trajectories 3/3

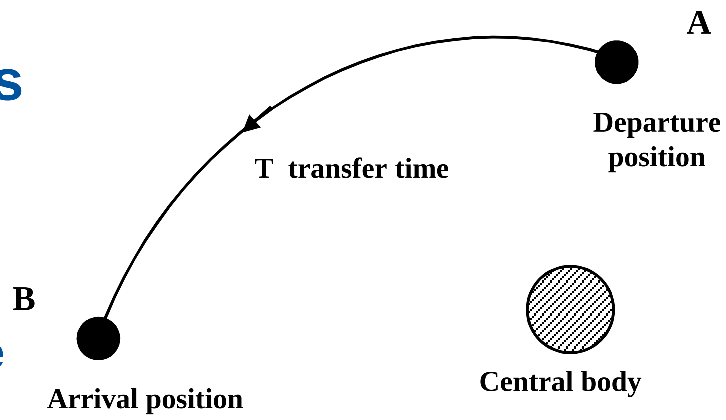
Solving Lambert's problem

Lambert's theorem

- "In a $1/r^2$ gravitational field with respect to a central body, given two points A and B and a certain duration T to go from A to B, there exists one and only one conic for a transfer requiring less than one loop and two conics for a transfer requiring one loop and more. The conic may be an ellipse, a parabola or a hyperbola

Computation of heliocentric trajectories

- **choice of departure and arrival dates**
- **deduce the positions of the planets**
- **choice of the number of loops**
- **compute the Lambert's transfer (ellipse around the sun)**



➔ Favorable dates in the vicinity of favorable planet configurations given by their synodic period

Synthesis of the patched conics method

1 - Heliocentric phase: Lambert's theorem allows to find the heliocentric ellipse based on the departure position and date from departure planet, and on the arrival position and date at the arrival planet (ballistic trajectory)

Rule: the heliocentric position of the probe is the same than that of the planet (given by ephemeris)

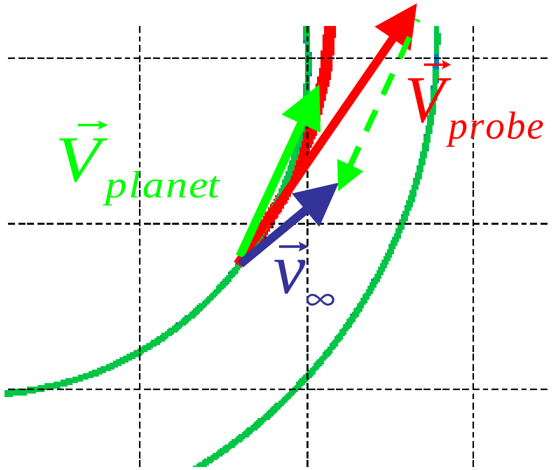
$$\vec{X}_{Planet\,d}(departure\ date)\ \&\ \vec{X}_{Planet\,a}(arrival\ date) \Rightarrow \overrightarrow{V_{Probe}}\ (departure\ date)\ \&\ \overrightarrow{V_{Probe}}\ (arrival\ date)$$

2 - Planetocentric phases

Rule: the hyperbolic excess velocity is equal to the relative velocity of the probe with respect to the planet

$$\overrightarrow{V_\infty^d} = \overrightarrow{V_{Probe}}(departure\ \ date) - \overrightarrow{V_{Planet}}(departure\ \ date)$$

$$\overrightarrow{V_\infty^a} = \overrightarrow{V_{Probe}}(arrival\ \ date) - \overrightarrow{V_{Planet}}(arrival\ \ date)$$

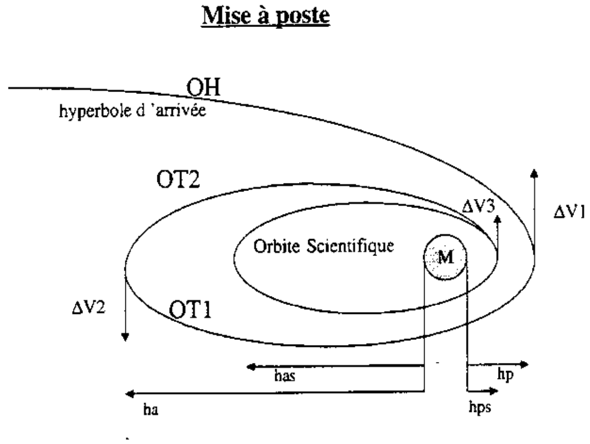


Giving the excess velocity, a periapsis altitude and a targeted inclination defines the escape or insertion hyperbola

⇒ An heliocentric transfer is defined by the departure and arrival dates. Optimal ones can be found through multiple computations (abacus, optimization process)

Insertion phase

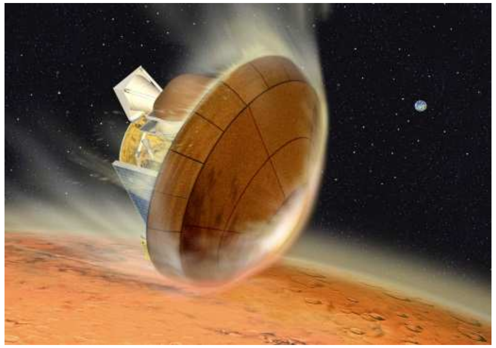
Transfer the probe from the arrival hyperbola to the scientific orbit (ellipse) around the target body



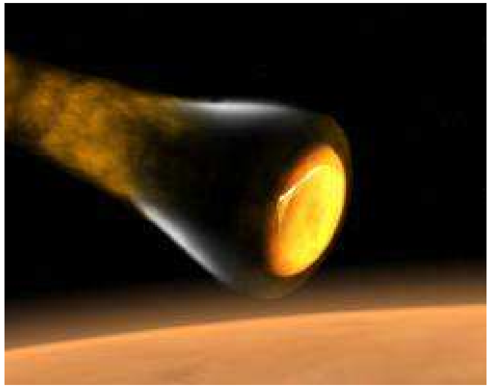
Very similar to the escape phase (duality)

Maneuvers located at the periapsis of the insertion orbit or continuous low thrust phases

$$\Delta V = \sqrt{V_\infty{}^2 + \frac{2 \times \mu_{planet}}{Rp}} - \sqrt{\mu_{planet}\left(\frac{2}{Rp} - \frac{1}{a}\right)}$$



Use of the atmospheric drag: aerobraking or aerocapture techniques



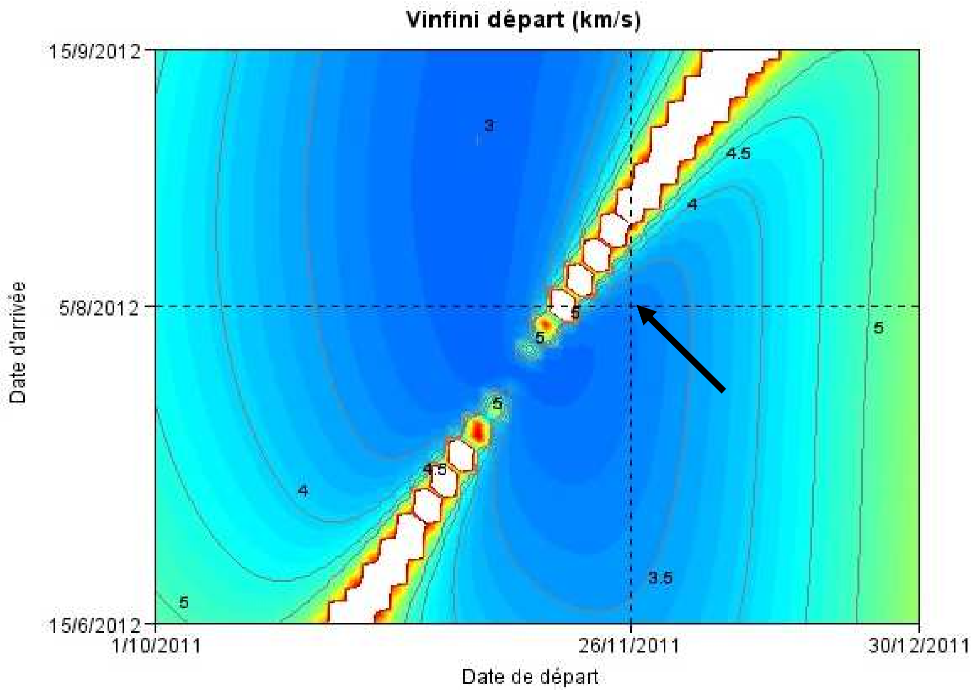
Re-entry phase:

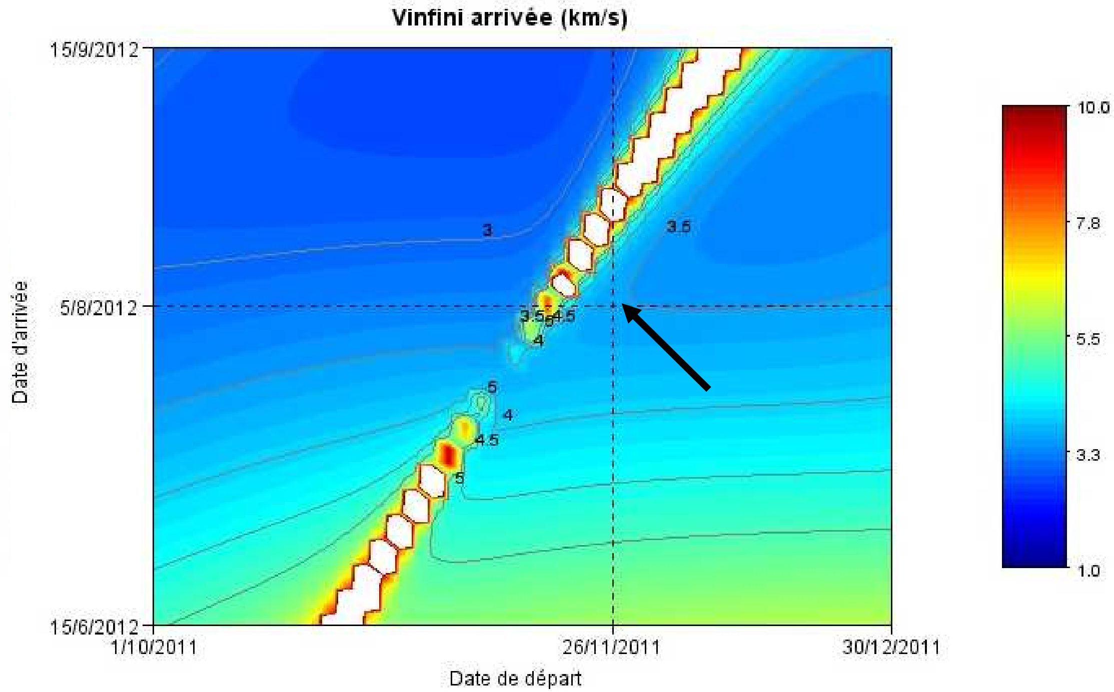
$$V_e{}^2 = V_\infty{}^2 + \frac{2 \times \mu_{planet}}{R_e}$$

