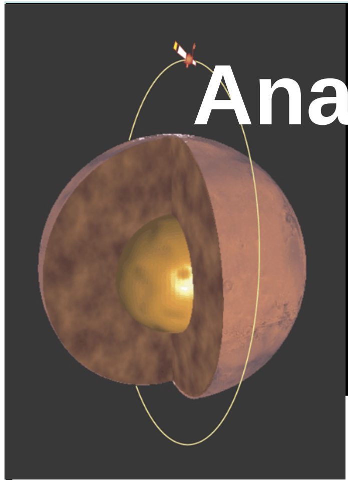
# *Trajectoires planétaires et interplanétaires*

## Analyses orbitales autour de Mars et Vénus

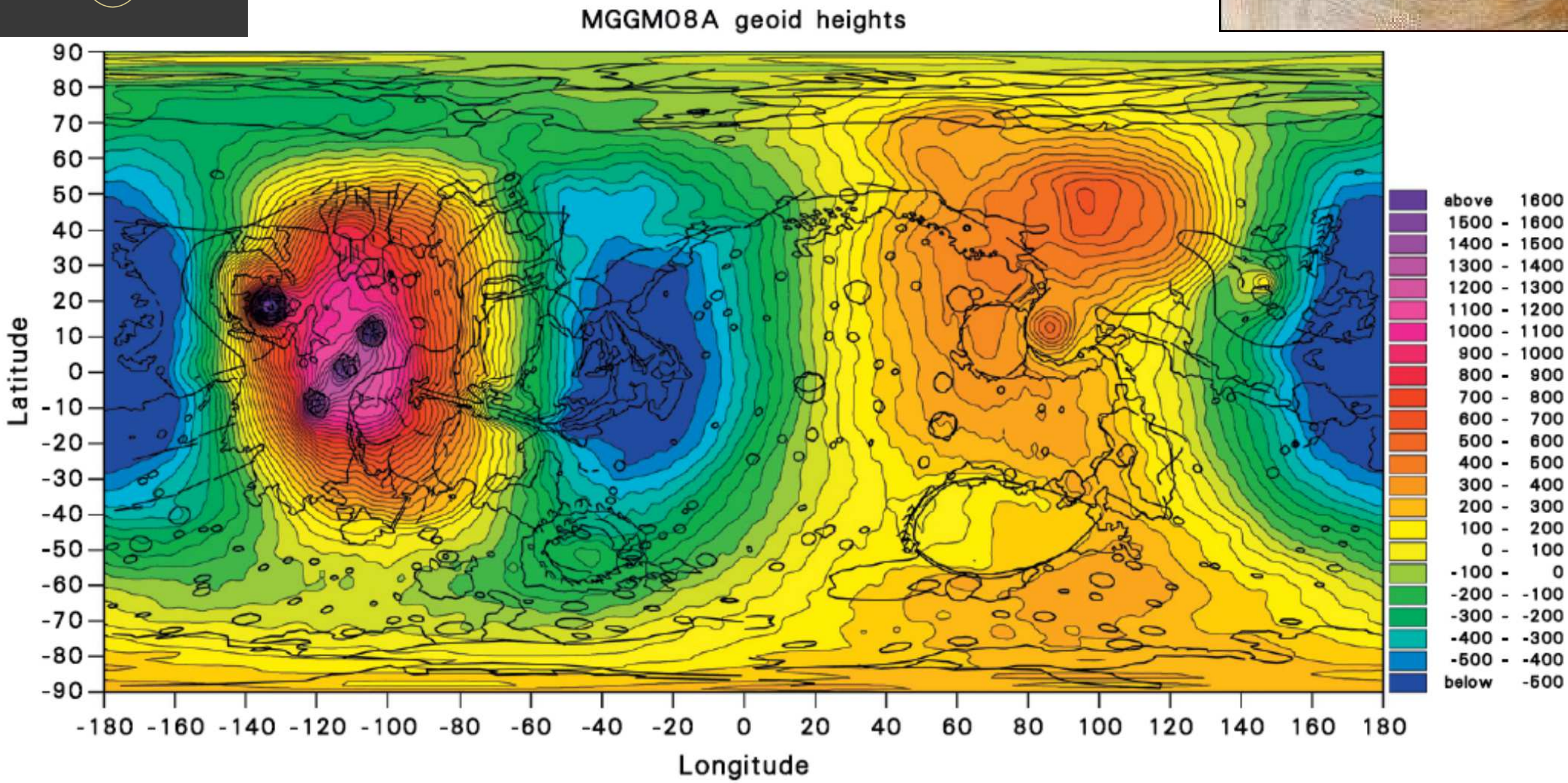
*Pascal Rosenblatt*

*Observatoire Royal de Belgique*



# Analyse orbitale autour de Mars et Vénus


**by Pascal Rosenblatt**
***(Royal Observatory of Belgium)***

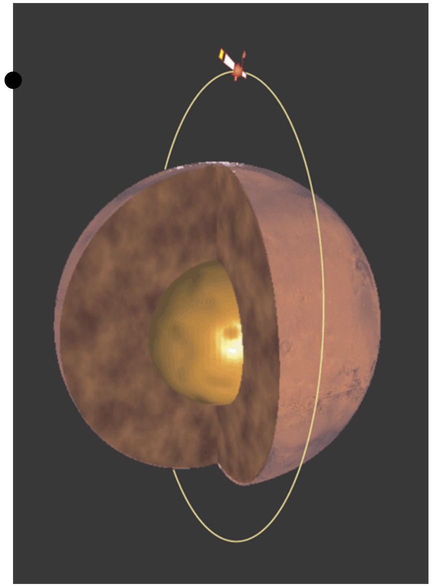



# Overview

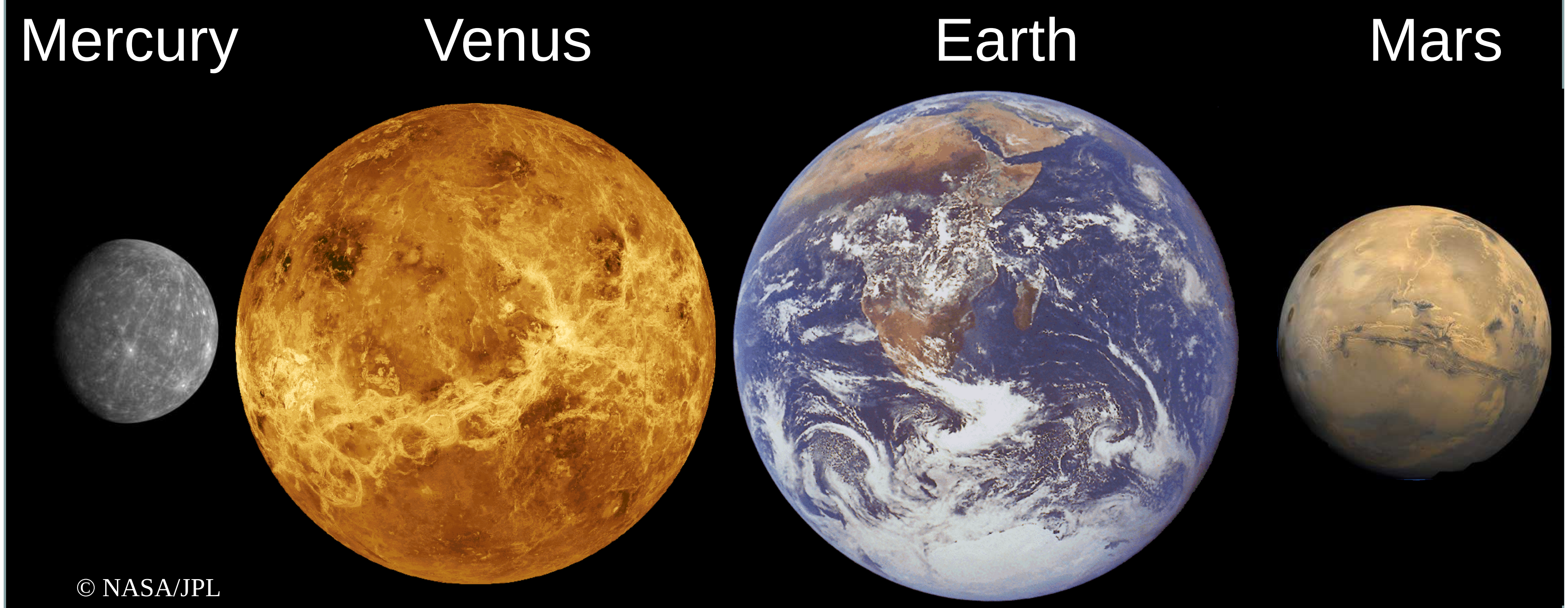
- Planetary geodesy provides a valuable way to get constrain on the interior as well as on the thermosphere of planets.

- We shall present the basic concepts of space geodesy and detail application involving orbiting spacecraft :

Mars' gravity field and its seasonal variations
Mars' deep interior and $k_2$ tidal Love number
Mass of the Martian moons, Phobos & Deimos
Venus' atmosphere density at high altitude



# Interior of terrestrial planets: a key to understand their evolution




- size
- composition
- physical state
- temperature

→

- formation
- evolution
- dynamics
- life

# Planetary atmospheres:
# study of 'school-case' for Earth's climate evolution

VENUS: 500° C

EARTH: 15° C

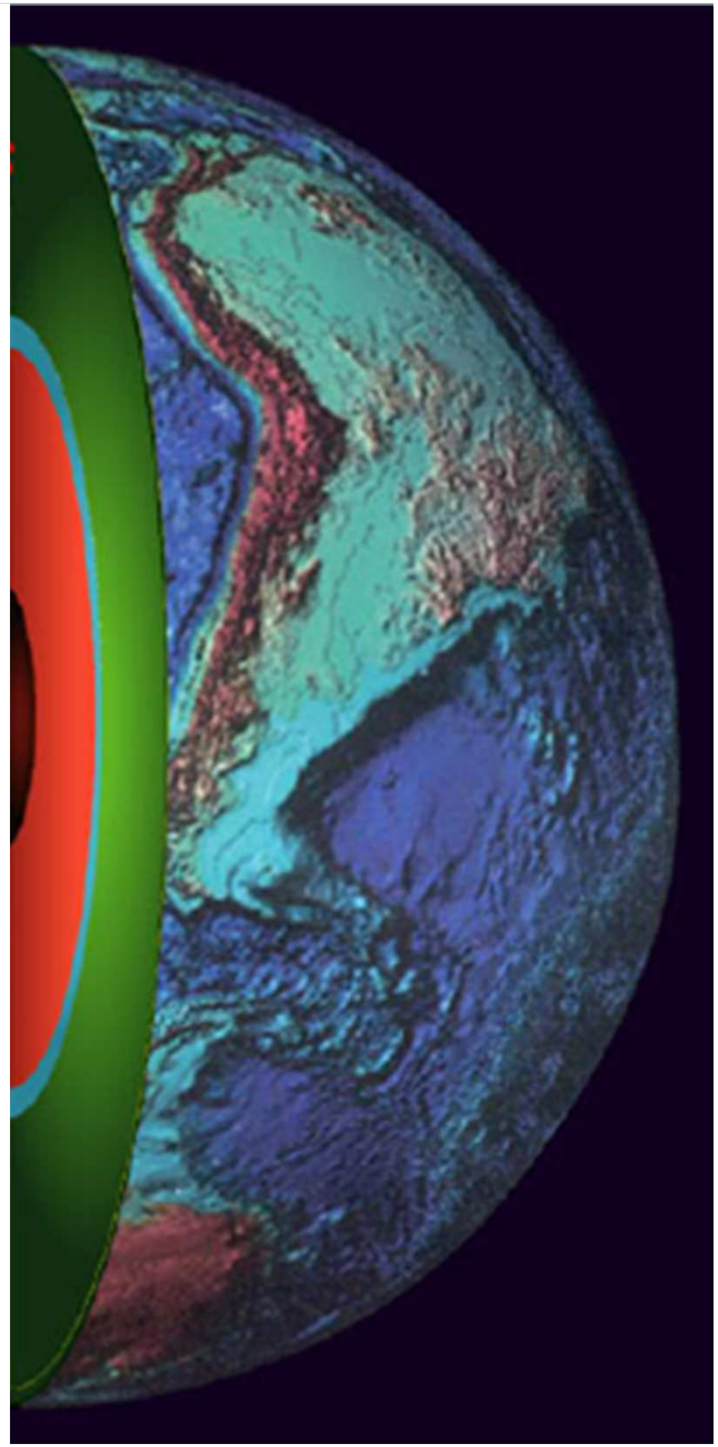
MARS: -50° C

- Which processes drive atmospheric dynamics, climate, … ?
- Determination of present dynamics and past and future evolution of the atmosphere.

- To date, Geodesy (and astrometry as well) is the single way to constrain planetary interior *(except for the Moon)*

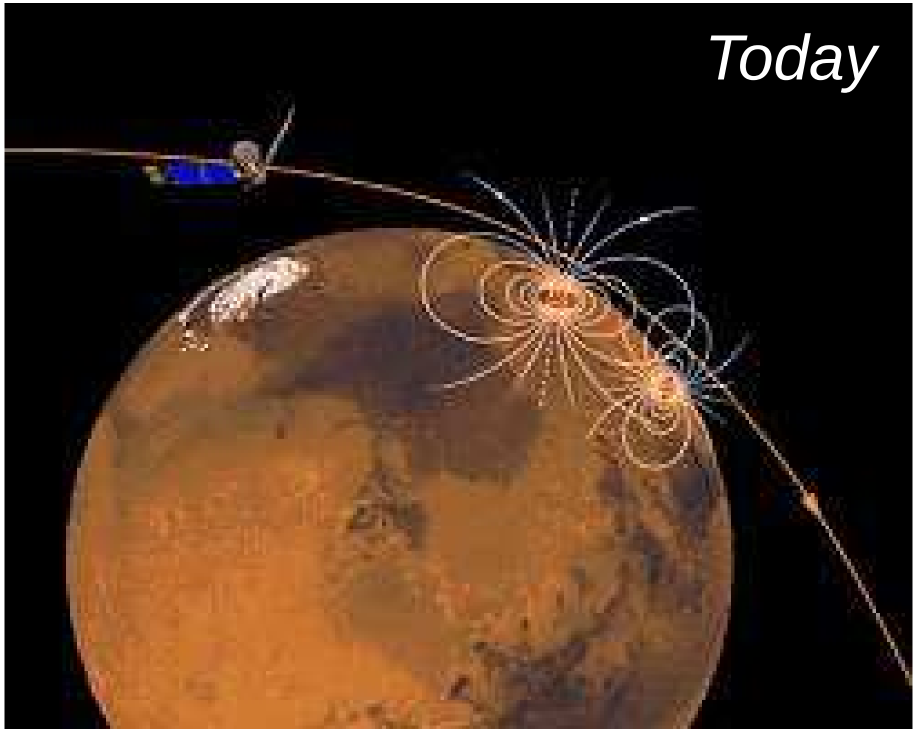
- Monitoring of the ***gravity field,*** rotation, tides, nutations.

- I will focus on the method, show the most recent results for Mars and Venus.

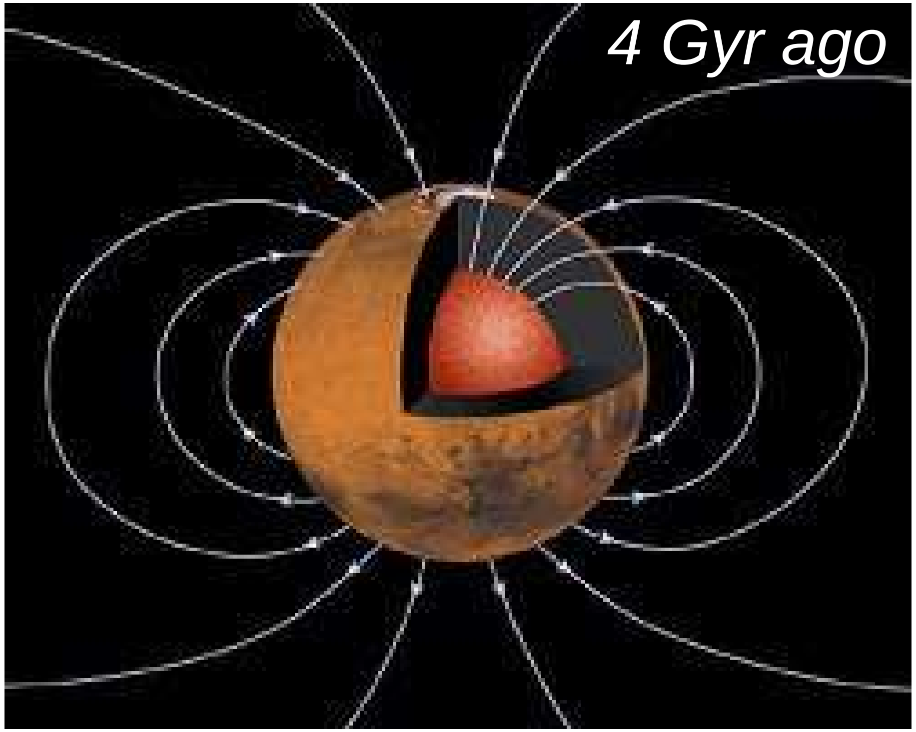


# Aréophysique: Mars deep interior

*Mars Global surveyor (2000)*



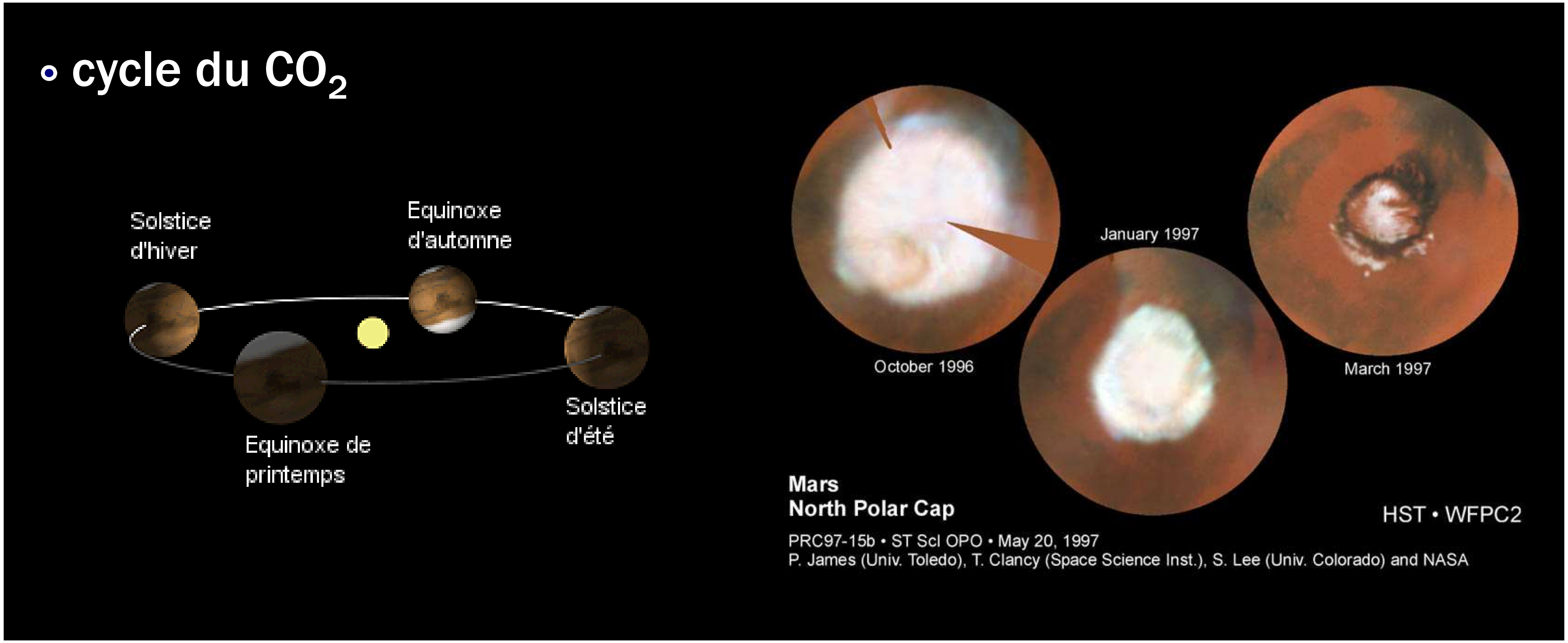

Crustal magnetic anomalies
on old terrain (4Gyr)




Mars had a Dipole Magnetic
field 4 Gyr ago

What is the state, size, composition of Mars’ core today?