- ballistic or controlled
- re-entry velocity into the atmosphere
- re-entry corridor: range of accessible re-entry slopes

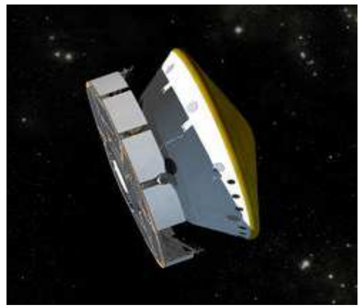
Example
Earth-Mars transfer MSL trajectory

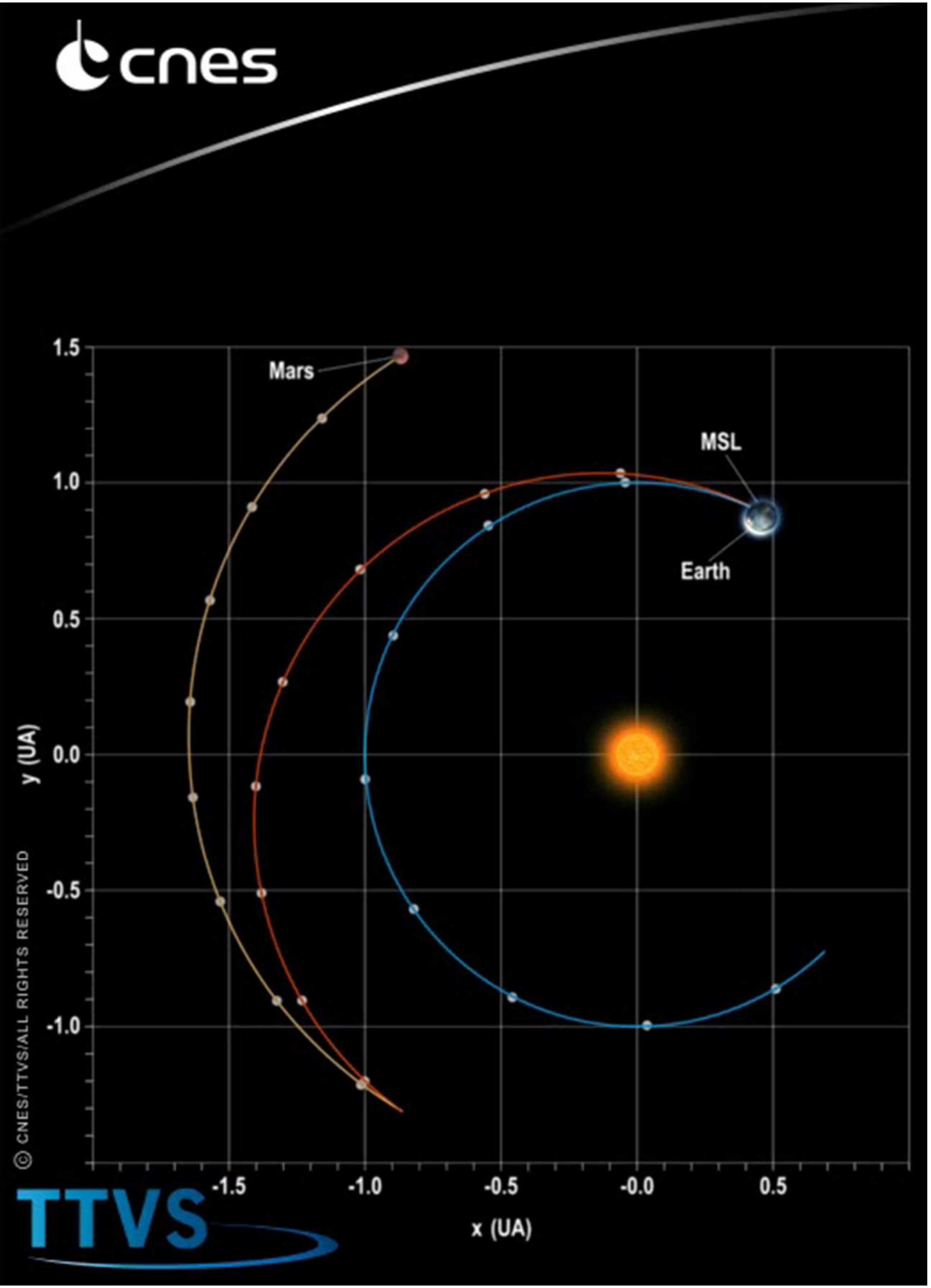
Selection abacus (Lambert transfer)





Computation using CNES Celestlab (Scilab) toolbox
http://atoms.scilab.org/toolboxes/celestlab



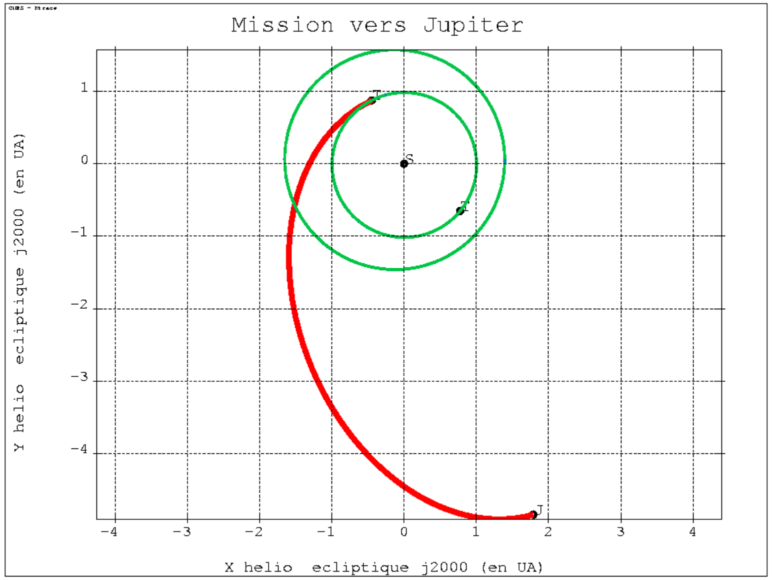


Mars MSL trajectory

- Departure:
  - November 26 2011
  - V∝ / Earth ≈ 3.2 km/s (associated heliocentric velocity: 34km/s)
- Arrival:
  - August 6 2012
  - V∝ / Mars ≈ 3.6 km/s (associated heliocentric velocity: 20km/s)
- Type I trajectory (< 180 deg)

Complex interplanetary trajectories

- Hyper energetic direct transfers: departure or/and arrival velocity increment(s) is (are) too large
  - Earth to Mercury transfer, arrival Delta-V
  - Earth to Jupiter transfer, departure Delta-V



Trajectory	Depart.	V∞	DLA	RLA	Arrival	V∞	DAP	RAP
Earth-Jupiter	08/20/1977	**10,2**	40.5	72.5	07/09/1979	**7.87**	-7.4	316.0

Solutions to be implemented

- **use of a propulsion system onboard the spacecraft**
  - **chemical propulsion: impulsive maneuvers**
  - **electrical propulsion: continuous low thrust arcs (high ISP value)**
- **advanced escape phase**
- **introduce gravity assist maneuvers into the mission scenario**
- **advanced insertion phase**

The gravity assist maneuvers 1

- Goal: increase (or decrease) the energy of the S/C trajectory.

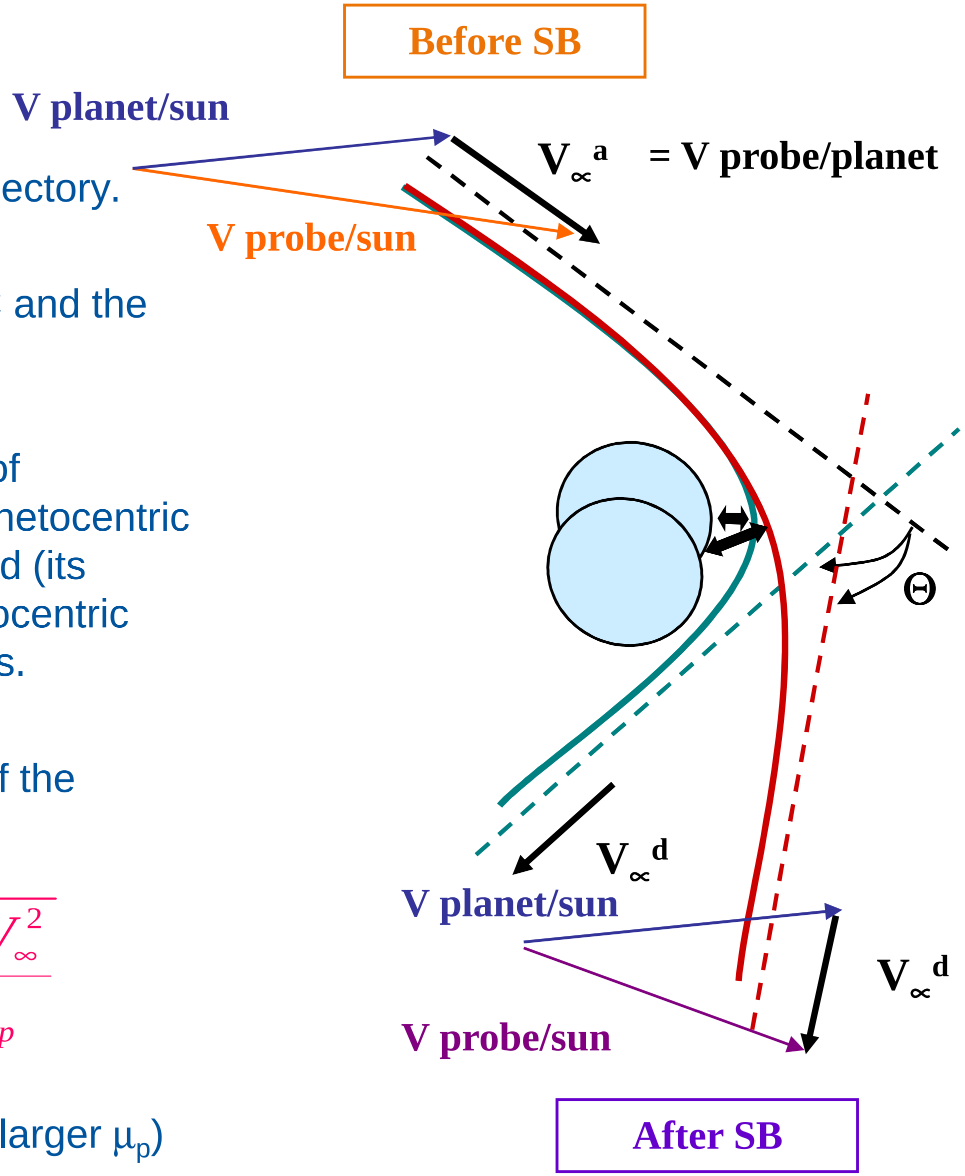
- Physical principle: exchange of energy between the S/C and the swing-by planet (constant kinetic momentum)

- Explanation: when the spacecraft goes into the sphere of influence, its trajectory is deviated with respect to the planetocentric hyperbola. The direction of the relative velocity is changed (its modulus as well if a maneuver is performed). So the heliocentric velocity is also changed in terms of direction and modulus.

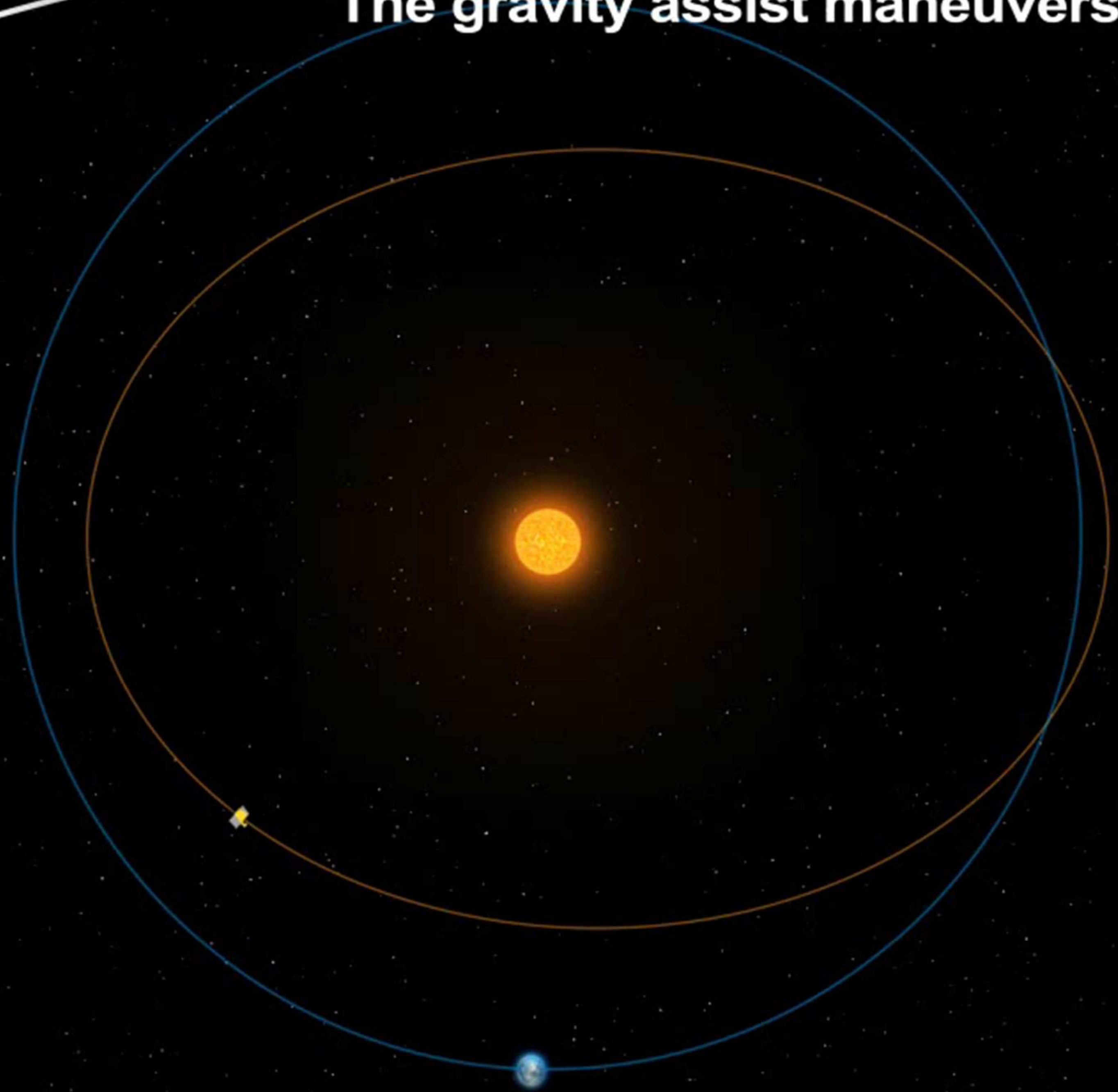
The deviation angle is adjusted thanks to the radius of the periapsis.

$$\sin\frac{\Theta}{2} = \frac{1}{1+\frac{r_p.V_\infty^2}{\mu_p}} \qquad \Delta V_{probe/sun} = \frac{2.V_\infty}{1+\frac{r_p.V_\infty^2}{\mu_p}}$$

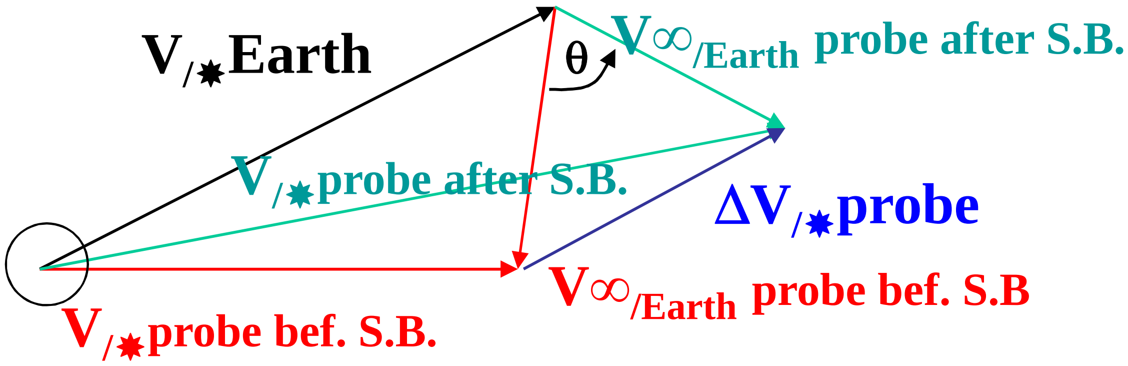
Remark: the larger the deviation is the smaller r_p (the larger μ_p) is.



The gravity assist maneuvers: swing-by



- Example of the 2nd swing-by of the Earth by Rosetta



- Fly-by distance: 3350 km
- Heliocentric velocity increased by 3.6 km/s

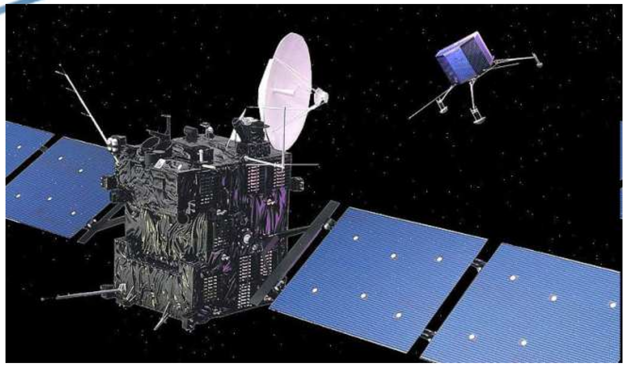
⇔ 2000 kg of propelant if chemical maneuver by the probe (2900 kg)

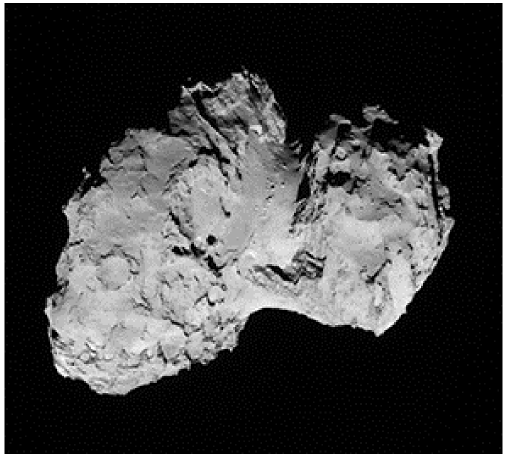
Trajectory selection principle:

- **Patching multiple heliocentric balistic phases and swing-by(s) (some Deep Space Maneuvers may be added to tune swing-by(s) conditions)**
- **Departure, arrival and swing-by dates and conditions are selected through an optimization process thanks to optimization software**