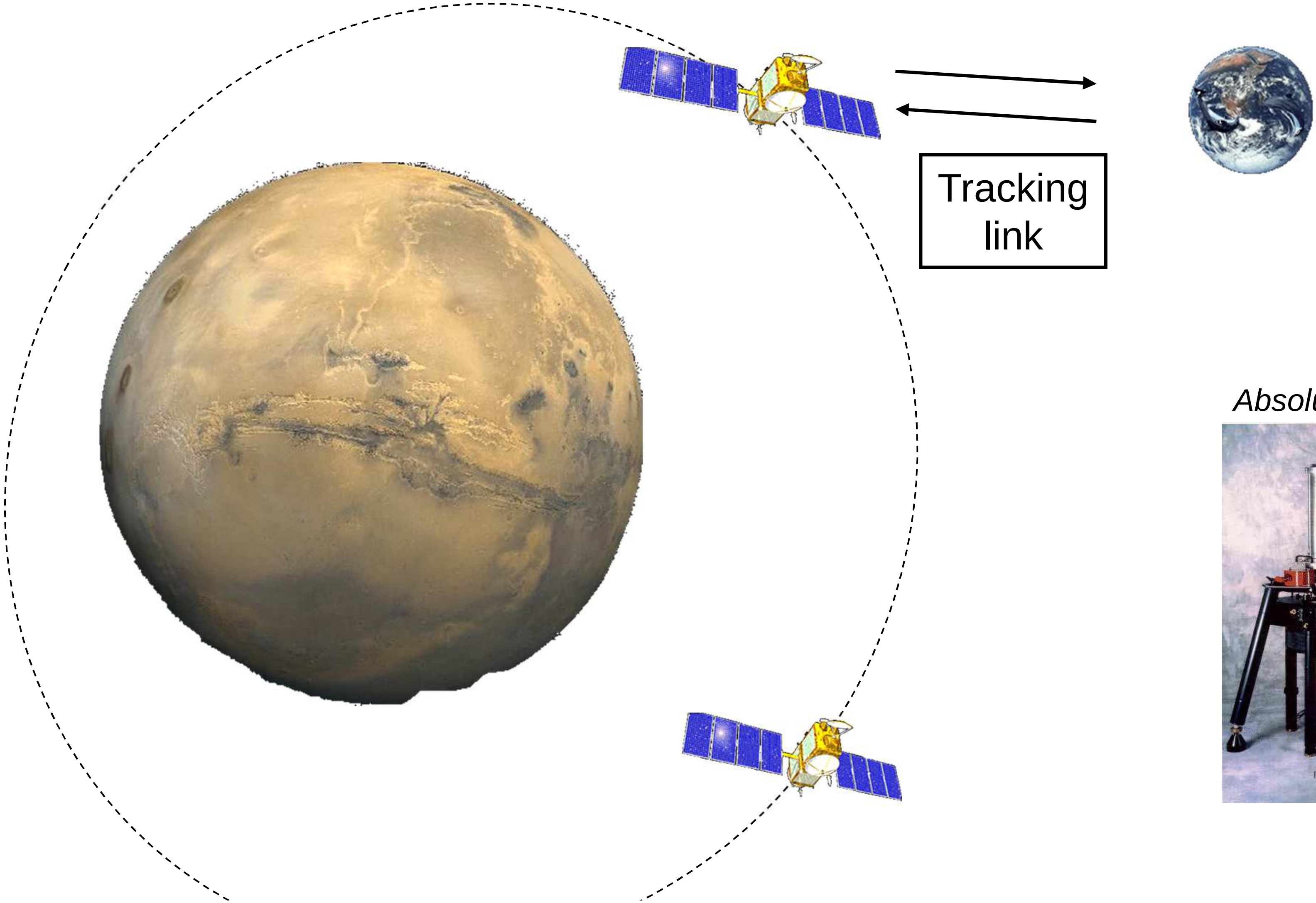
# Aréophysique: Mars' $CO_2$ seasonal cycle




- Up to 25% of total mass of the Martian atmosphere.

→ Seasonal gravity field variations

# How to measure gravity field with a spacecraft?




*Absolute Gravimeter*

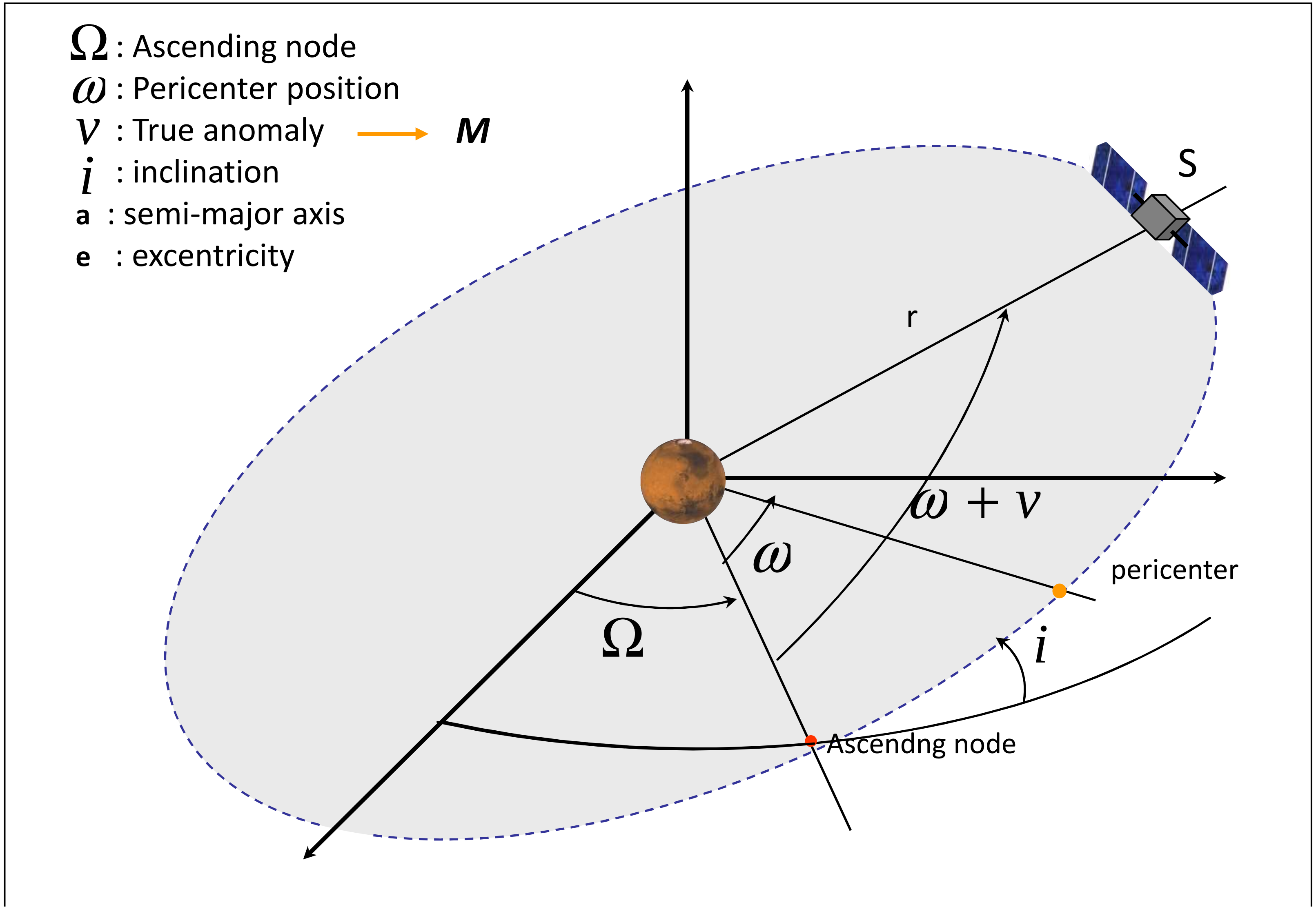

Principle: Monitoring the free-fall motion of the spacecraft
as for the falling-ball of an absolute gravimeter

# Spherical and homogeneous planet: Keplerian trajectory


- The trajectories of planets around the Sun follow ellipses around the Sun (*Kepler's laws*).
- Theory of gravitation (Newton).
- The trajectory of an artificial satellite around a spherical and homogeneous planet follow Kepler's laws (*first approximation*)

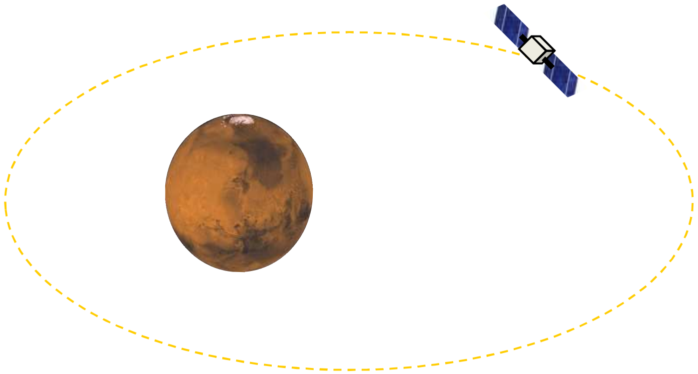
→ *The artificial satellite or orbiter trajectory follows an ellipse whose focus is at the center of mass of the planet.*
*This elliptical trajectory remains in a plane fixed in space.*

# Positioning in space: 6 Orbital Elements (OE):




- **Ω** and ***i*** allow positioning the orbital plane in space
  ***a** and **e*** provide the shape of the ellipse and ***ω*** allows positioning it in the orbital plane
  *v* or ***M** (mean anomaly)* allow positioning the orbiter on the ellipse at any given time *t*.

Spherical and homogeneous planet:



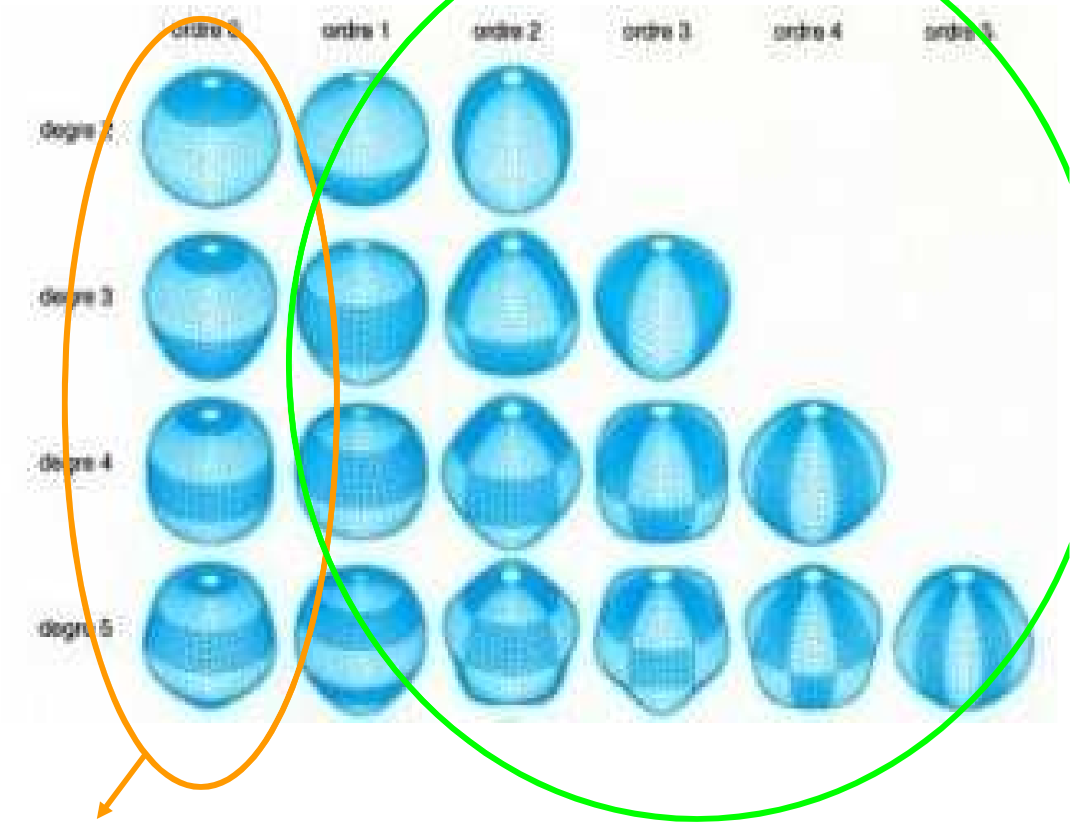
All Orbital Element do not change except the mean anomaly *M*

→ Kepler motion

Density anomalies inside the planet:

Tesseral terms

$C_{l,m}$ , $S_{l,m}$




Zonal terms : $J_l = - C_{l,0}$

All Orbital Elements vary

→ Perturbation of Keplerian motion

# Representation of planet gravity field (1)

✓ *Spherical harmonics expansion of gravity field at any point out of the planet*

$$U = \frac{GM}{r} + GM \sum_{l=1}^{l=\infty} \sum_{m=0}^{m=l} \frac{a_e^l}{r^{l+1}} P_{lm}(\sin(\varphi))(C_{lm}\cos(m\lambda) + S_{lm}\sin(m\lambda))$$

Central Potential $U_0$:
(Point-mass representation)
Keplerian motion

Perturbing potential $U_p$:

That potential is completely defined by the set of harmonics coefficients:

$C_{lm}$, $S_{lm}$, as well as by $a_e$ and $GM$.

With:
$(r,\phi,\lambda)$ spherical coordinates of a point P outside the planet
$a_e$ equatorial radius of the planet

$P_{lm}(\sin(\varphi))$ Legendre Polynomials

$C_{lm}$ et $S_{lm}$ Stokes or harmonics coefficients

*l and m* degree and order of the spherical harmonics expansion
Practically it is truncated at a maximal degree: *L.*
*GM (=$C_{00}$) Gravitational constant times Planet's mass*.

➢ The precise reconstruction of the perturbed orbital motion of the spacecraft allows retrieving the harmonics coefficients: $C_{lm}$, $S_{lm}$

# Representation of planet gravity field (2)

✓ W. Kaula has developed the perturbing gravitational potential $U_p$ as a function of the Orbital Elements: *a, e, i, ω, Ω et M*.