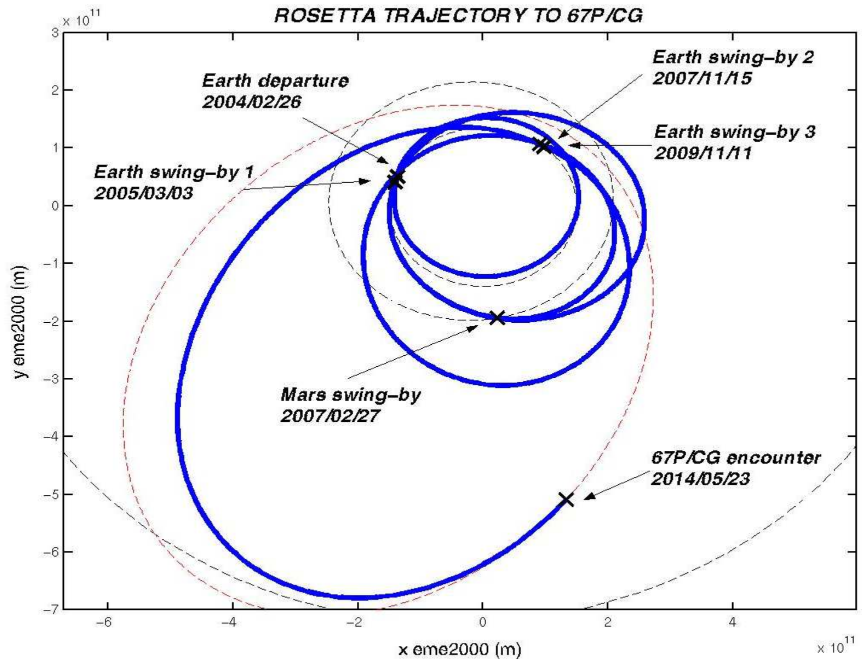
Gravity assist (swing-by) maneuvers 4

The ROSETTA mission

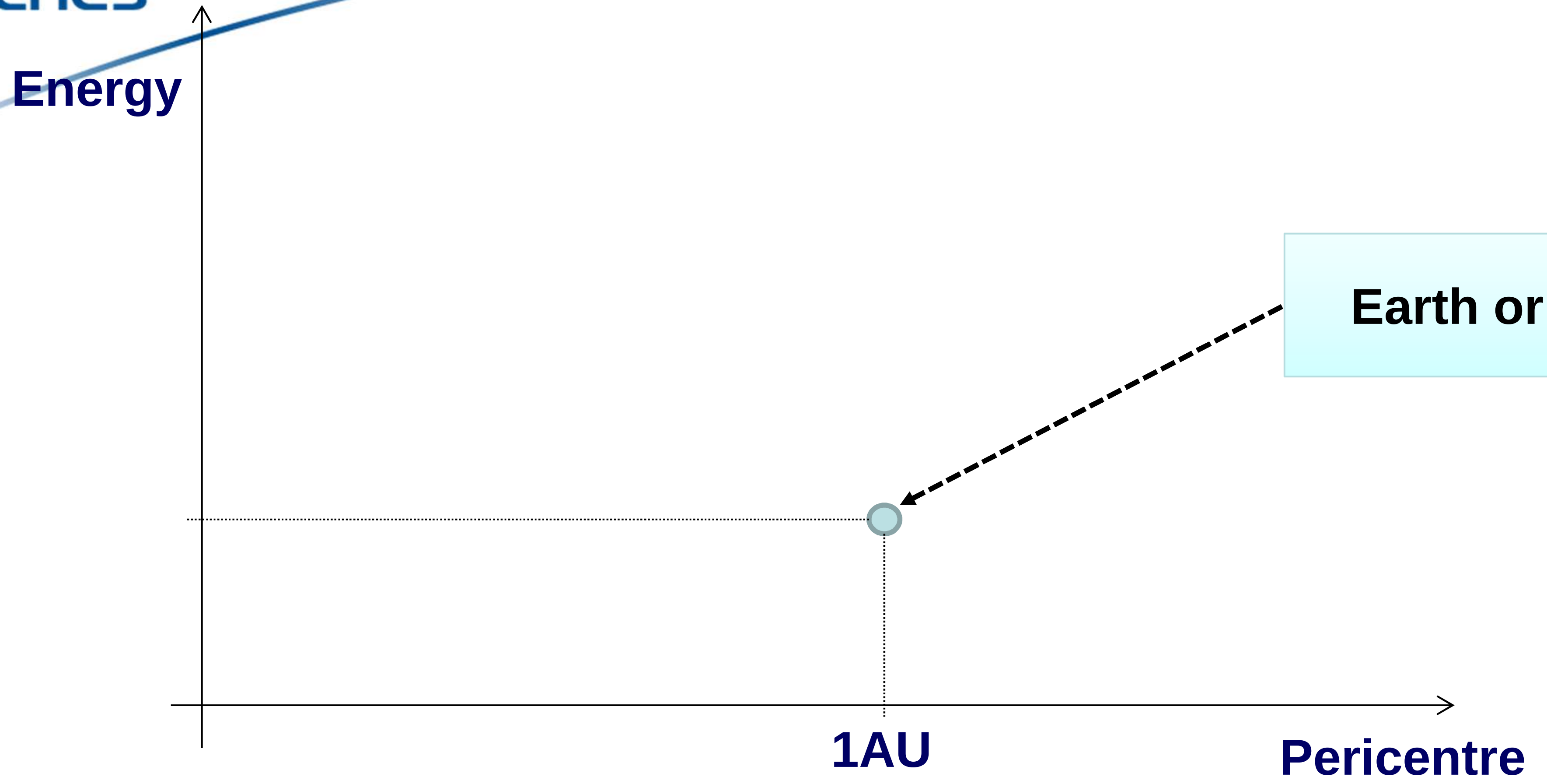




- Cooperation between ESA, CNES, DLR, ASI…
- Towards the comet 67P/Churyumov-Gerasimenko
- Launch performed the 02/03/2004 on an Ariane 5 LV (v_∞=3.5km/s, δ_∞=2deg)
- Cruise duration: 10 years
- 4 gravity assists maneuvers: Earth (2005, 2007, 2009) and Mars (2007)
- 2 asteroid flybys: Steins (2008) and Lutetia (2010)
- 4 Deep Space Maneuvers (ΔV = 204m/s)
- 2 Rendezvous maneuvers (ΔV = 1583m/s)
- Lander release on November 2014

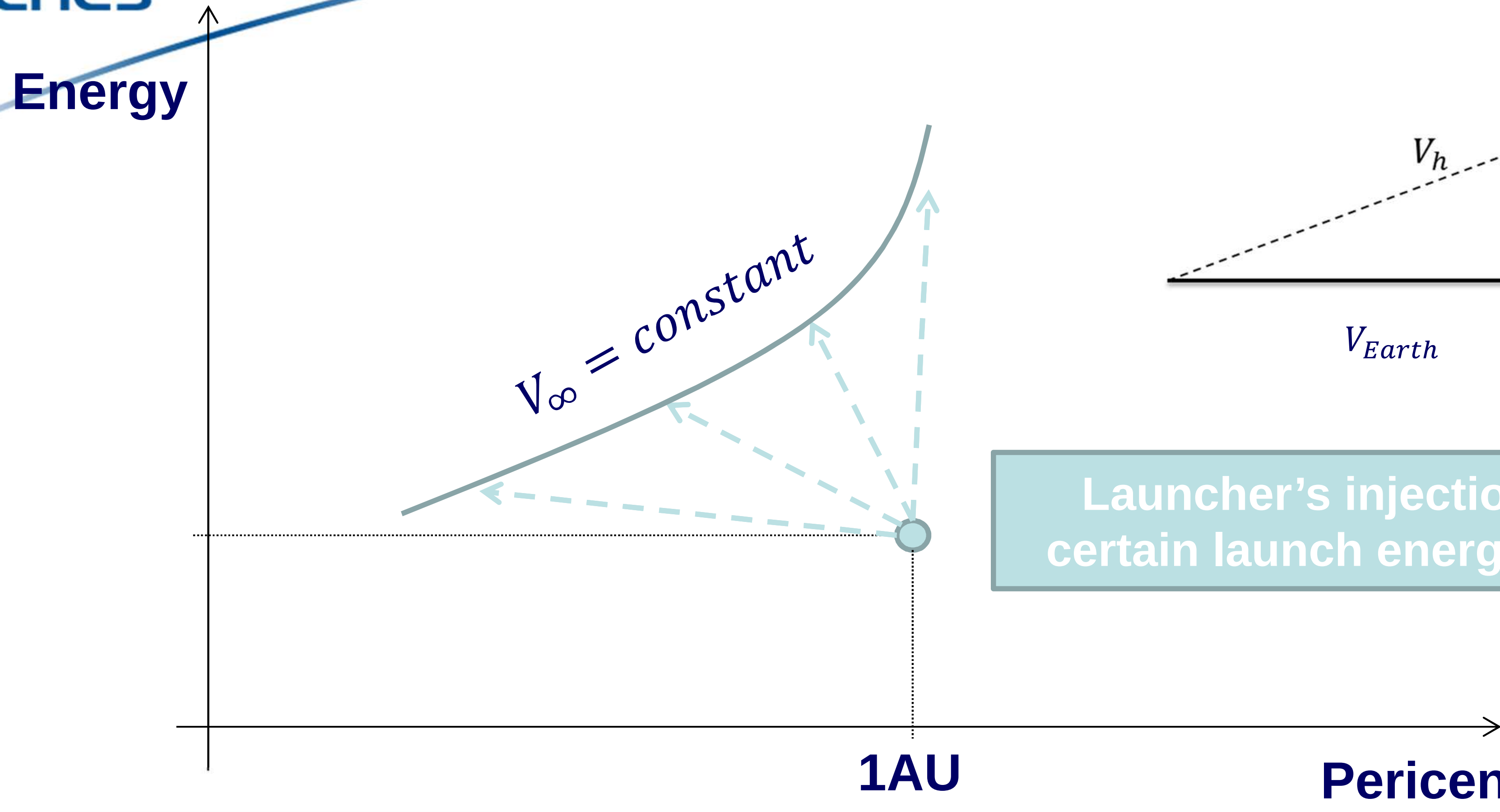


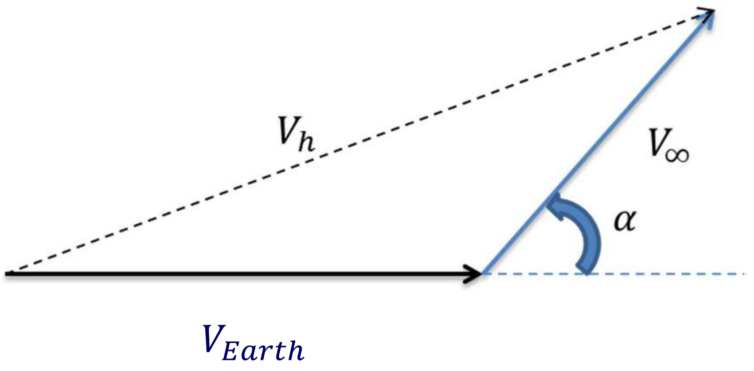
Tisserand graph principle



- **One point, one orbit**
- **Usually two variables are plotted (2D graphs)**
  - **Apocentre VS Pericentre**
  - **Pericentre VS Period**
  - **Pericentre VS Energy**
- **Gravity assist bodies are supposed to lie on the same plane ($i = 0$) and to have circular orbits**

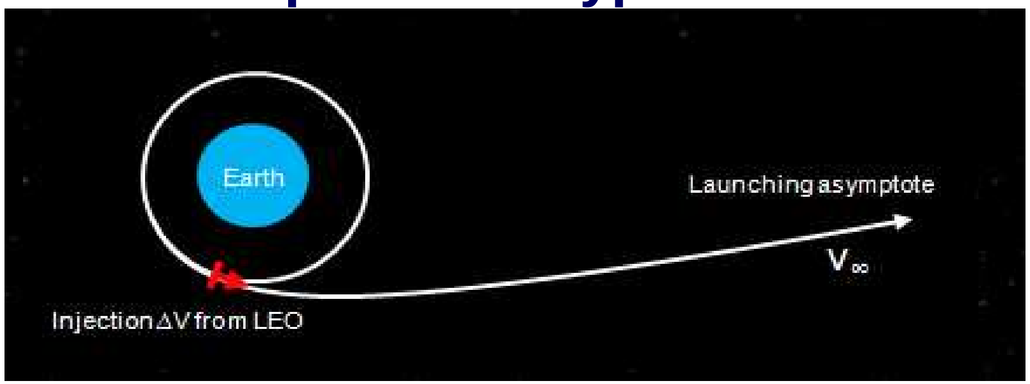
Tisserand graph principle





Launcher's injection: it gives a certain launch energy ($C3 = (V_\infty)^2$)

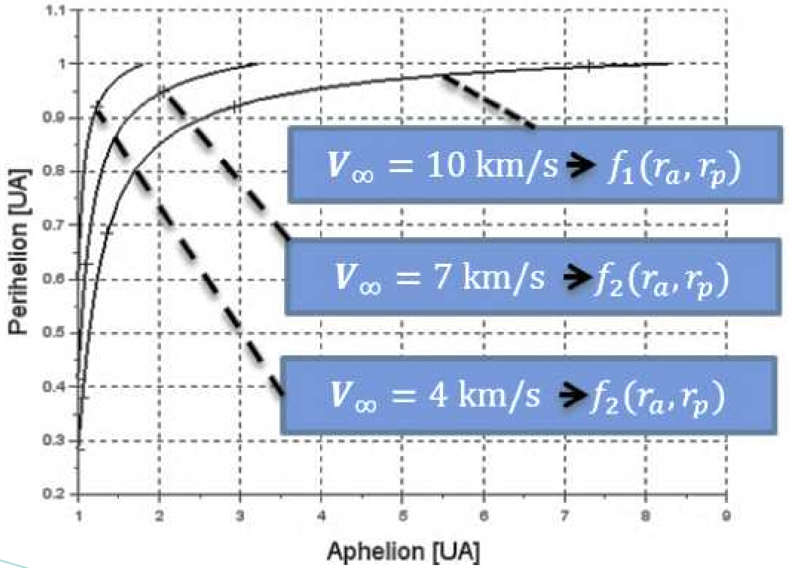
Departure's hyperbola

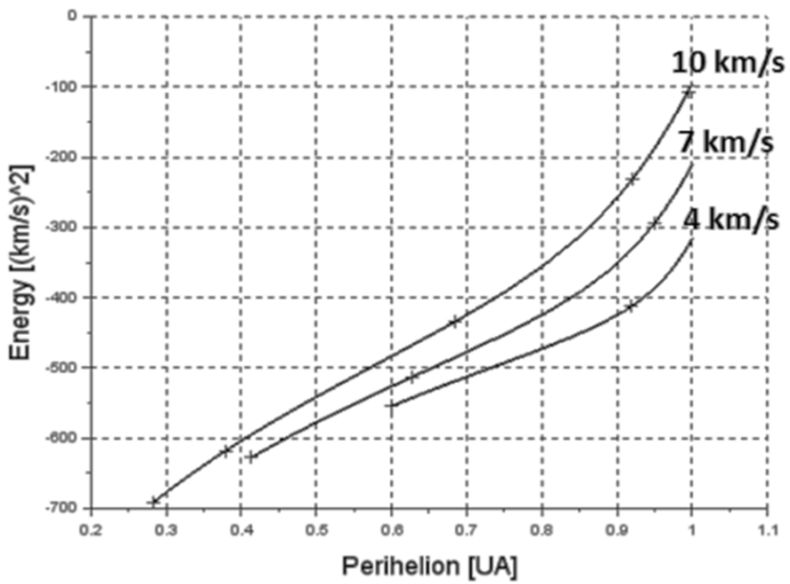


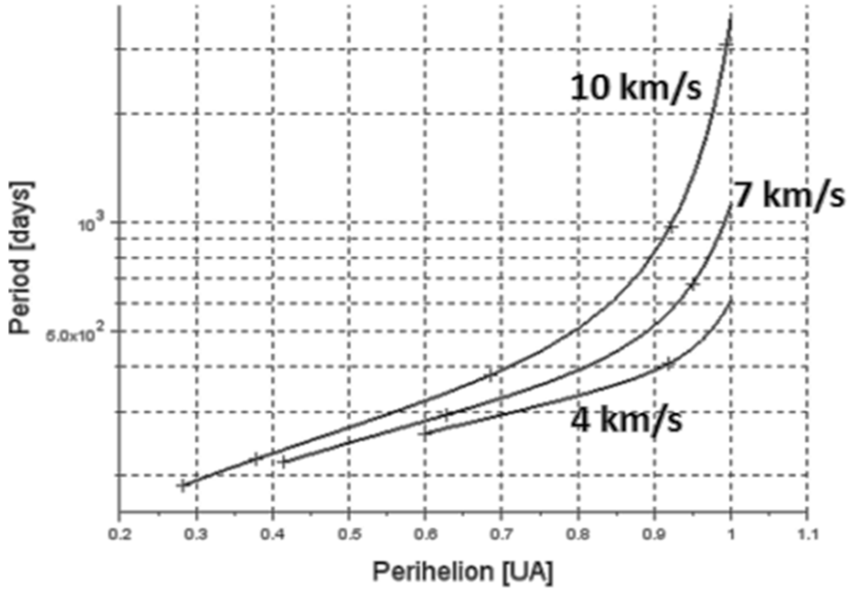
Tisserand graph principle

$$f(E, r_p) \leftrightarrow f(a, e) \leftrightarrow f(r_a, r_p) \leftrightarrow f(P, r_p)$$

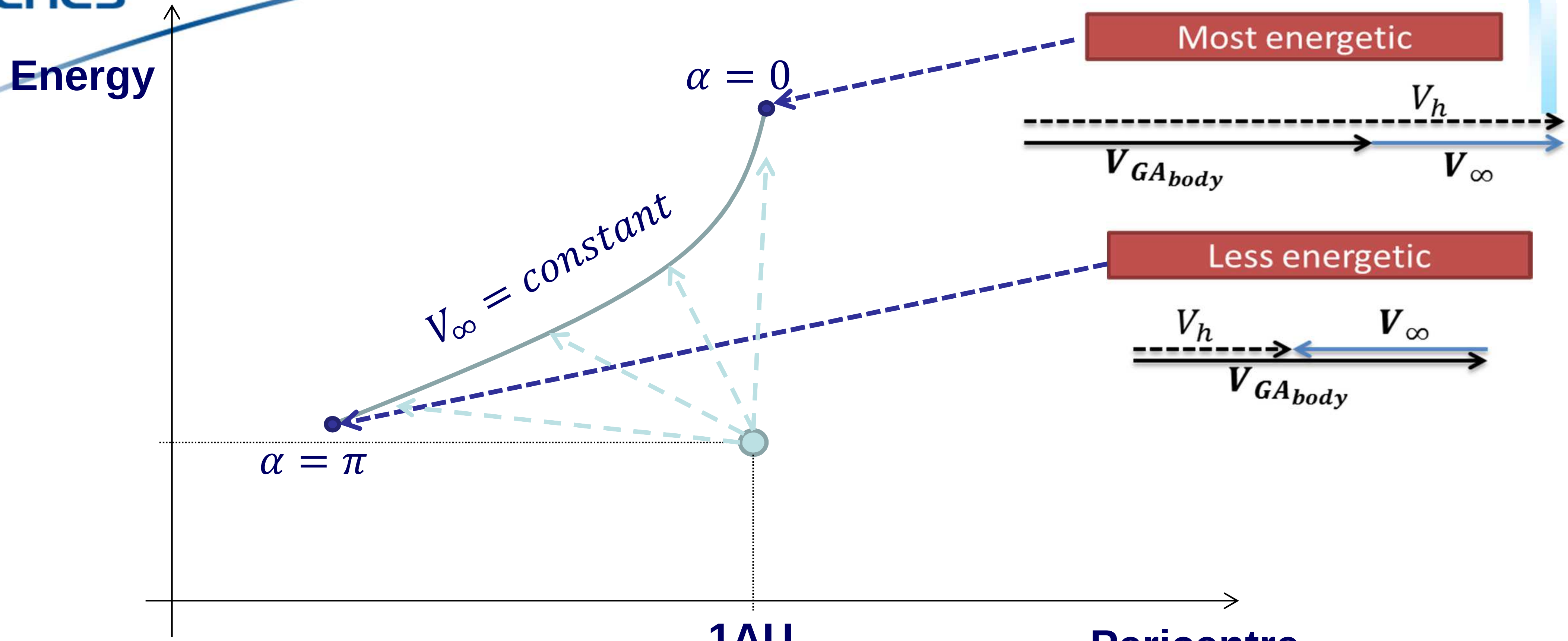
Equivalent plots (Tisserand graphs)