Inclination functions only depend on "*i*"

Eccentricity functions only depend on "*e*"

$$U_p = GM \sum_{l=2}^{l=L} \sum_{m=0}^{m=l} \frac{a_e^l}{a^{l+1}} \sum_{p=0}^{p=l} F_{lmp}(i) \sum_{q=-\infty}^{q=+\infty} G_{lpq}(e)\, S_{lmpq}(\omega, M, \Omega, \theta)$$

$$S_{lmpq}(\omega, M, \Omega, \theta) = \begin{bmatrix} C_{lm} \\ -S_{lm} \end{bmatrix}_{l-m\ impair}^{l-m\ pair} \cos(\psi_{lmpq}) + \begin{bmatrix} S_{lm} \\ C_{lm} \end{bmatrix}_{l-m\ impair}^{l-m\ pair} \sin(\psi_{lmpq})$$

$$\psi_{lmpq} = (l-2p+q)M + (l-2p)\omega + m(\Omega - \theta)$$ "gravity" argument of Kaula

*θ is the sideral angle of reference longitude at planetary surface (spin)*

➢ This new expression of the gravitational potential allows deriving the variations of the Orbital Elements due to each harmonics coefficients $C_{lm}$ et $S_{lm}$

→ Lagrange's equations

# Equation of motion: Lagrange's equations (central and perturbed gravitational potential)

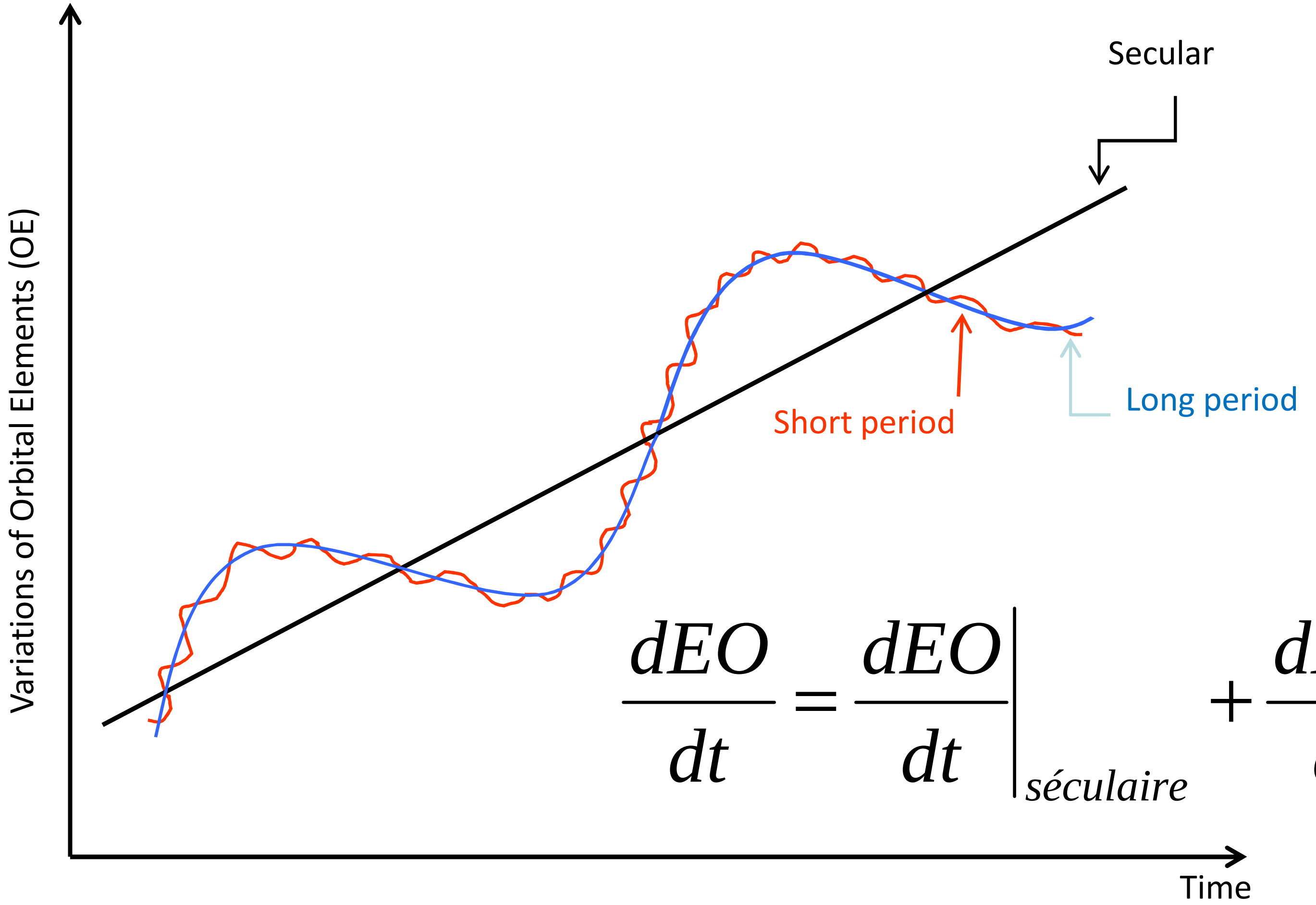
$$
\begin{aligned}
\frac{da}{dt} &= \frac{2}{na}\frac{\partial U_P}{\partial M} \\
\frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial U_P}{\partial \omega} + \frac{1-e^2}{na^2 e}\frac{\partial U_P}{\partial M} \\
\frac{di}{dt} &= \frac{-1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial U_P}{\partial \Omega} + \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial U_P}{\partial \omega} \\
\frac{d\Omega}{dt} &= \frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial U_P}{\partial i} \\
\frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial U_P}{\partial e} - \frac{\cos i}{na^2\sqrt{1-e^2}\sin i}\frac{\partial U_P}{\partial i} \\
\frac{dM}{dt} &= n - \frac{2}{na}\frac{\partial U_P}{\partial a} - \frac{1-e^2}{na^2 e}\frac{\partial U_P}{\partial e}
\end{aligned}
$$

Unperturbed Keplerian motion: *Up=0*
therefore: $M=n(t-t_p)$

- Lagrange's equations for a motion driven by forces derived from a potential.
- '*a, e ,i ,Ω ,ω* , et *M*' are the Orbital Elements (OE)
- 'n' is the mean motion of the unperturbed Keplerian motion.
- '*Up*' is the perturbing potential accounting for the non-spherical terms of the gravititaional potential of the planet.

- Perturbed Keplerian motion: $Up \neq 0$
- One cannot obtain accurate solution of Lagrange's equations using the perturbation method. Hereafter, is presented the Kaula's method (1966).

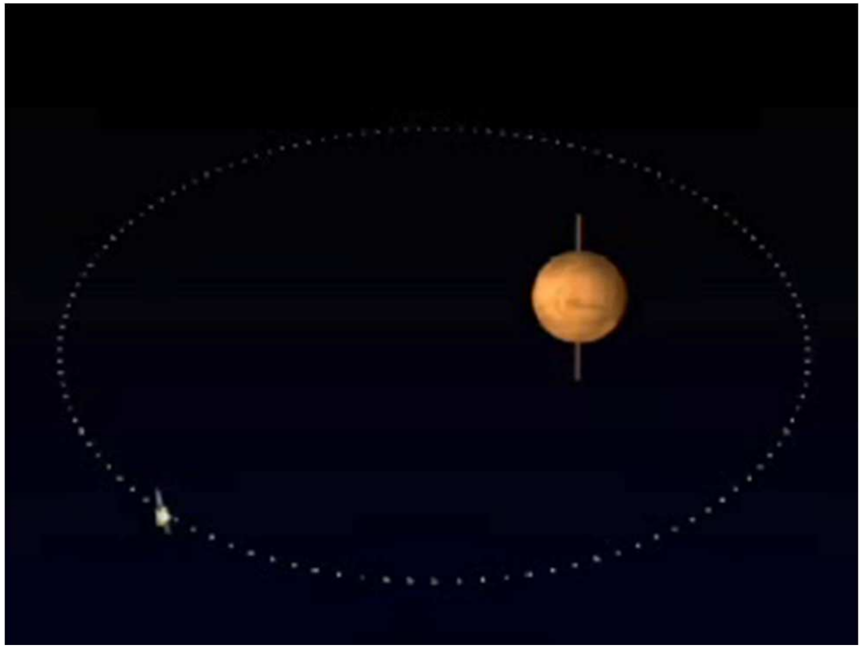
# Kaula's solution (1966)




$$\frac{dEO}{dt} = \left.\frac{dEO}{dt}\right|_{séculaire} + \left.\frac{dEO}{dt}\right|_{périodique}$$

- Integration of Lagrange's equation into two steps:
  1) Secular drifts then 2) Périodic terms

# First order perturbation: Precessing Orbit

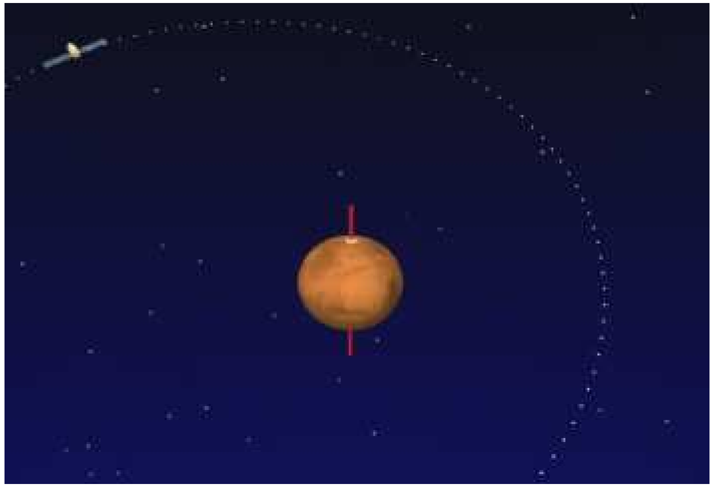


- Secular drift of the ascending node around the rotation axis of the planet:

$$\boxed{\frac{d\Omega}{dt} = \frac{3n\cos(i)}{2(1-e^2)^2}\left(\frac{a_e}{a}\right)^2 C_{20}}$$

- Proportional to $C_{20}$ and $\cos(i)$

# First order perturbation: Precessing Orbit
# Drift of percienter argument: $\omega$



- Secular drift of the pericenter argument (rotation of the ellipse in the orbital plane:

$$\boxed{\frac{d\omega}{dt} = -\frac{3n}{(1-e^2)^2}\left(\frac{a_e}{a}\right)^2\left(1-\frac{5}{4}\sin^2(i)\right)C_{20}}$$

- Proportional to $C_{20}$

# Longest period perturbations:
# Oscillation of pericenter position and excentricity

*Again from Kaula's linear theory:*

$$\boxed{\frac{dp}{dt} + j\dot{\omega}_S p = -\frac{3n}{2(1-e^2)^3}\sin(i)\left(\frac{a_e}{a}\right)^3\left[\left(\frac{5}{4}\sin^2(i)-1\right)+\left(\frac{1}{2\sin^2(i)}-\frac{31}{8}+\frac{15}{4}\sin^2(i)\right)e^2\right]C_{30}}$$

<u>Important remarks:</u>

*(1) "$p=p_0\,exp(-j\omega)$"* is the excentricity vector. It follows periodic variations: $T_S = \dfrac{2\pi}{\dot{\omega}_S}$
The period depends on the secular drift of pericenter position
(i.e. a, e, i, and $C_{20}$)

*(2)* The amplitude of this periodic variation varie as $(1/a)^3$ so less than as $(1/a)^2$

*(3)* This amplitude is proportional *to sin(i)*.
So it is zero for equatorial orbit, $i=0°$
and maximal for polar orbits, $i=90°$

# Others periodic perturbations

$$\psi_{lmpq} = (l-2p+q)M_s + (l-2p)\omega_s + m(\Omega_s - \theta) \neq 0$$

- The gravitational potential depends on all Orbitla Elements (OE) and is expressed through the summation over all the indexes *l,m,p,q of* $\psi_{lmpq}$

$$\frac{dOE}{dt} = GM\sum K(a,e,i)\cos(\psi_{lmpq})$$

Pour *a,e,i.*
En intégrant on a:

$$\Delta OE = GM\sum K(a,e,i)\frac{\sin(\psi_{lmpq})}{\dot{\psi}_{lmpq}}$$

$$\frac{dOE}{dt} = GM\sum K(a,e,i)\sin(\psi_{lmpq})$$

Pour *Ω,ω,M.*
En intégrant, on a:

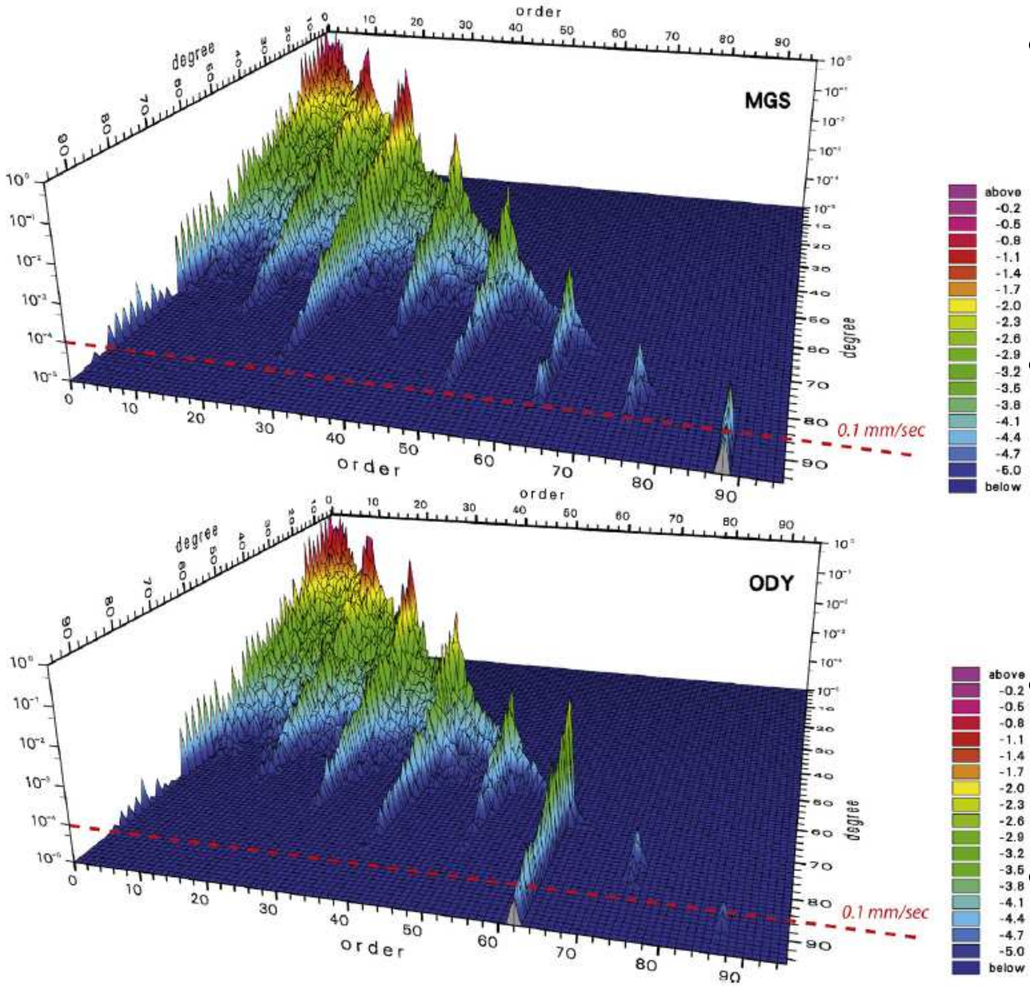
$$\Delta OE = -GM\sum K(a,e,i)\frac{\cos(\psi_{lmpq})}{\dot{\psi}_{lmpq}}$$

With: $\boxed{\dot{\psi}_{lmpq} = (l-2p+q)\dot{M}_s + (l-2p)\dot{\omega}_s + m(\dot{\Omega}_s - \dot{\theta})}$

- Important result:
  The amplitude of periodic variations is proportional to $1/\dot{\psi}_{lmpq}$
  Short periodic perturbations are lower than long period perturbations.

- Resonances: $\dot{\psi}_{lmpq} = 0$ . Kaula's approach can only allow detecting those resonances for any orbit (*a,e,i*) for each associated harmonics coefficients $c_{lm}$ et $s_{lm}$.

# Orbit filters gravity field through resonances




Marty et al., 2009

- Kaula's solution
  → Orbital velocity perturbation on radial direction due to each degree & order: $C_{lm}$ et $S_{lm}$
- Comparison with error on velocity measurements
  --> Resolution of gravity field (SNR=1)
- Example: MGS/ODY
  → Maximal degree Lmax: 60.
- First idea of gravity field resolution from a given orbit.

*Mean OE: a=400 km, e=0.01, i=92.9°*

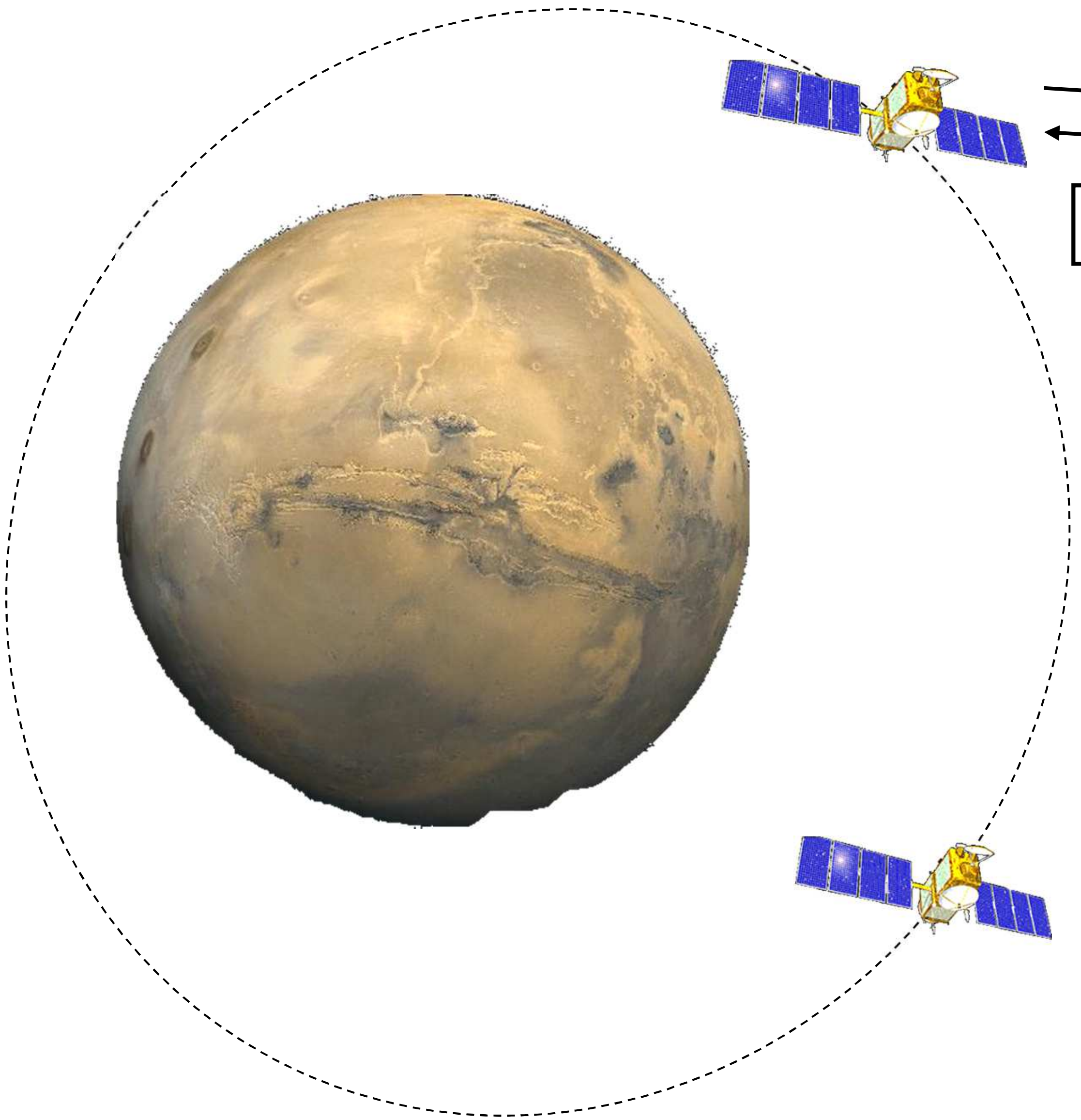
# From theory … to practice

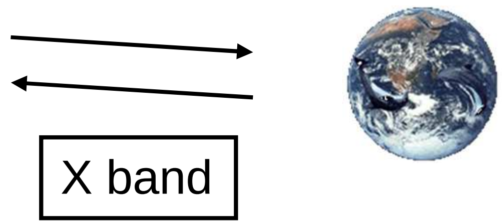
- Kaula's approach allows studying the sensitivity of a given orbit to harmonics of gravity field $C_{lm}$ et $S_{lm}$ (i.e. given $a,e,i$).