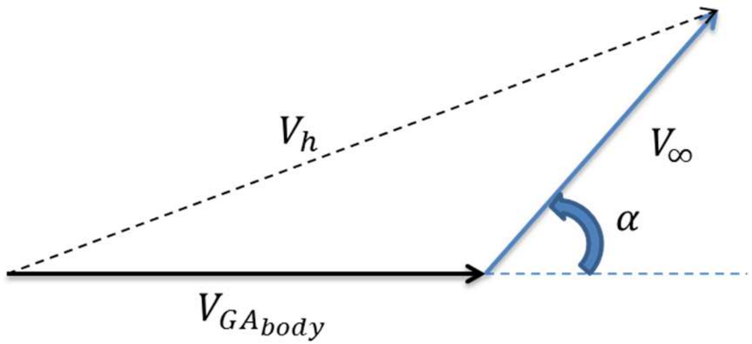






Tisserand graph principle

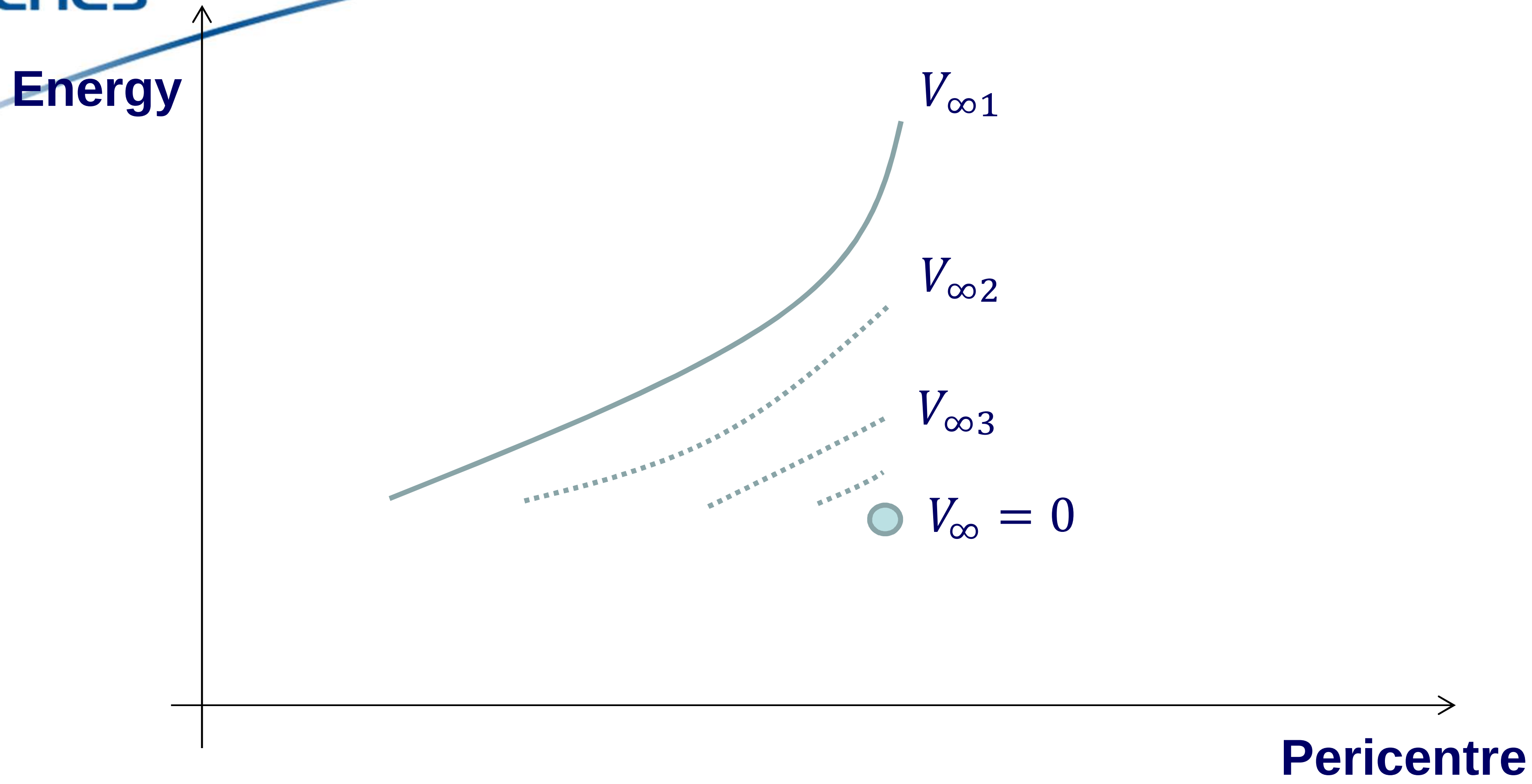




$$V_h = \sqrt{V_{GA_{body}}{}^2 + V_\infty{}^2 + 2V_{GA_{body}} V_\infty \cos\alpha}$$

$$\boldsymbol{E} \longrightarrow a \longrightarrow h \longrightarrow e \longrightarrow \boldsymbol{r_p}$$