- Nevertheless, it cannot allow (as for any other analytical approach) to obtain an accurate orbit reconstruction (*perturbation method*, *non-conservative forces*, …).

- *In practice*, one numerically solve for the fundamental equation of dynamics.
- The gravity field harmonics are then estimated through the fit of an initial gravity field to **tracking data** of the orbiting spacecraft.
- The non-conservative forces must also be estimated, because they are not well known. This is the major limitation of the method.

# Precise determination of the spacecraft orbital perturbations






Tracking data:
Radio-link between the Earth and spacecraft (generally 2-way X band)

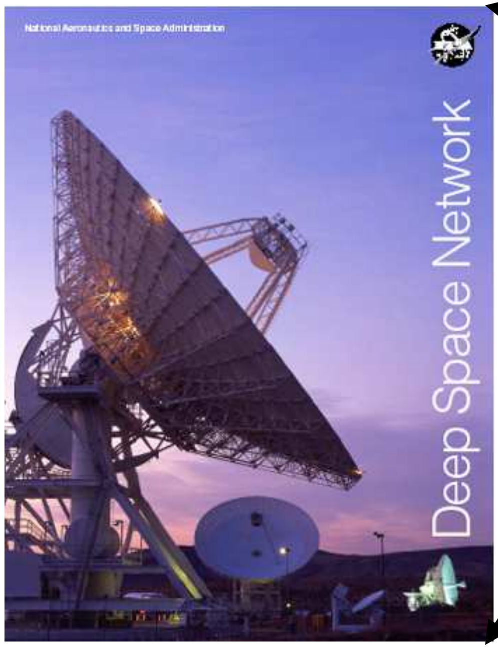
Precise Orbit Determination (POD)
GINS[(1)] : orbitography software:

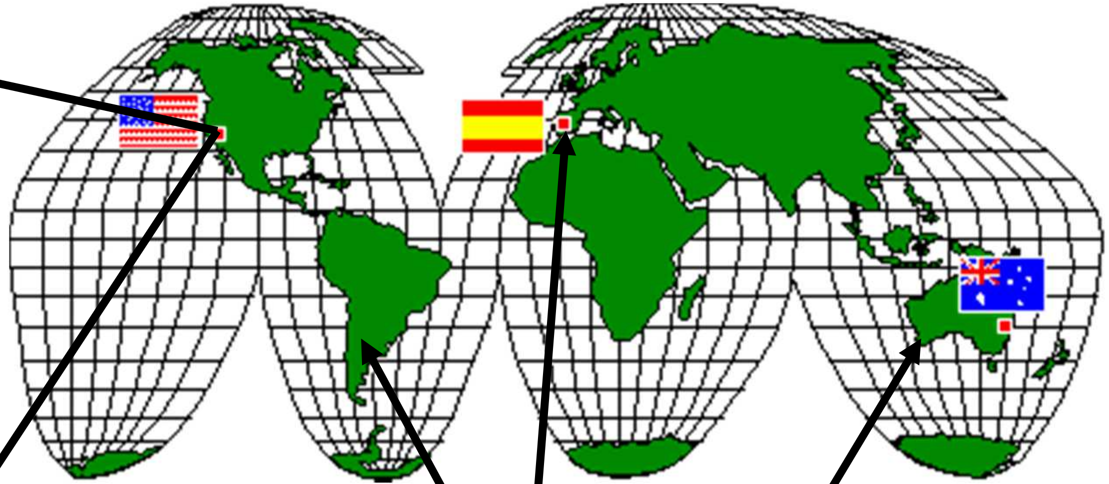
- Calculation of orbit from least squares fitting of a dynamical model of motion to tracking data
- Determination of the planetary gravity field (including its time variations), $k_2$ tidal Love number, *GM* of the moons, …

*(1) Géodésie par Intégration Numériques simultanées, developed by the French space agency (CNES) and further adapted to planetary geodesy applications by the Royal Observatory of Belgium (ROB).*

# NASA's DSN and ESA's ESTRACK networks of tracking stations

➢To track any spacecraft or Lander in the solar system



ESA's ESTRACK: 34 m. antennas at New Norcia, Cebreros & Malargüe. (Mars Express, Venus Express, Rosetta)

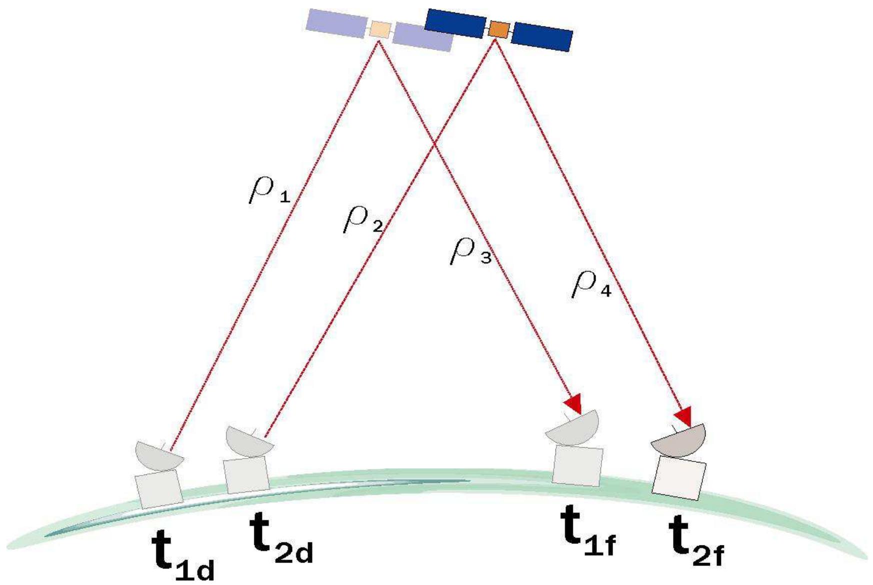
Diameter of 25 to 70 meters.

Precision:
0.02-0.05 mm/s (1-3 mHz)
< ~1 m. But bias of ~3-4 m.
Dating: Maser clocks

- Radio-link (transmitting & receiving) for data & telemetry
- Doppler & range radio-tracking mainly at frequencies of 8.4 Ghz (X band)

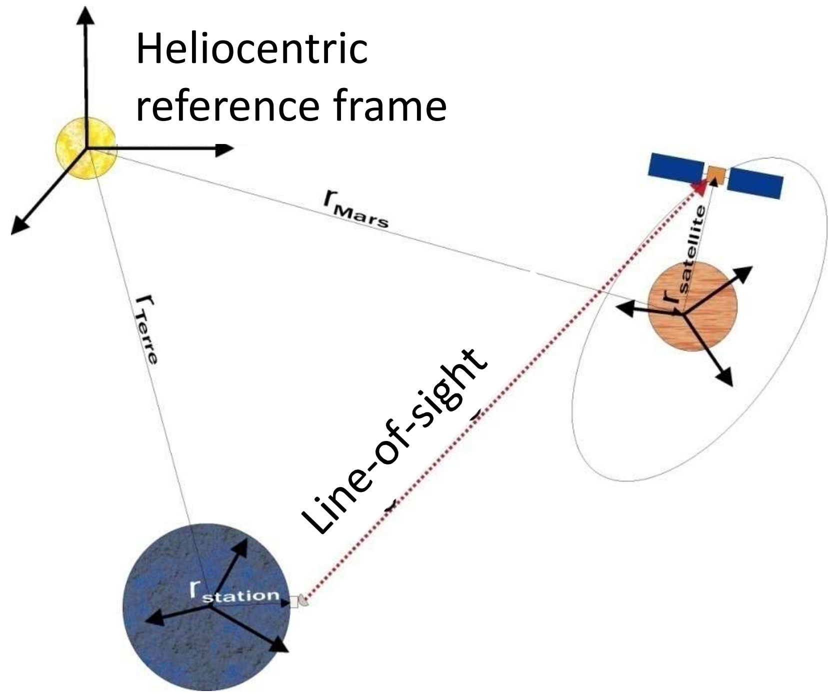
# Application of Doppler effect to spacecraft velocity determination

- **Doppler effect: range-rate variations**




$$\Delta F = 2\frac{v_{LOS}}{c}F_{ref}$$

- **relative velocity: Earth-spacecraft**

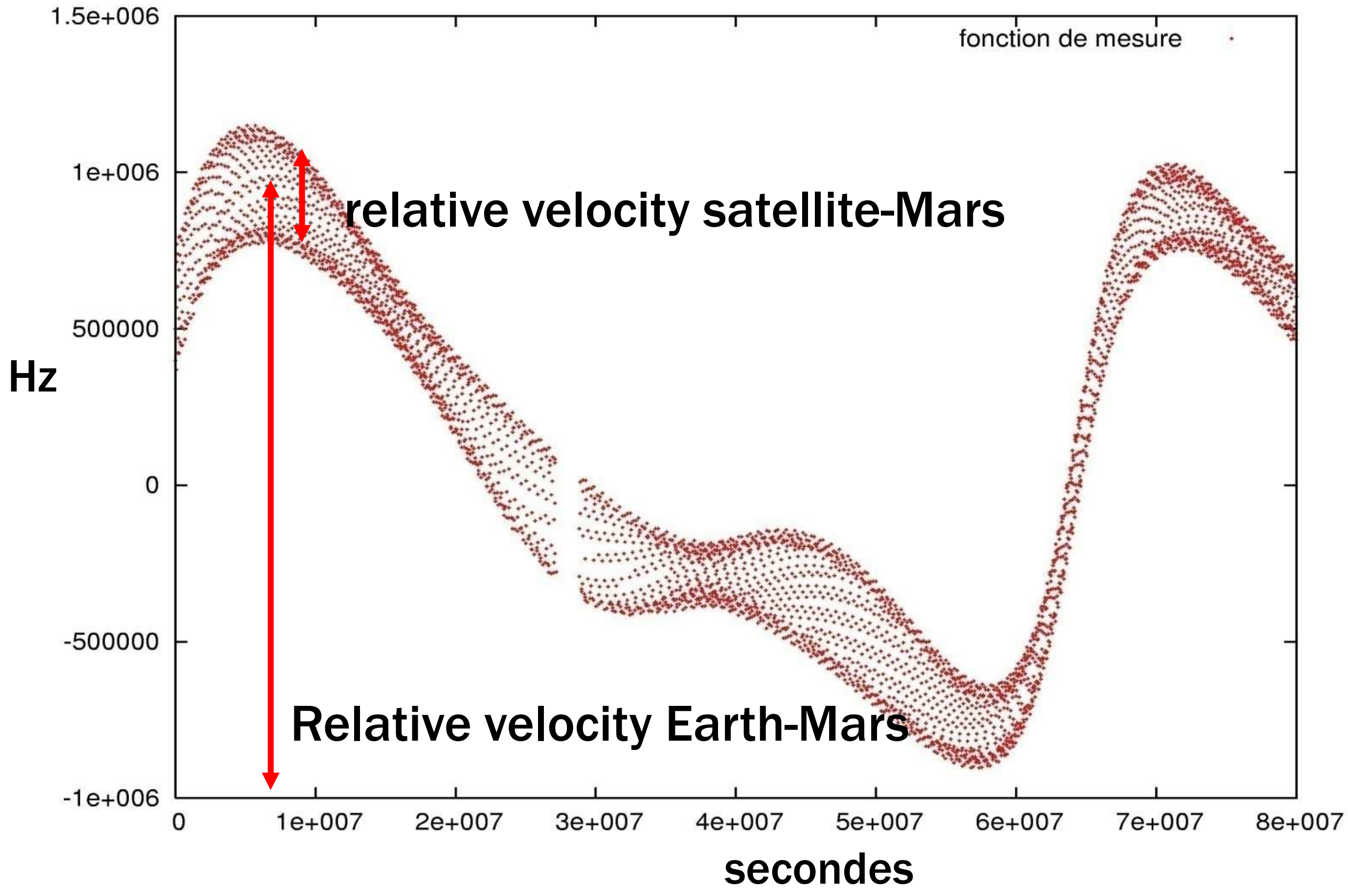



$$\vec{v}_{rel} = \vec{v}_{sat}|_{Mars} + \vec{v}_{Mars}|_{sun} - \vec{v}_{sta}|_{Terre} - \vec{v}_{Terre}|_{sun}$$

Important remark: Doppler effect = 0 when the Line-Of-Sight direction is perpendicular to the spacecraft velocity direction

# Doppler signal variations

## *Earth-Mars radio-link*



# Precise orbit determination: *in some words*

- To model all the forces acting on the spacecraft and to numerically integrate its motion (***not only gravity***)

- To calculate the associated predicted tracking data

- To perform an iterative least squares process on the radio-science tracking data (Doppler and ranging) in order to adjust parameters related to the data and to *dynamical model of spacecraft motion*

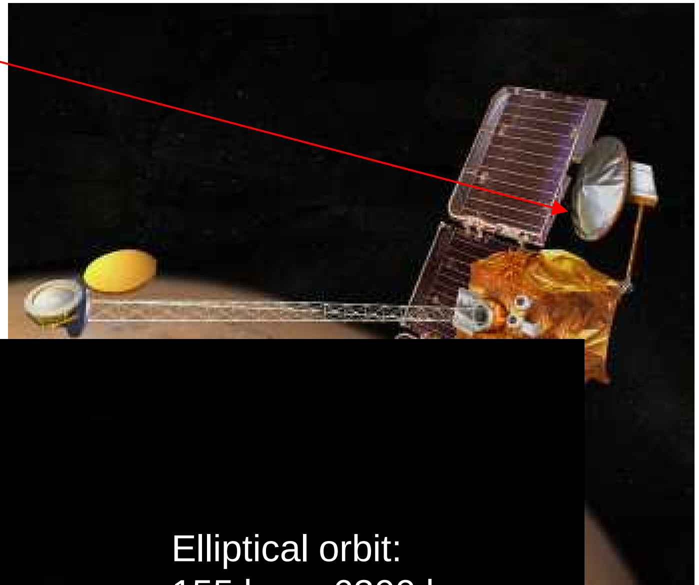
- We use a dedicated software ***GINS***[1].
  Two others software are able to do same calculations *DPODP* at JPL (USA) and *GEODYN* at GSFC (USA). Others operational or in development in Europe.

[1] *Géodésie par Intégrations Numériques Simultanées, developped by CNES and applied by ROB for planetary geodesy applications*

# Martian spacecraft tracking data

Steerable antenna.
Continuous tracking
from the Earth.


Fron



Near-circular

om NASA



From NASA

From ESA

ain Antenna !
ous tracking
from the Earth.

Elliptical orbit at
altitude 250x10400 km

# Spacecraft dynamical model

- Model of all the forces acting on the spacecraft

- Gravitational forces:
  - Mars' static gravity field (JGM95J from JPL)
  - Point mass representation of other solar system bodies using JPL planetary and Martian moon ephemerides.

- Non-gravitational forces :
  - Atmospheric drag (atm. Density model)
  - solar pressure radiation, albedo & IR radiations .
  - Residual accelerations induced by each unbalanced wheel off-loading (WoL) or angular moment desaturation event.

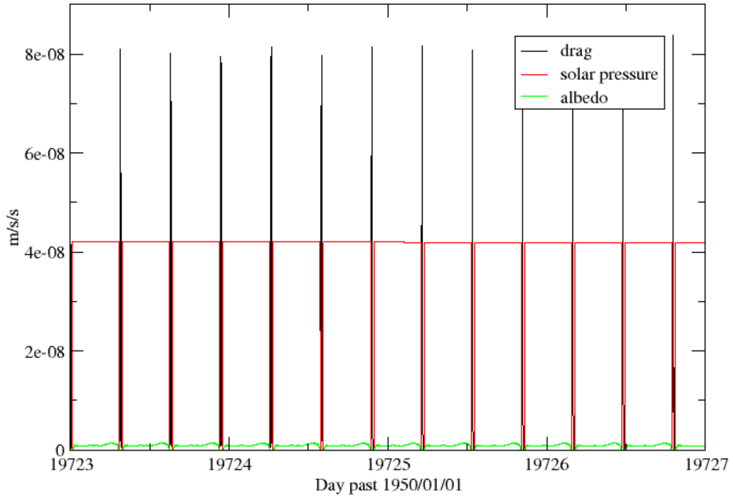
*Most unknown forces* to be adjusted:
Drag (specially MEX pericenter altitude at 250 km) and WoL accelerations.
Additional difficulty for MEX: *Non-continuous tracking.*

# Non-gravitational accelerations

Altitude range: 250x11000 km

**Peri. at 250 km**

Non-gravitational forces acting on MEX




<u>Most perturbing forces on MEX:</u>

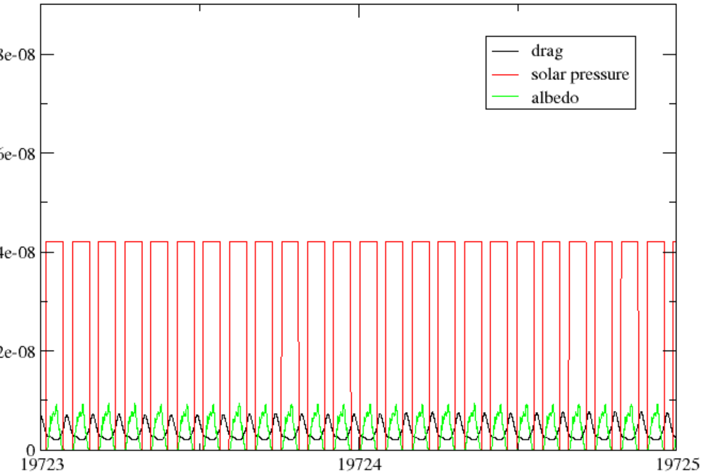
Solar pressure: up to 4. $10^{-08}$ m/s$^2$
Drag at pericenter: up to 8. $10^{-08}$ m/s$^2$

Wheel Off Loading (WoL) events near apocenter (~1 per day or per 4 orbits) (*not on the graph*).
~$10^{-4}$ – $10^{-5}$ m/s$^2$

Altitude range: 400x400 km

**Peri. at 400 km**

Non-gravitational forces acting on MGS

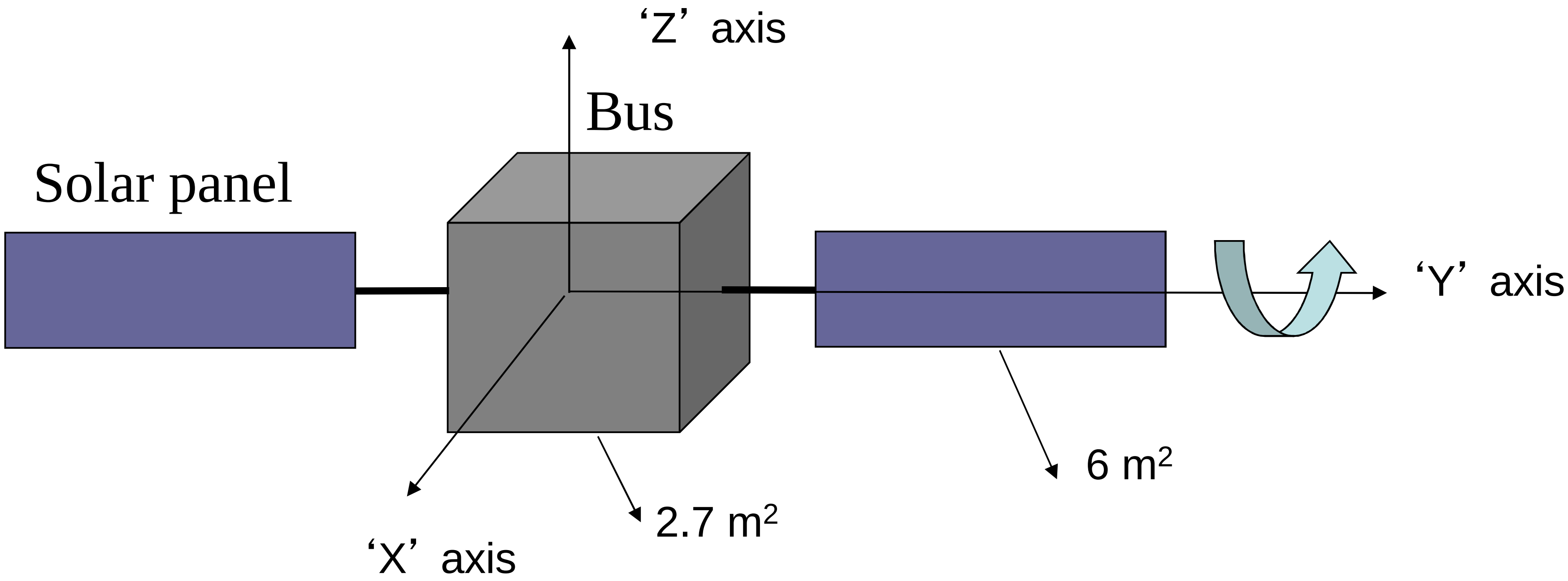



<u>Most perturbing forces on MGS:</u>

Solar pressure: up to 4. $10^{-08}$ m/s$^2$

Angular Momentum Desaturation (AMD) events (~1 per day or per 12 orbits) (*not on the graph*):
~$10^{-4}$ – $10^{-5}$ m/s$^2$

# Non-gravitational forces act any surface area of the s/c



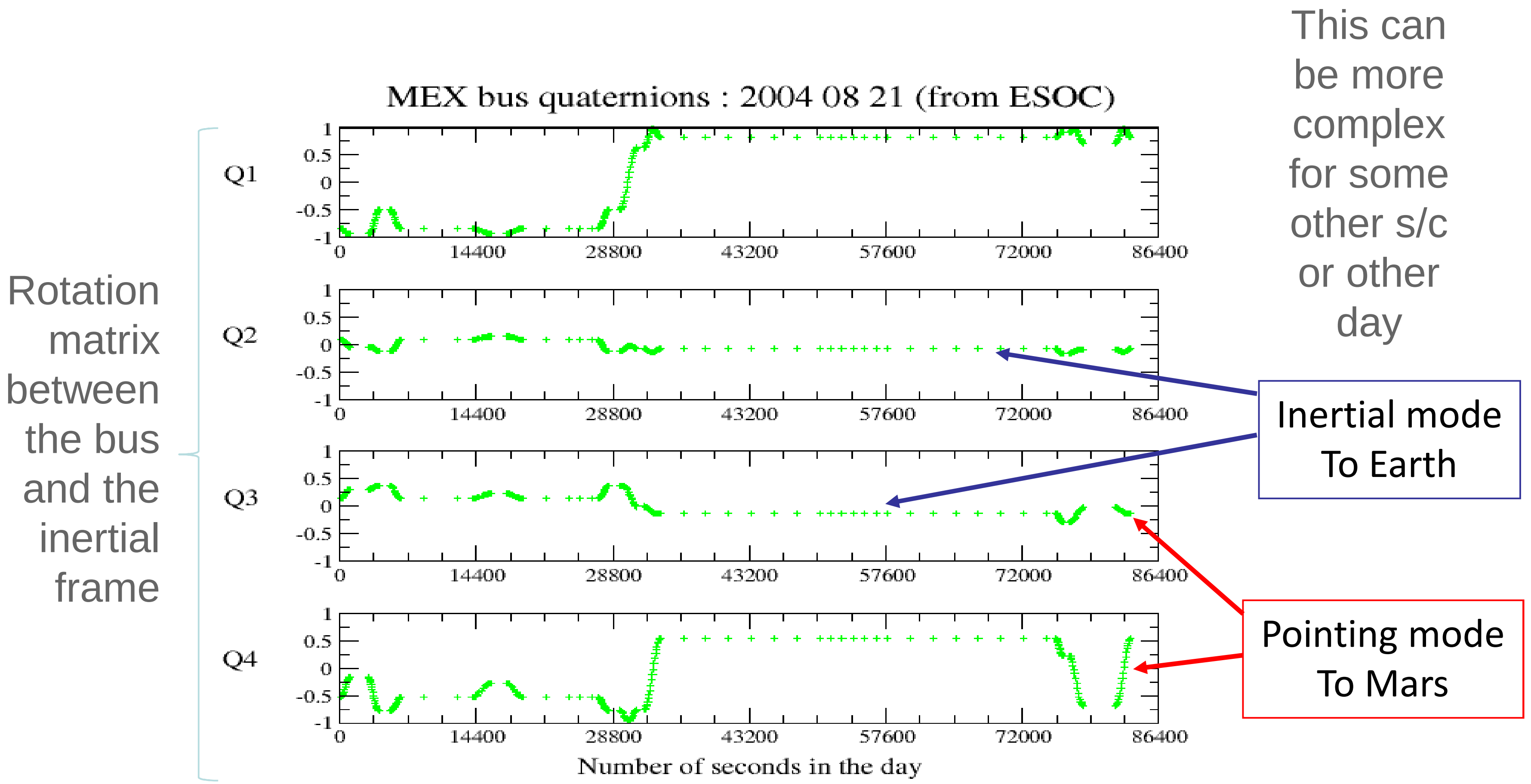

"Macro-Model" (*box-and-wings model)*

Spacecraft as flat plates (6 for the bus & 4 for solar panels) with known area and optical properties (reflectivity and absorptivity).

The orientation of this model in space is given by quaternions (bus, …).

*Shift between center of mass and the HGA center of phase. Mass history.*

# Example of quaternions of the bus of MEX




Attitude mode : Earth or Mars pointing or fixed inertial modes.

# Precise Orbit Determination: POD

**INPUT**

Modeling all forces:

- Gravitational
- Non-gravitational (the most ill-known)

Pre-processing of:

- Initial state vector
- Attitude mode of spacecraft
- Desaturation events
- (CoM s/c) / (CoP HGA)
- Tracking data

**GINS software**

Numerical integration of motion
&
Iterative least-squares fit on successive data-arcs of 4-6 days long

**OUTPUT**

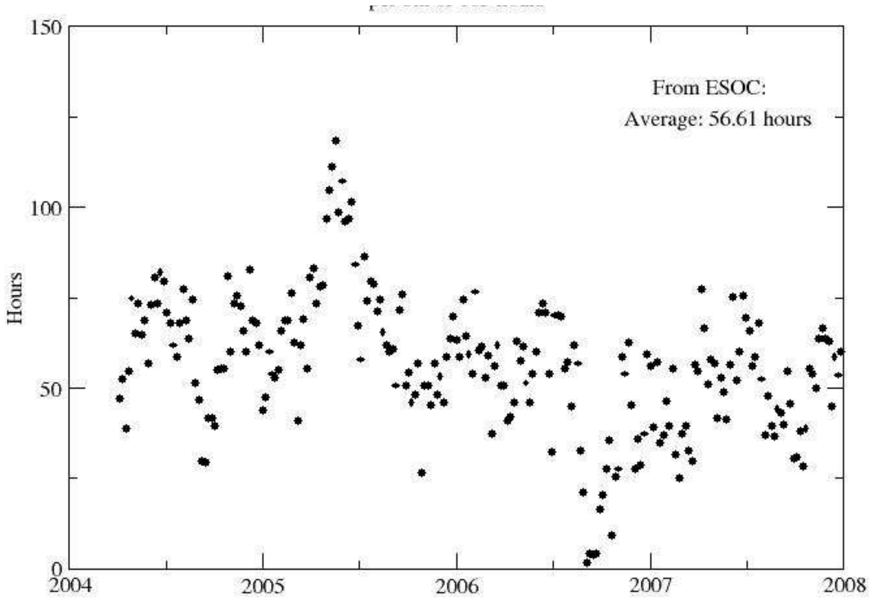
- Accurate position & velocity of spacecraft
- Doppler range residuals
- Scale factor (drag, …)
- Normal matrix ith information s’ geophys ameter (Gra d and its se al changes, mass of th moons…)

Challenge !

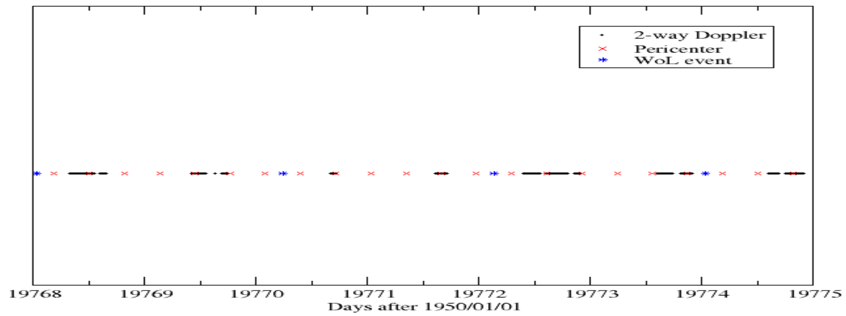
The quality of the estimated geophysical parameter strongly rely on the precision of the reconstructed orbit of the spacecraft.

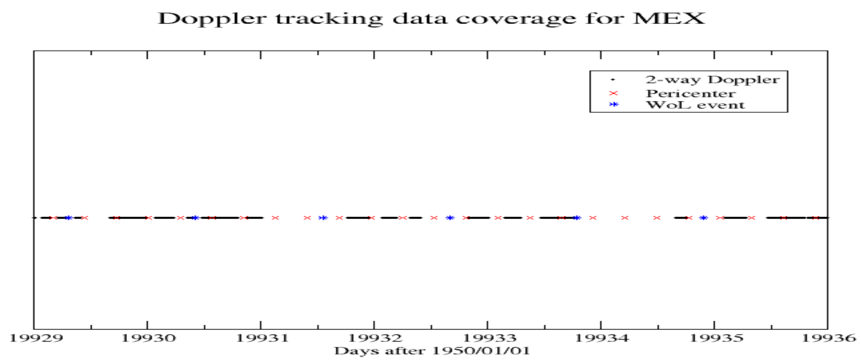
# Non-continuous tracking of MEX. A problem !

MEX tracking mean duration per week




Examples of MEX tracking coverage per week

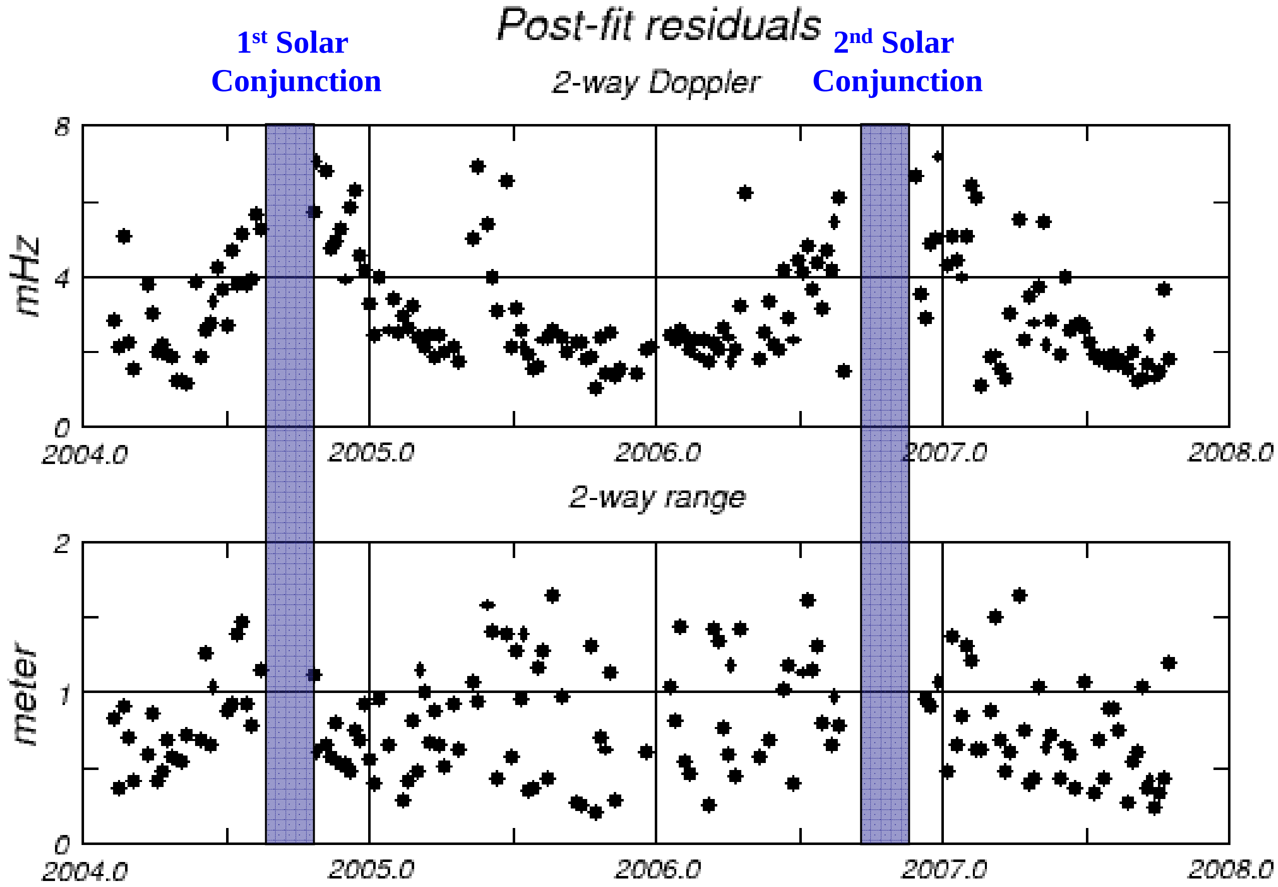



- Depending on the data-arc, successive MEX revolutions around Mars are not tracked at all.
- Difficulty to correctly determine the effect of the desaturation (WoL) and of the drag at each pericenter pass (altitude of 250-700 km).

# Post-fit residuals of tracking data: MEX

*rms value for each data-arc*



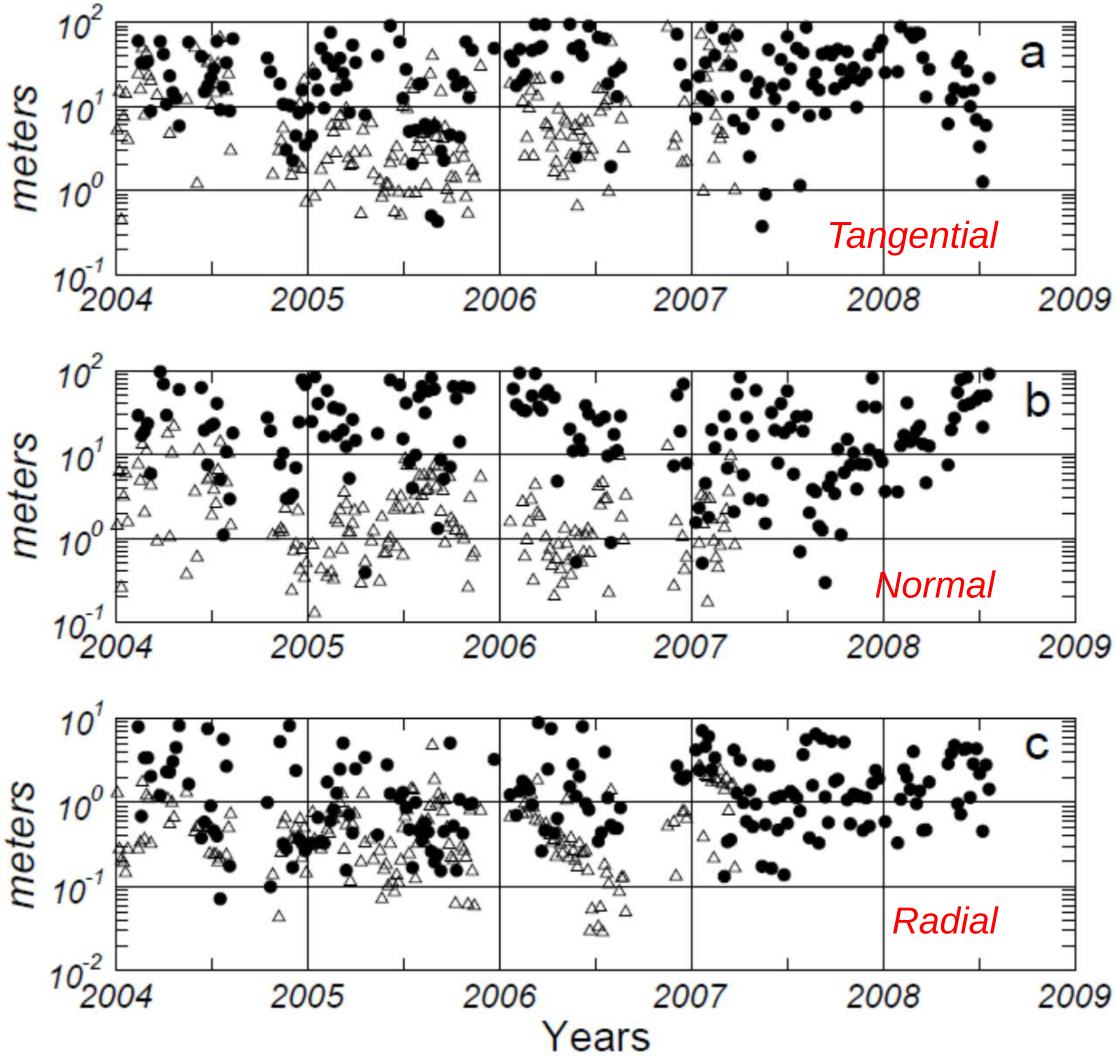

MEX modeled velocity residuals relative to Mars

MEX modeled ranging residuals relative to Mars

- Good residuals in Doppler and range. Effect of interplanetary plasma on signal at solar conjunction (September 2004 and October 2006).