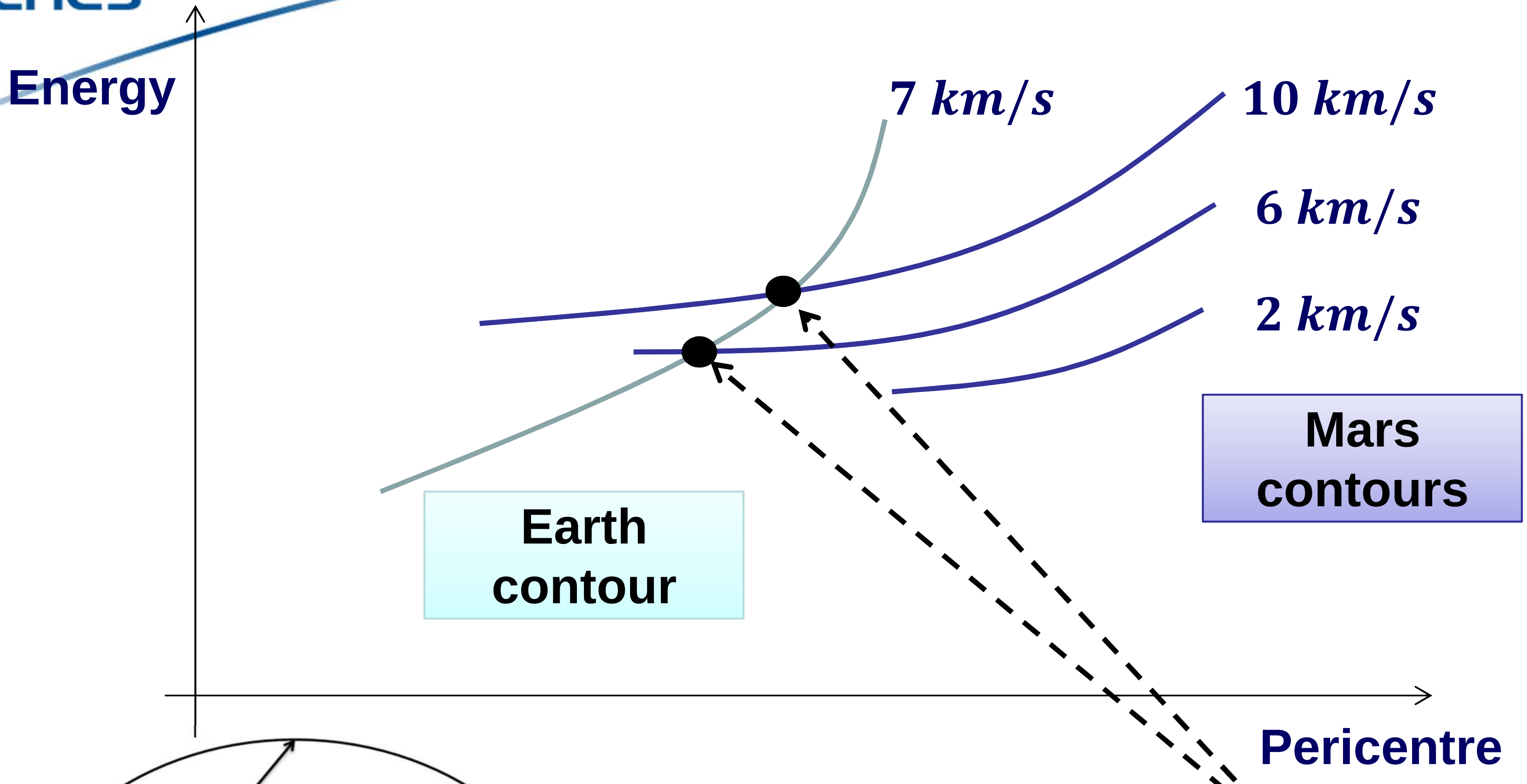
Tisserand graph principle

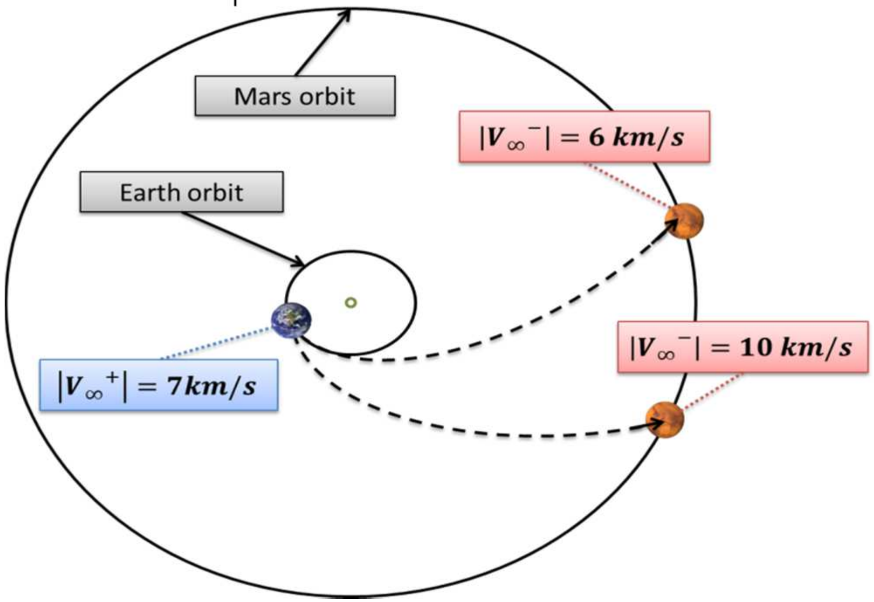


$V_{\infty 1} > V_{\infty 2} > V_{\infty 3} > 0$

- **Infinite number of contours might be plotted**

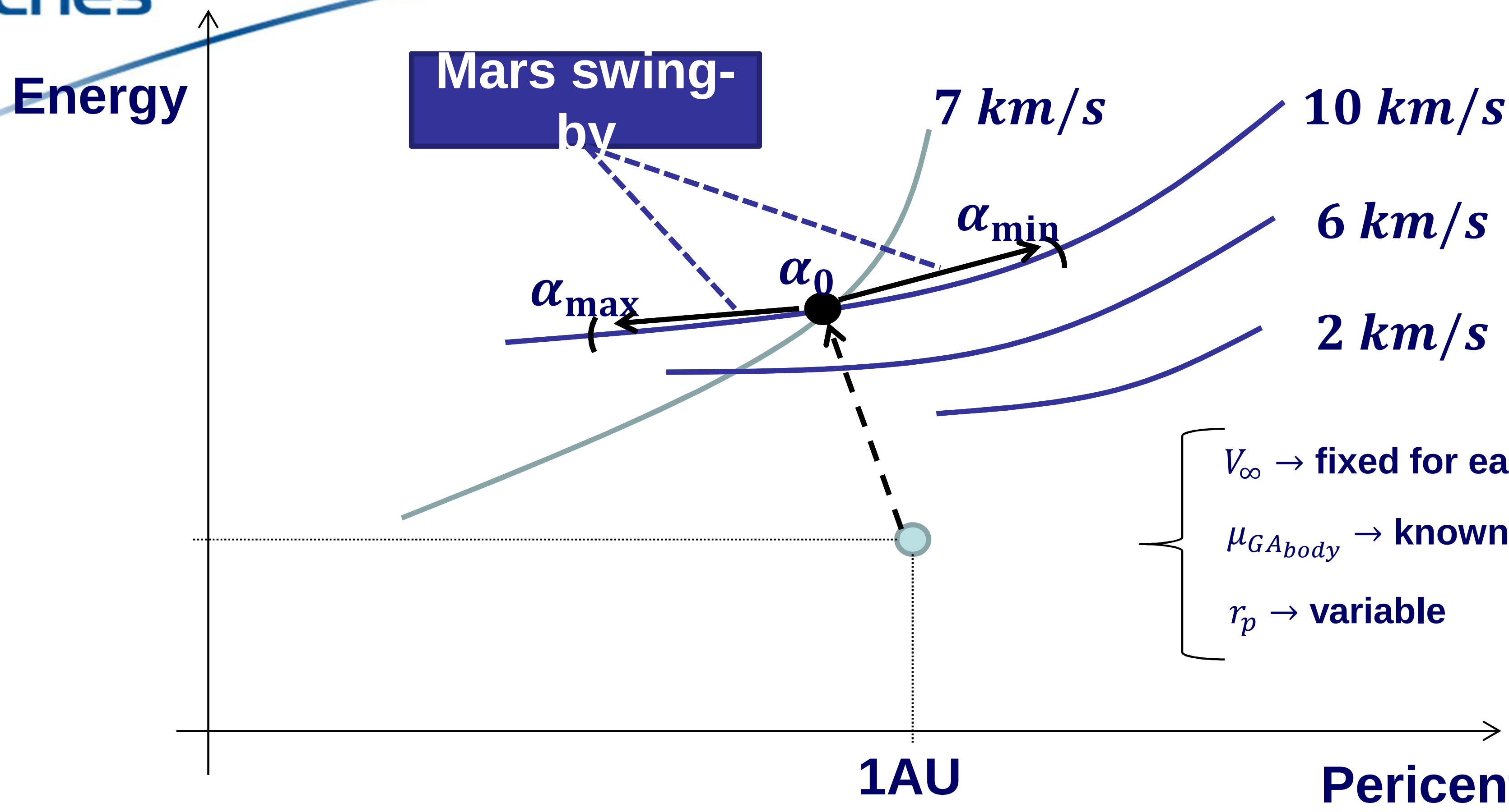
Tisserand graph principle



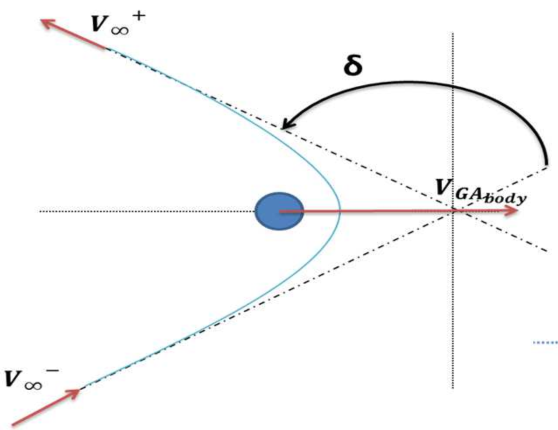


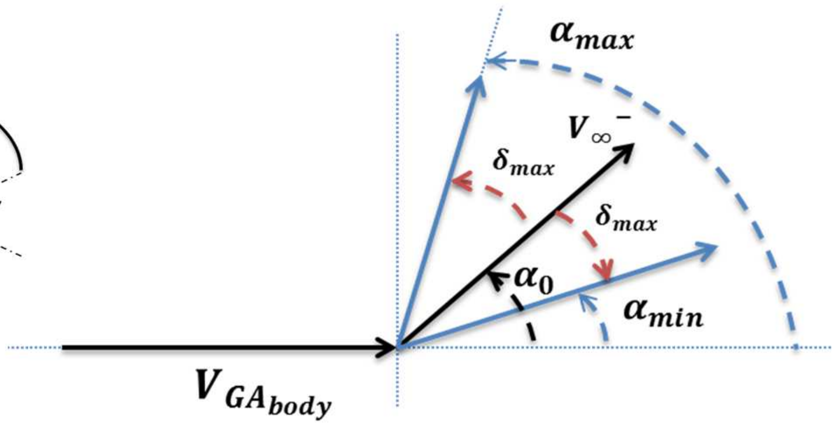
If one point belongs simultaneously to two different contours it means that this orbit intercepts both planets' orbits with the corresponding values of v-infinity

Tisserand graph principle



- V_∞ → fixed for each contour
- $\mu_{GA_{body}}$ → known for each GA body
- r_p → variable





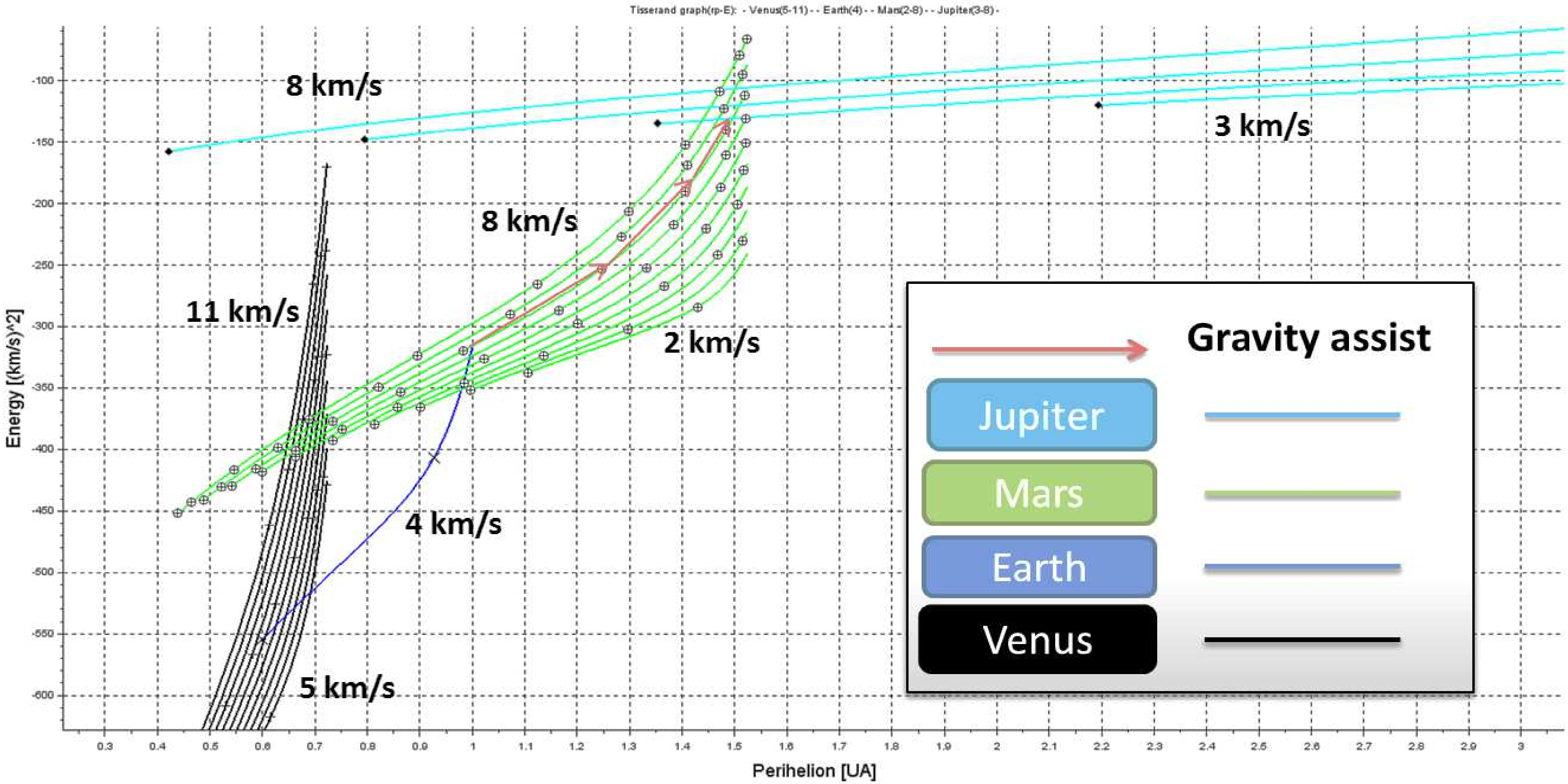
$$\delta = 2\,\mathrm{asin}\left(\frac{1}{1+\frac{r_p V_\infty{}^2}{\mu_{GA_{body}}}}\right)$$

Using Tisserand graphs

Example: Mission to Jupiter. Constraints:

- $h_{min} = 300\ km$
- $\left(V_{\infty Earth}\right)^2 = C3 = 16\ (km/s)^2$

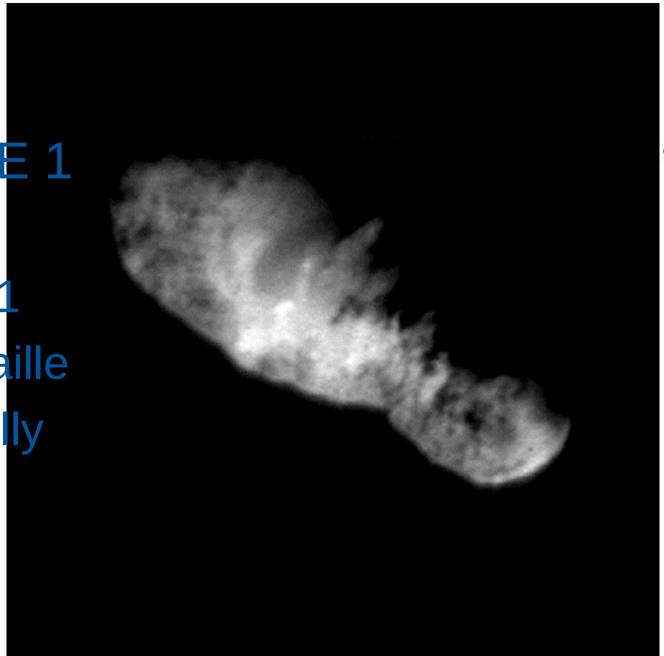
Feasible sequences	Non-feasible sequences
$E-M-M-M-J = E-M^3-J$	$E-J$
-	$E-M-J$
-	$E-M^2-J$
-	$E-V-J$
-	$E-V^n-J$