- Test of orbit quality: Orbit overlap (test of internal coherency).

# Precision on MEX & Odyssey reconstructed orbit




<u>Orbit overlap results:</u>

*Black dots: MEX*
*(average 20 meters)*

*Open triangle: Mars Odyssey*
*(average 2 meters)*

*Same results for MGS*
(Marty et al., 2009)

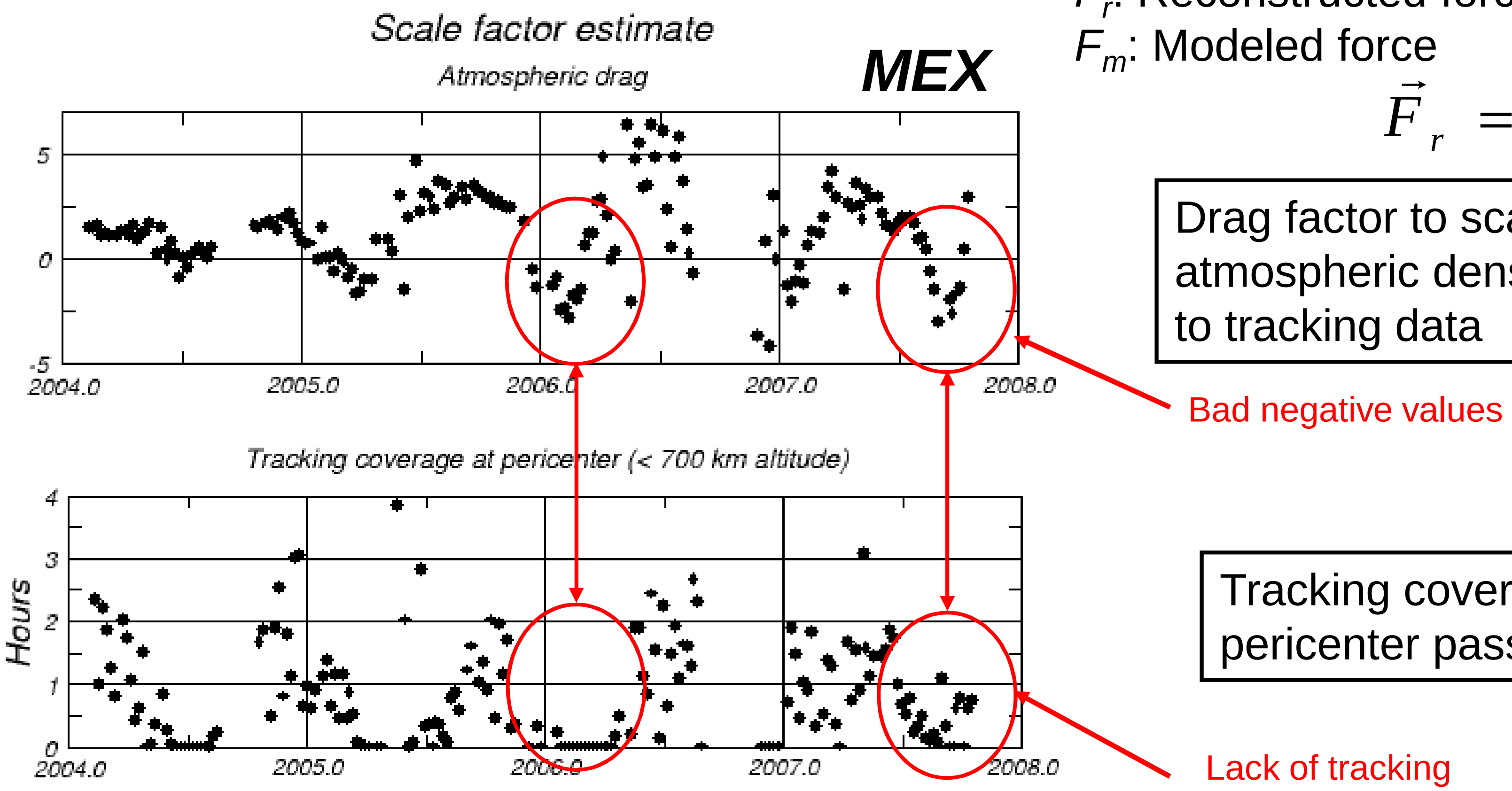
- MEX orbit precision is about ten times worse than Odyssey (MGS) orbit precision.
- This is mainly due to the lack of tracking data at MEX pericenter which does not permit to precisely estimate the atmospheric drag orbital perturbations

# Estimation of the atmospheric drag effect: Drag scale factor $F_D$ (1)

$F_r$: Reconstructed force

$F_m$: Modeled force

$$\vec{F}_r = F_D \ \vec{F}_m.$$

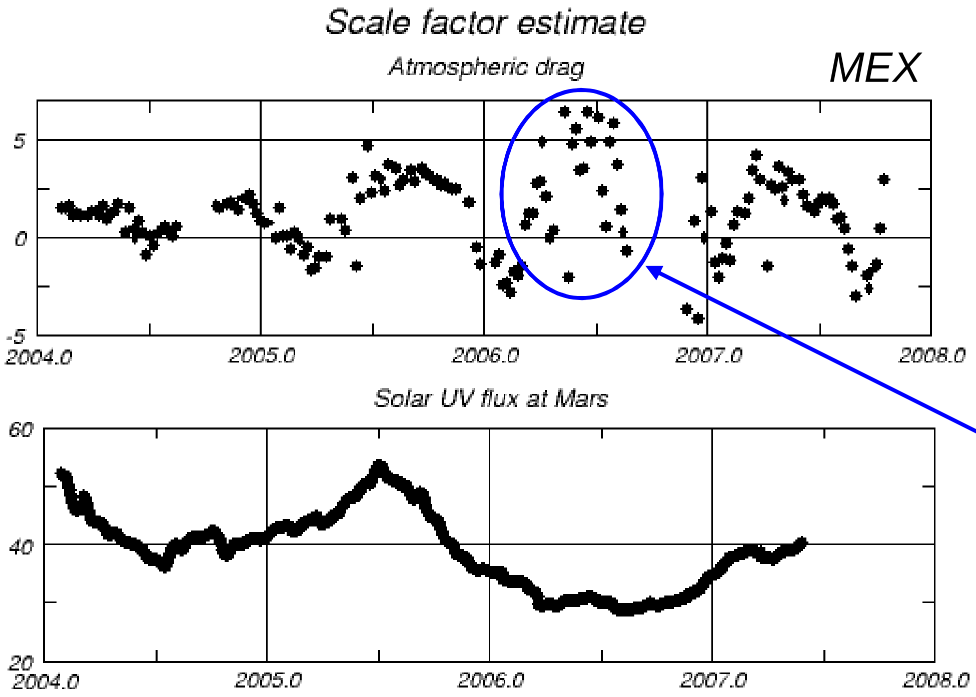



Drag factor to scale high atmospheric density model to tracking data

Tracking coverage at pericenter passes

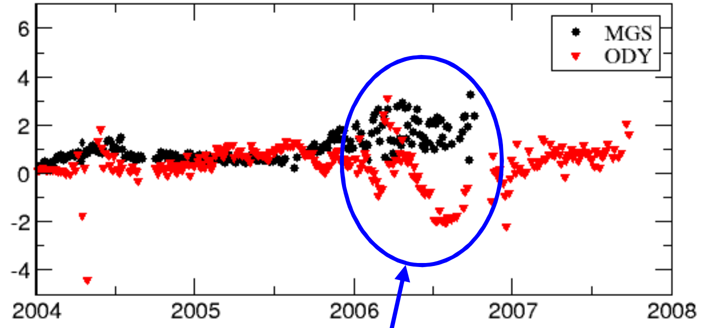
- Very poor tracking coverage at MEX pericenter passes occurs during repeated period of several weeks.
- This limits the orbit accuracy in the along-track direction.

# Estimation of the atmospheric drag effect: Drag scale factor $F_D$ (2)




*Period of minimum*

Drag scale factor estimate for MGS and ODY *(Duron et al., AGU fall meeting, 2007).*

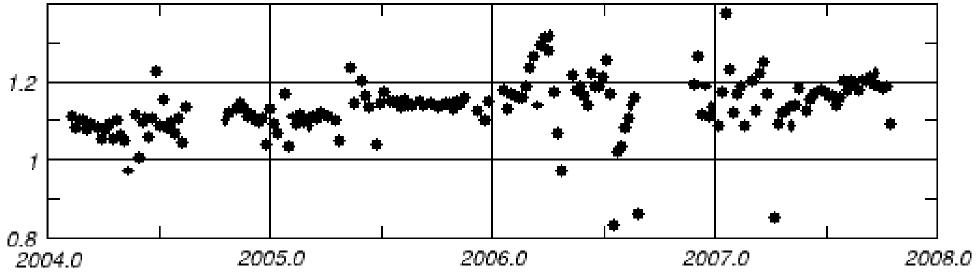



Less reliable estimates of the drag factor for MEX/MGS/ODY in 2006

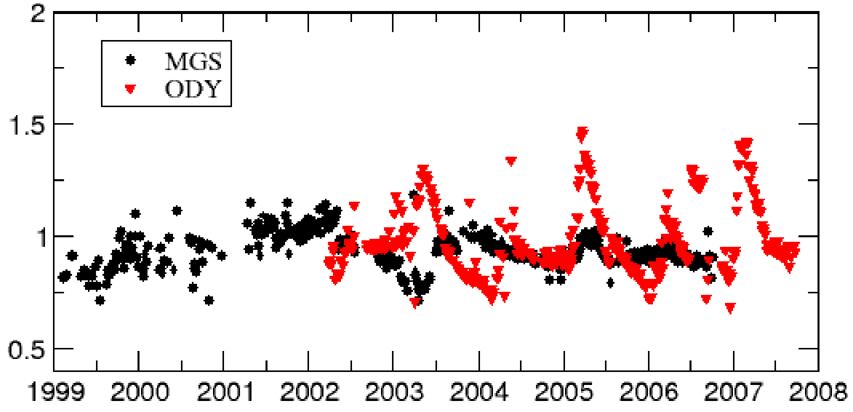
- Solar UV flux minimum decreases the density of Mars high atmosphere.
- Thus, this decreases the drag force onto MEX, and makes more difficult to estimate the drag scale factor for this period.
- Similar behavior for MGS and ODY spacecraft

# Estimation of the solar pressure scale factor: $F_s$

Solar pressure scale factor estimate for MEX




Solar pressure factor estimate for MGS and ODY

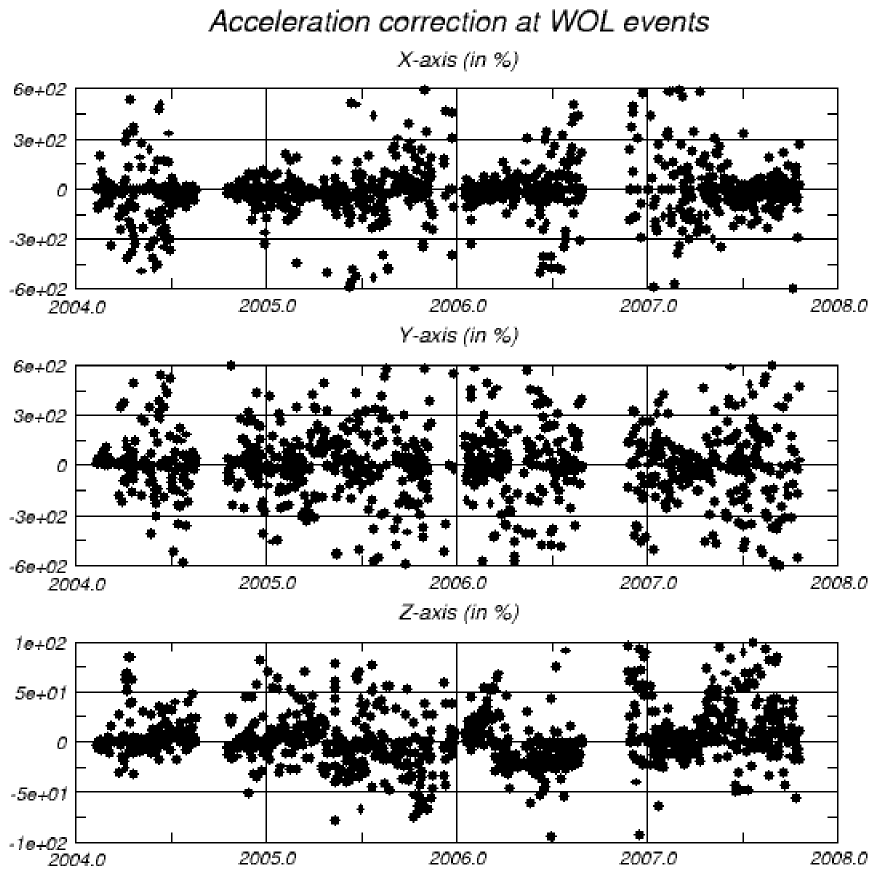



$F_r$: Reconstructed force
$F_m$: Modeled force

$$\vec{F}_r = F_S \vec{F}_{m.}$$

- Increasing MEX solar pressure scale factor with time.
- Similar behavior for MGS between 1999-2002.
- Darkening of the faces of the spacecraft with time?

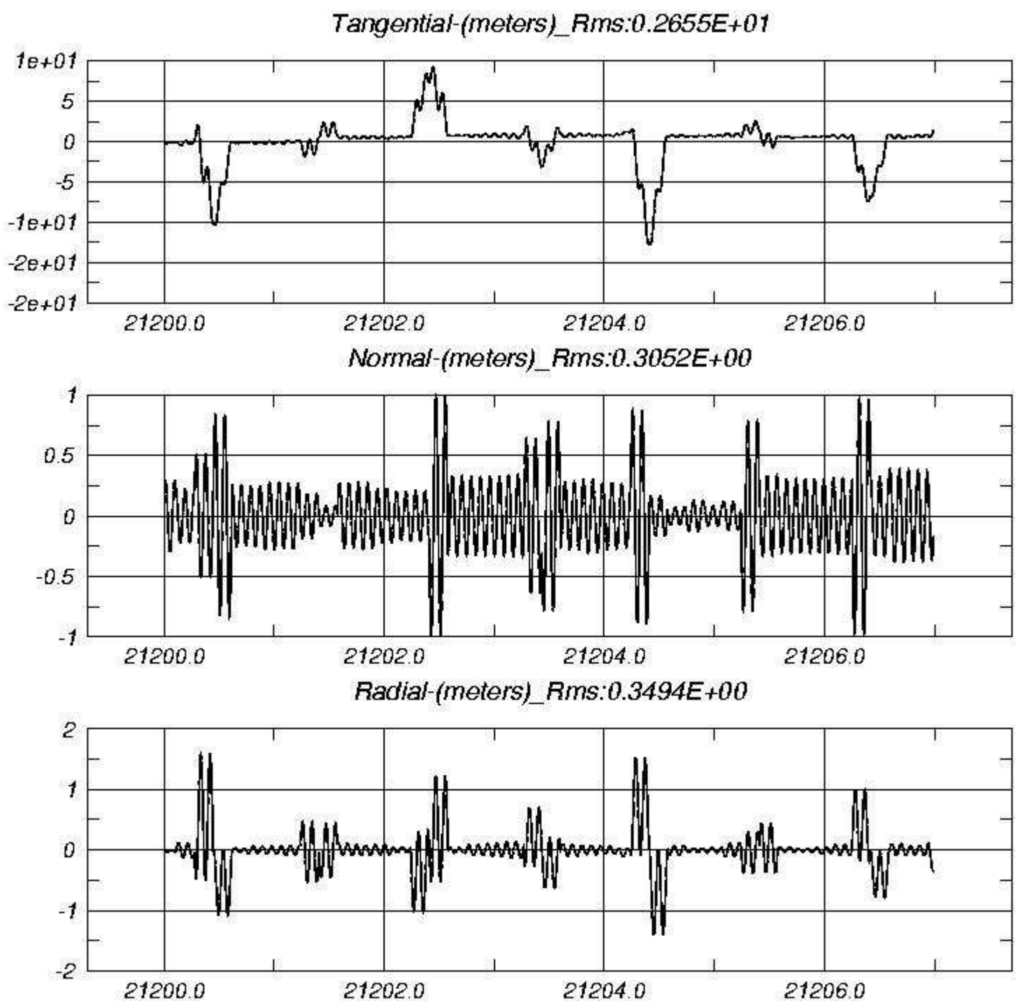
# Corrections to the WoL residual accelerations: MEX




Toward the along-track direction

- ➢ Accelerations associated with WoL events not well resolved along the directions of the X and Y axes of the spacecraft frame.
- ➢ This degrades the determination of MEX orbit in directions normal to the along track direction.

# Effect of attitude maneuvers on the orbit



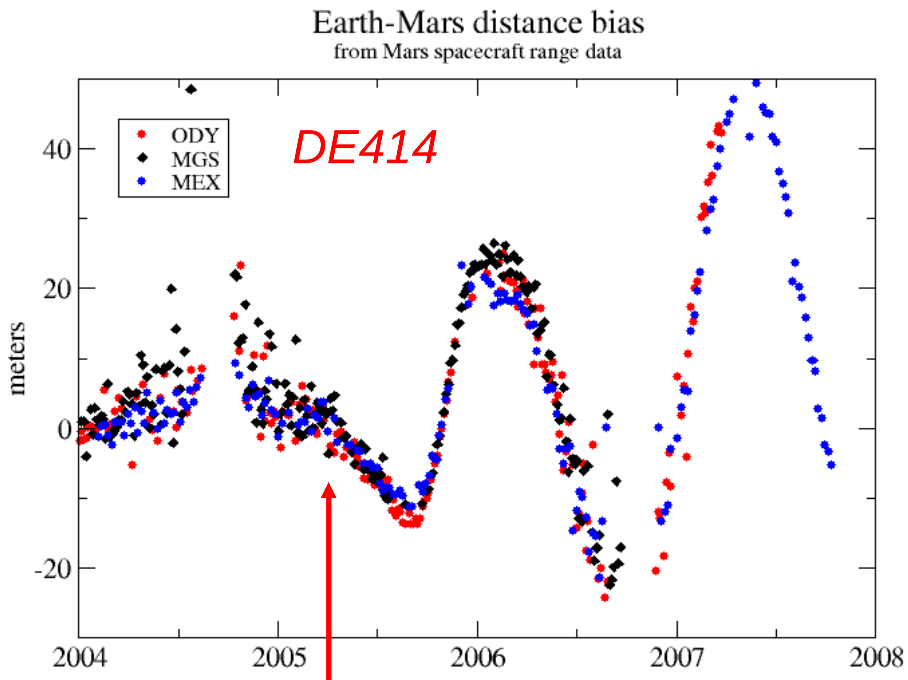

*Along-track*

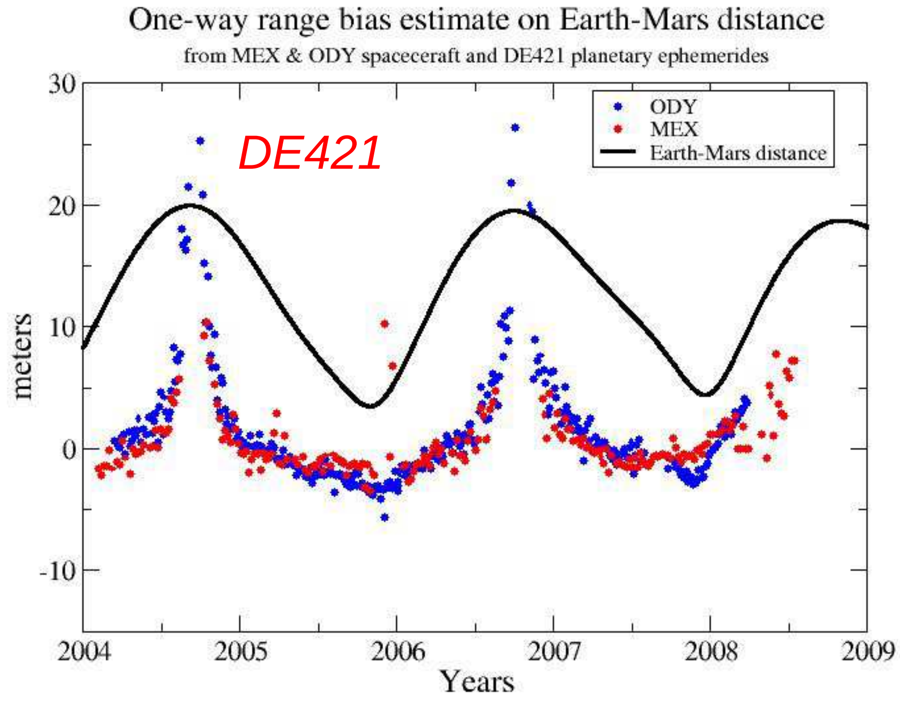
*Normal to orbital plane*

*Radial direction*
*In-plane toward the planet*

- A few meters at each attitude maneuver event
  (1 mm/s of Delta_Velocity at eahc maneuver in 3 directions

# Spacecraft range tracking and planetary ephemerides



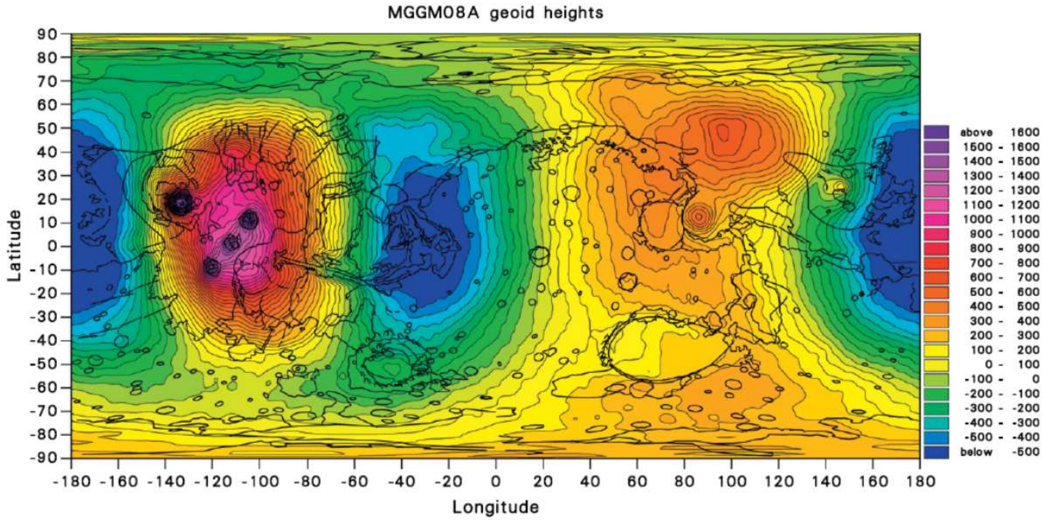

April 2005 = epoch of last spacecraft range data included in DE414 planetary ephemeris

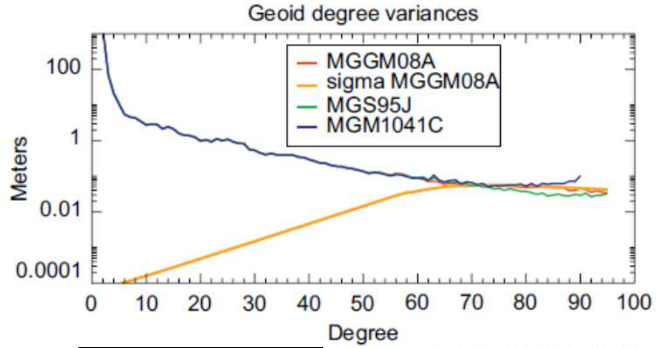
- The range tracking data allows planets ephemerides to be updated. *Especially for Mars.*

# Current Mars' static gravity field solutions

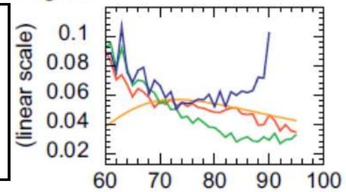
*GRGS/ROB solution, Marty et al., 2009*

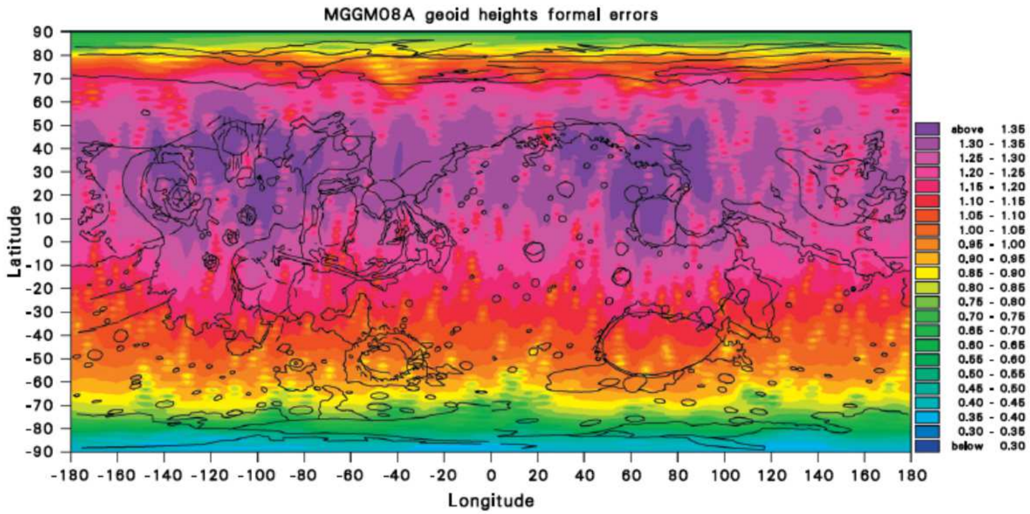
*Spatial resolution about 300 km*




Solutions from:
GRGS/ROB
JPL
GSFC




Limitation due to the orbit sensitivity to Doppler tracking precision and to non-gravitational forces:
**Degree strength around 70.**

**Challenge**: To detect fine temporal variations of the first zonal harmonics and the $k_2$ tidal Love number

# Degree strength of gravity field solution

$$U = \frac{GM}{r} + GM \sum_{l=1}^{l=\infty} \sum_{m=0}^{m=l} \frac{a_e^l}{r^{l+1}} P_{lm}(\sin(\varphi))(C_{lm}\cos(m\lambda) + S_{lm}\sin(m\lambda))$$

Intensité du potentiel en $1/r^{l+1}$

→ Sensibilité à un degré maximal $L$ donné par l'altitude au périgée $h_p$ :

$$L \approx \frac{2\pi a_e}{h_p}$$

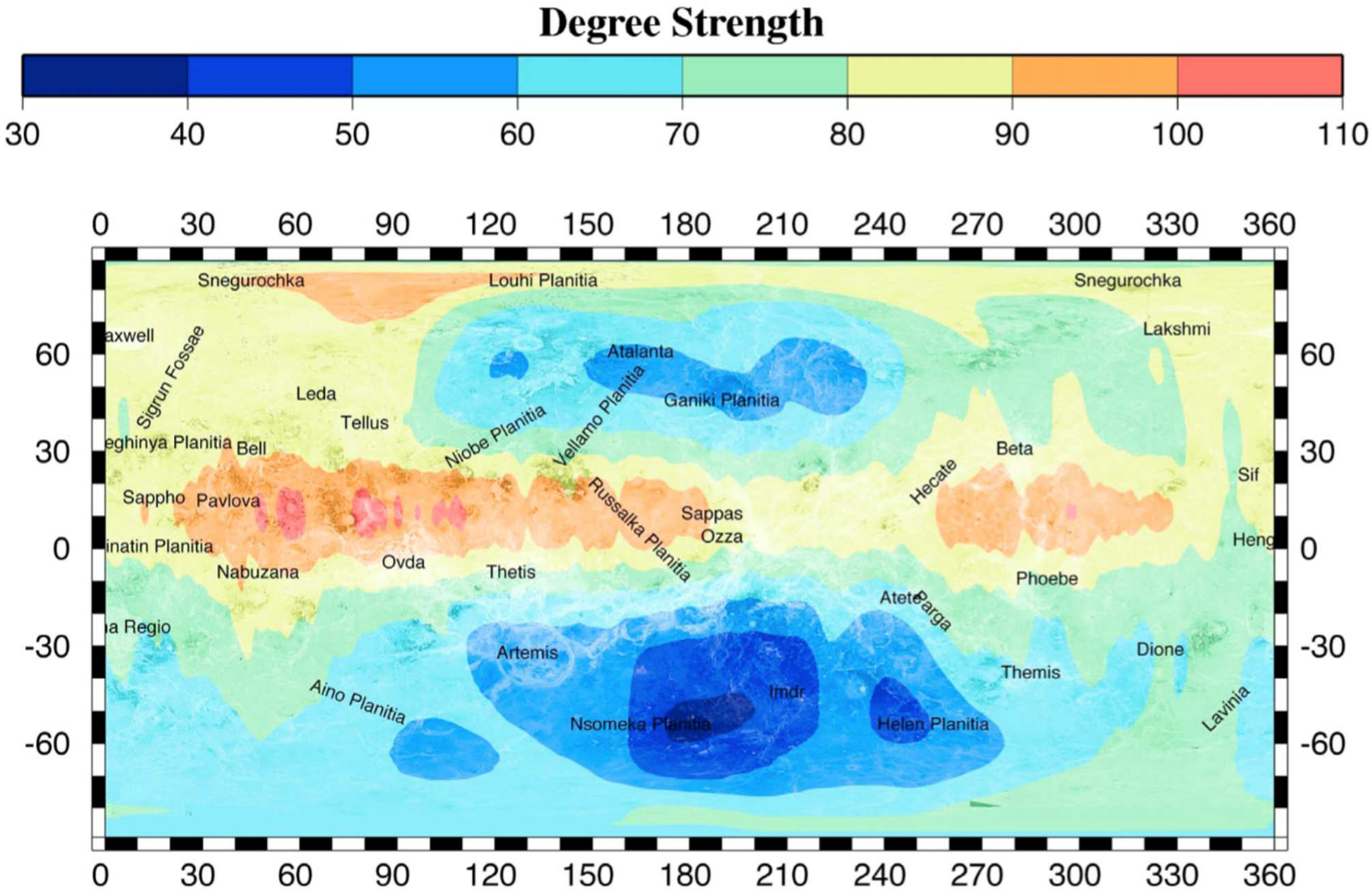
*Règle du pouce de Kaula*

✓<u>Exemple</u>: Mars Global Surveyor (MGS): $h_p$=380 km → $L$=56

($a_e$=3400 km) Mars Express (MEX): $h_p$=250 km → $L$=85

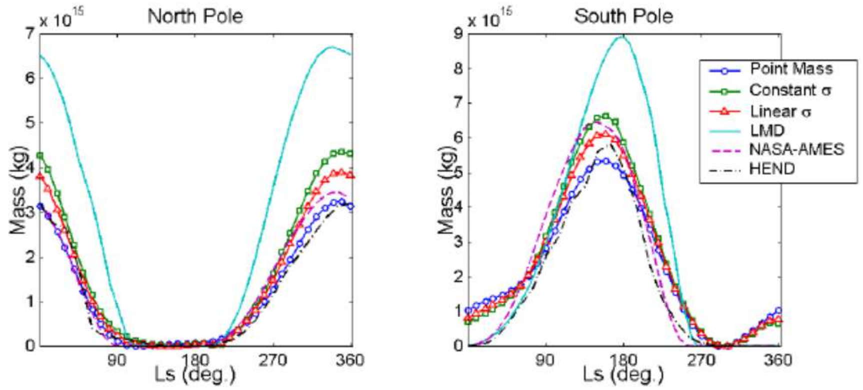
- ✓ Kaula's rule of thumb provides a good approximation of the degree strength "L" of the gravity field solution.
- ✓ Elliptical orbit offers lower altitude than circular orbits but only at pericenter, thus generally doesn't cover the entire planetary surface (like for MEX, Magellan).

# Degree strength variations: Venus gravity field solution

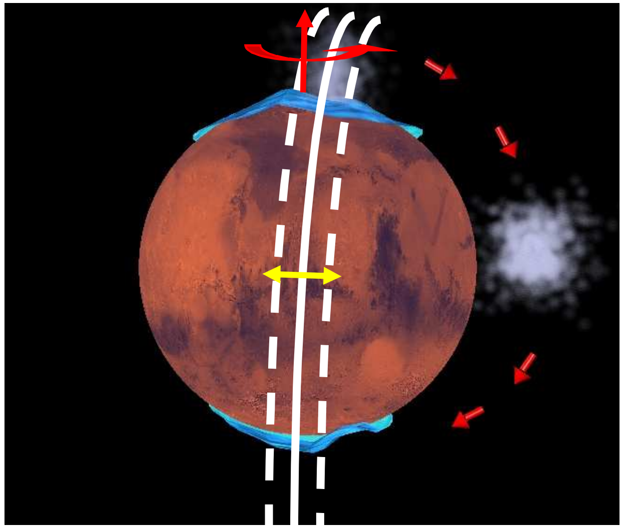



*Anderson & Smrekar, 2006*

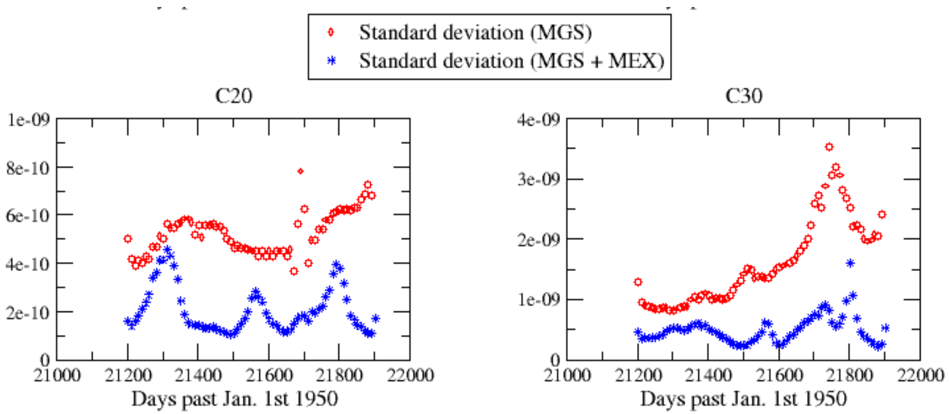
# Gravity field variations and Mars' $CO_2$ seasonal mass budget




Seasonal gravity field to determine mass transfer budget, but insufficient precision to constrain the models of $CO_2$ seasonal deposits (*Karatekin et al., JGR, 2006*).



*Mars' atmosphere $CO_2$ seasonal cycle*



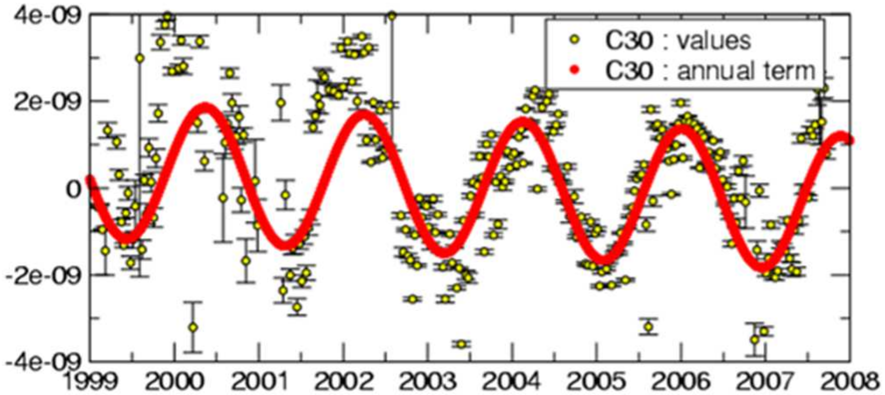

*GINS simulations (Rosenblatt et al., AGU fall meeting 2005)*

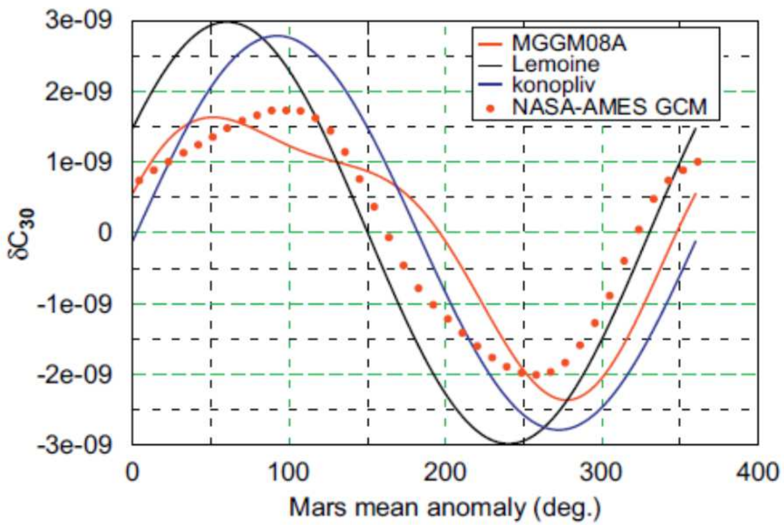
MEX + MGS can improve the solution of first zonal harmonics variations, thus the seasonal mass budget, given the orbits are well resolved.