Low thrust interplanetary missions 1

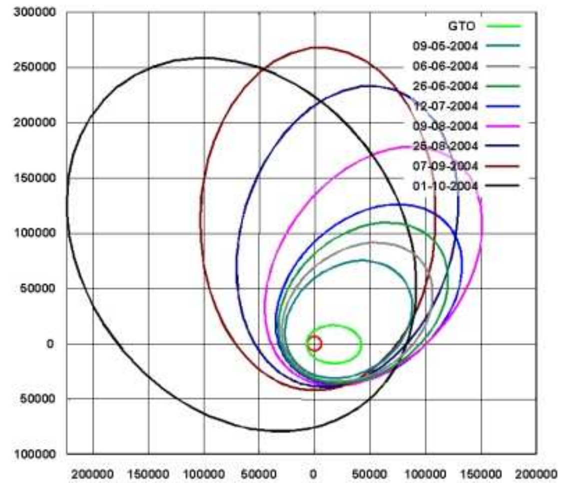
DEEP SPACE 1

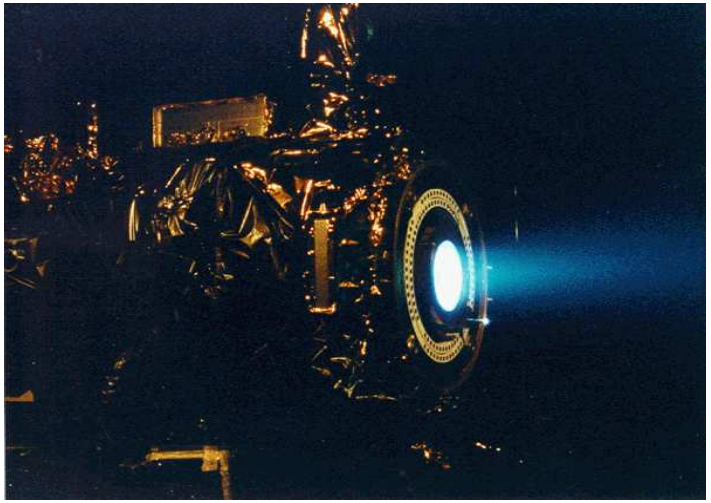
- NASA
- 1998 – 2001
- asteroid Braille
- comet Borelly

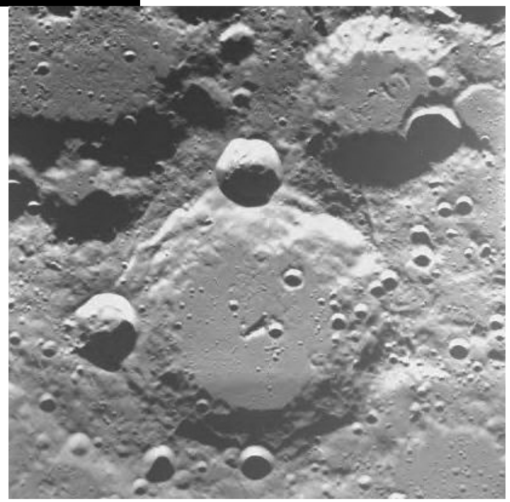


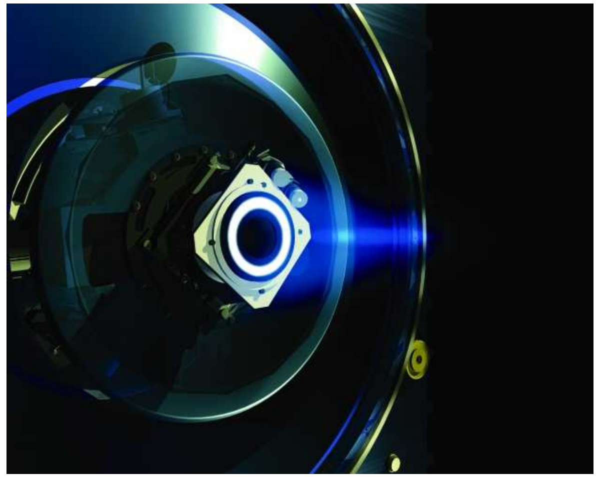
SMART 1

- ESA
- 2003-2006
- Moon



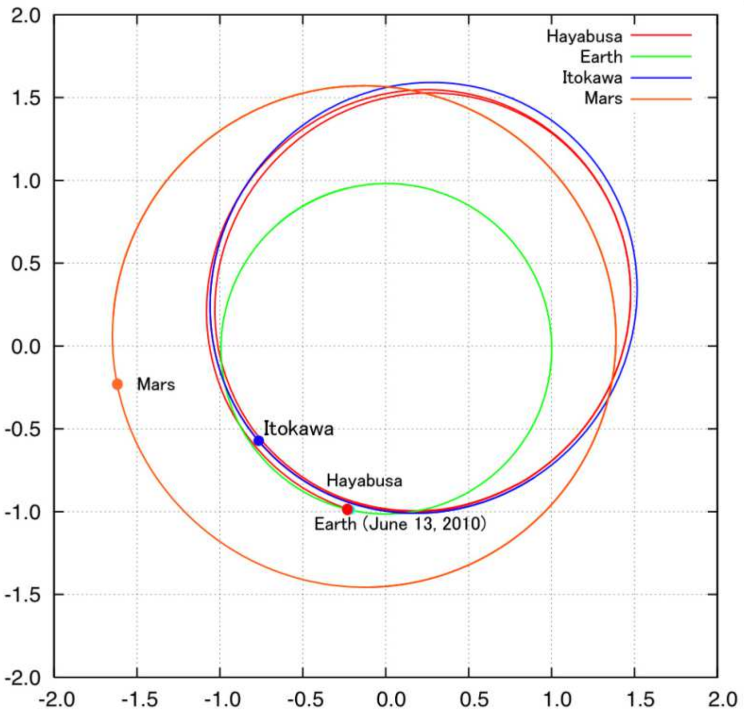


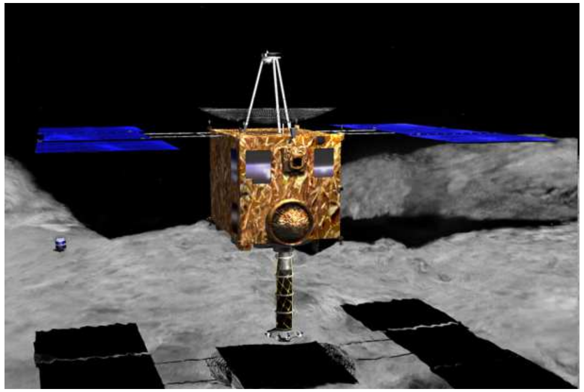




Low thrust interplanetary missions 2

- HAYABUSA 1
  - ISAS/JAXA
  - 2003-2010 (june 13th)
  - asteroid Itokawa (NEO)
  - sample return
- HAYABUSA 2
  - Mascot (DLR, CNES)
  - Arrival 2018
  - Asteroid Ryugu

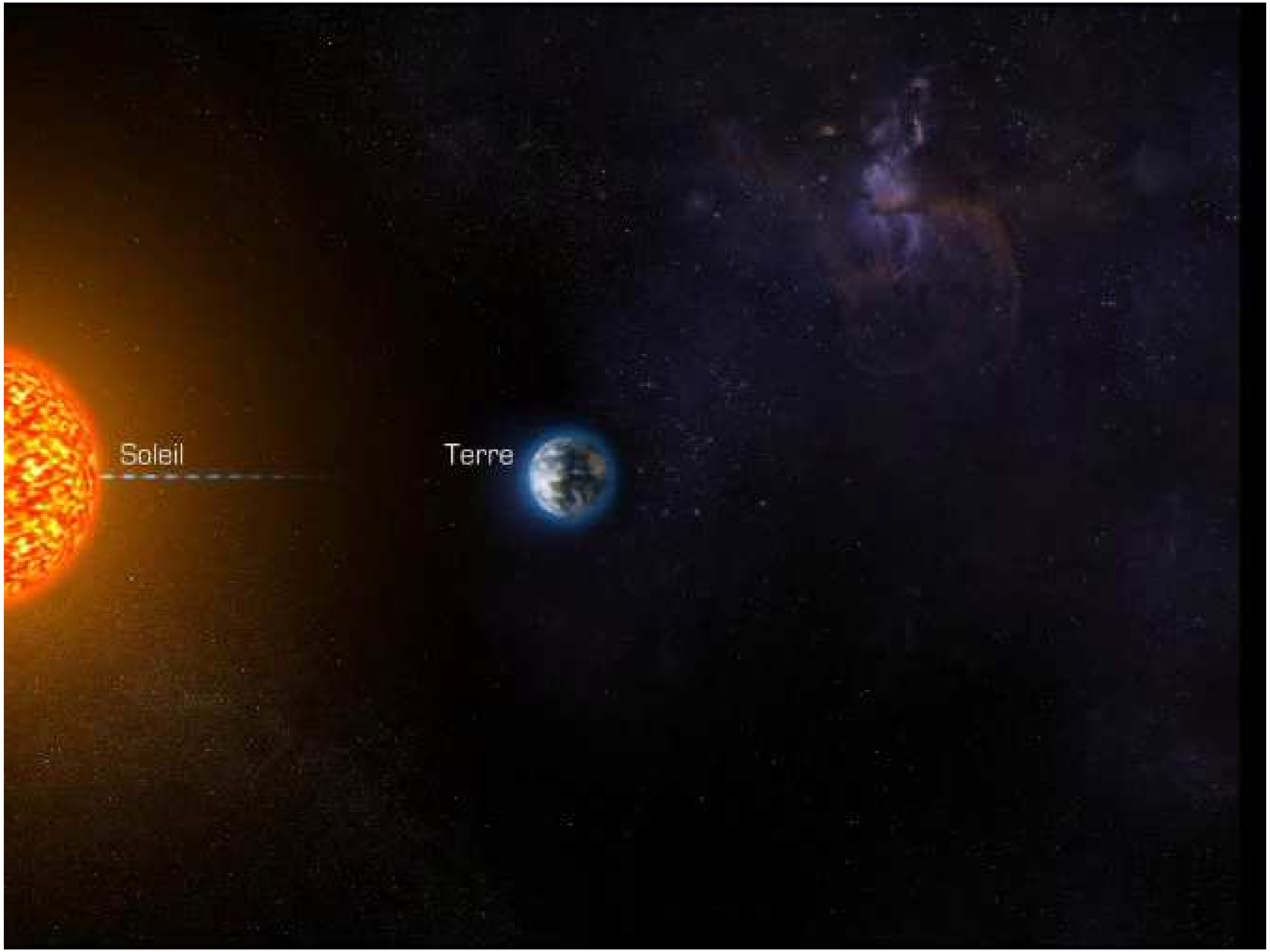




Lagrangian Points (1/3)

Lagrangian Points (2/3)

Equilibrium points (which does not mean stable points)



Lagrangian Points (3/3)