We need to perform an accurate spacecraft orbit.
**Yes, but how much accurate?**

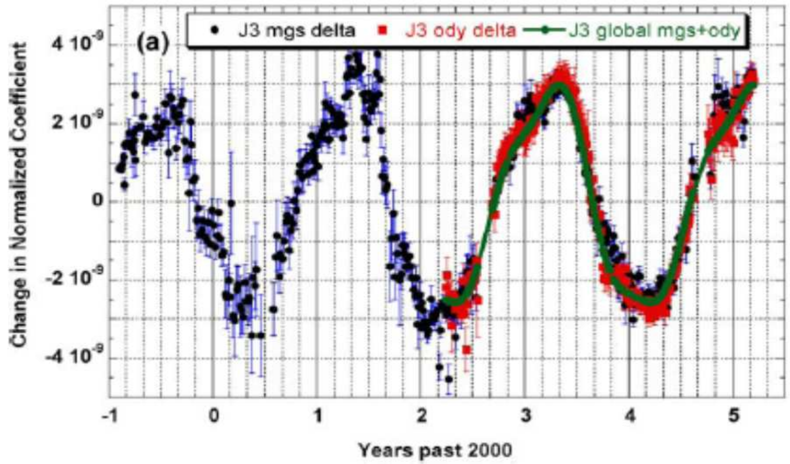
# Current solutions of Mars' $C_{30}$ seasonal variations from MGS/ODY tracking data

**GRGS/ROB solution**
(Marty et al. 2009)



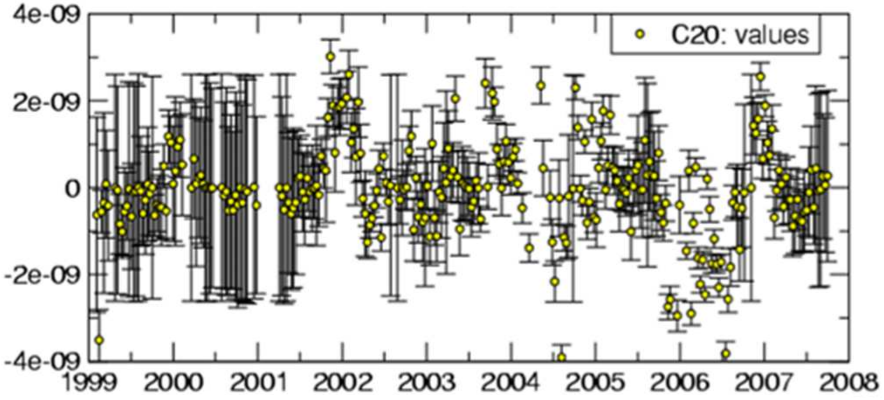

**JPL solution**
(Konopliv et al., 2006)



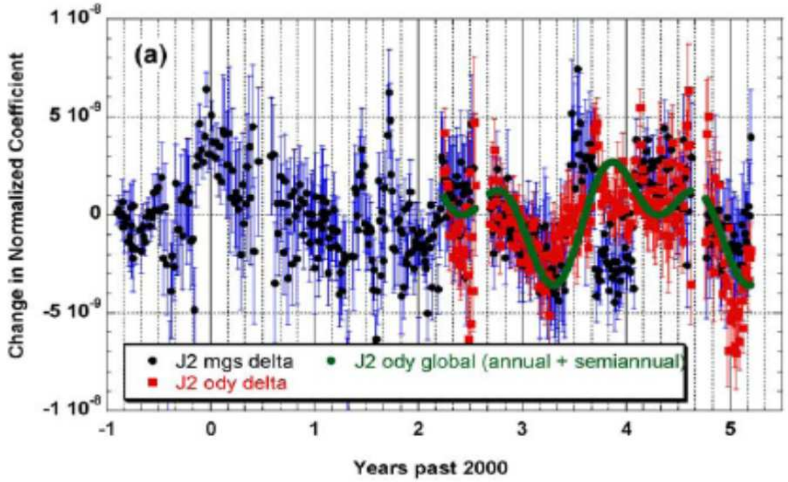

- $C_{30}$ seasonal variations determined from MGS and ODY tracking data.
- Slight differences in amplitude between JPL (Konopliv), GSFC (Lemoine) and GRGS/ROB solutions.

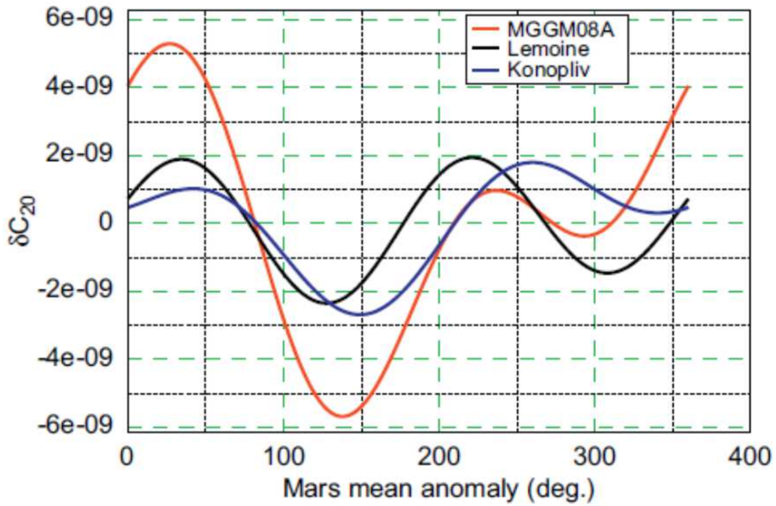
# Current solutions of Mars' $C_{20}$ seasonal variations from MGS/ODY tracking data

**GRGS/ROB solution**
(Marty et al. 2009)



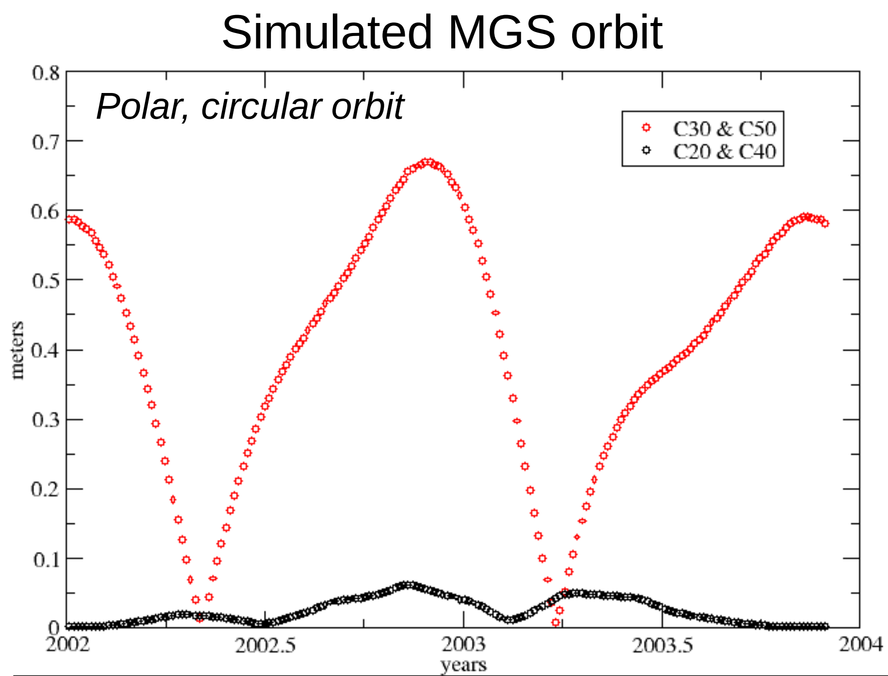

**JPL solution**
(Konopliv et al., 2006)

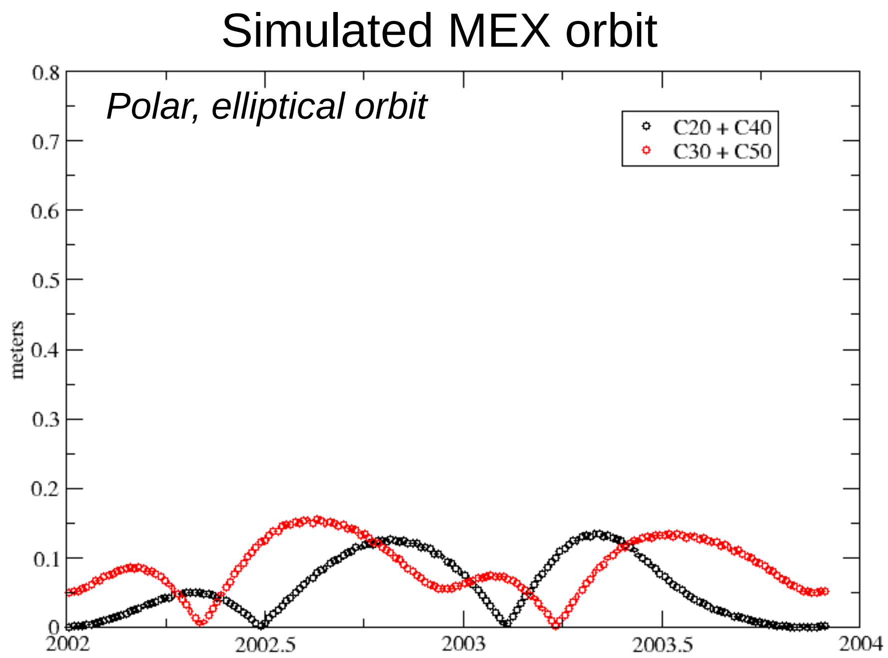



- $C_{20}$ seasonal variations determined from MGS and ODY tracking data.
- Significant differences in amplitude between JPL (Konopliv), GSFC (Lemoine) and GRGS/ROB solutions.
- Can MEX tracking data help to improve the MGS/ODY solution?

# Perturbation of spacecraft position due to Mars' time variable gravity field



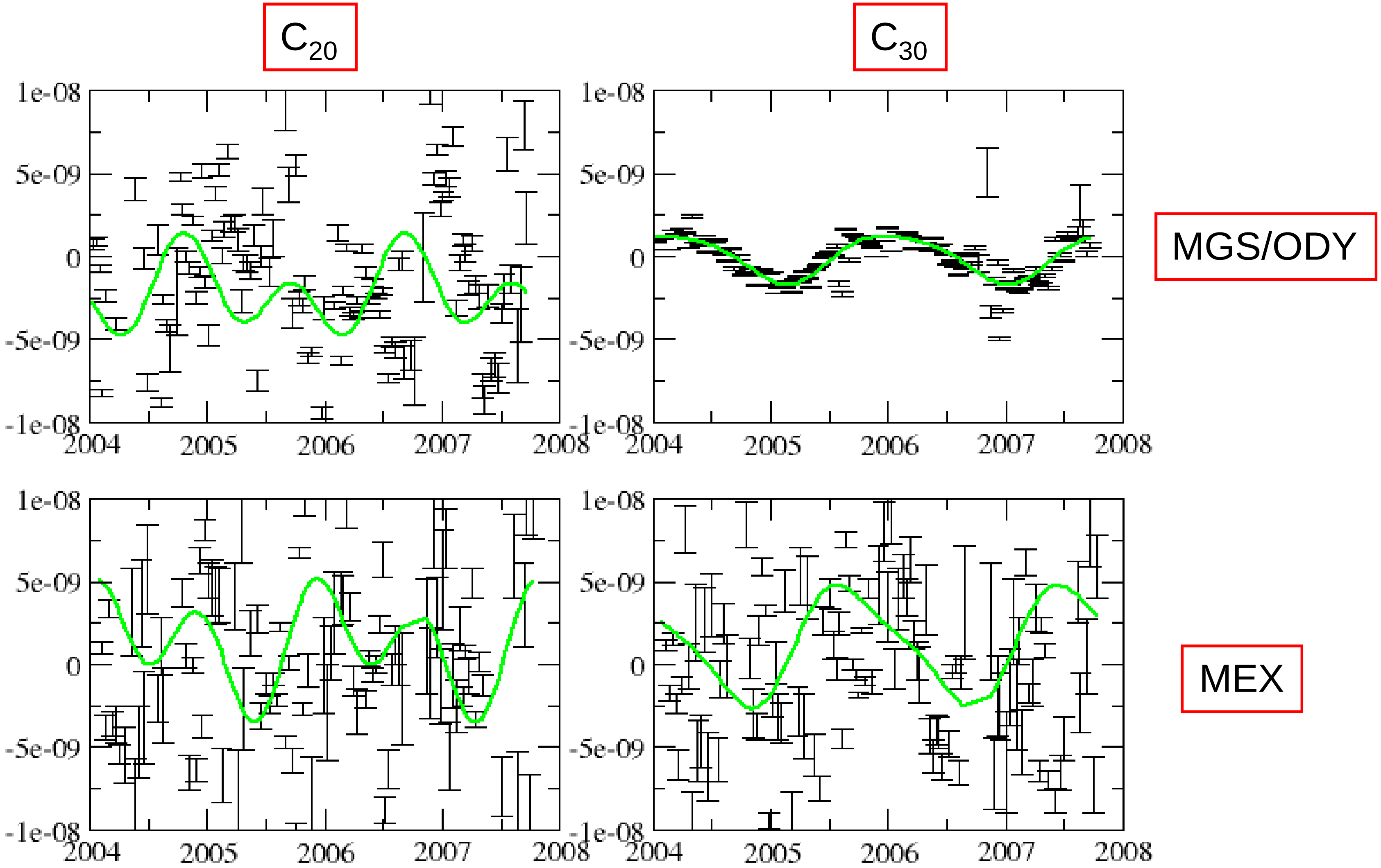




- GINS procedure: Fit of spacecraft simulated positions computed with time variable gravity on spacecraft simulated positions computed without time variable gravity on successive 4-days over 1 Martian year.

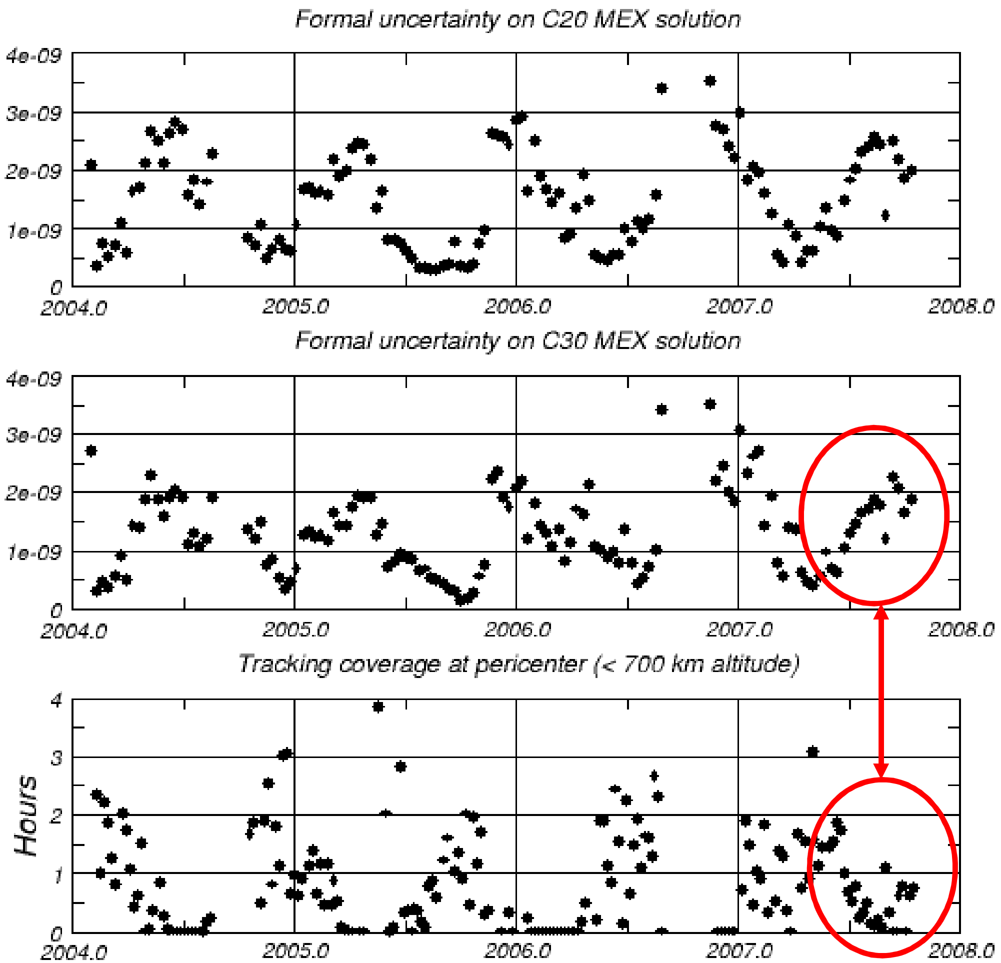
- **MGS orbit** (actual orbit **accuracy is 1-2 meters** on average): Signature of **odd zonal harmonics < 1 meter & even zonal harmonics < 10 cm**

- **MEX orbit** (actual orbit **accuracy is 20 meters** on average): Signature of time variable **zonal harmonics ~10 cm !**

# Fit of time variable $C_{20}$ & $C_{30}$ from MGS/ODY and from MEX



# Effect of poor MEX tracking coverage on time variable $C_{20}$ & $C_{30}$ solutions




- The formal uncertainty on $C_{20}$ & $C_{30}$ solutions from MEX are systematically higher when tracking data are not available at pericenter passes, i.e. when the drag cannot be well resolved !

- This limits our ability to retrieve the information about the time variable gravity from MEX orbits

# How to improve Mars seasonal gravity solutions? (1)

*Odd zonal harmonics variations detection favored by polar orbit:*

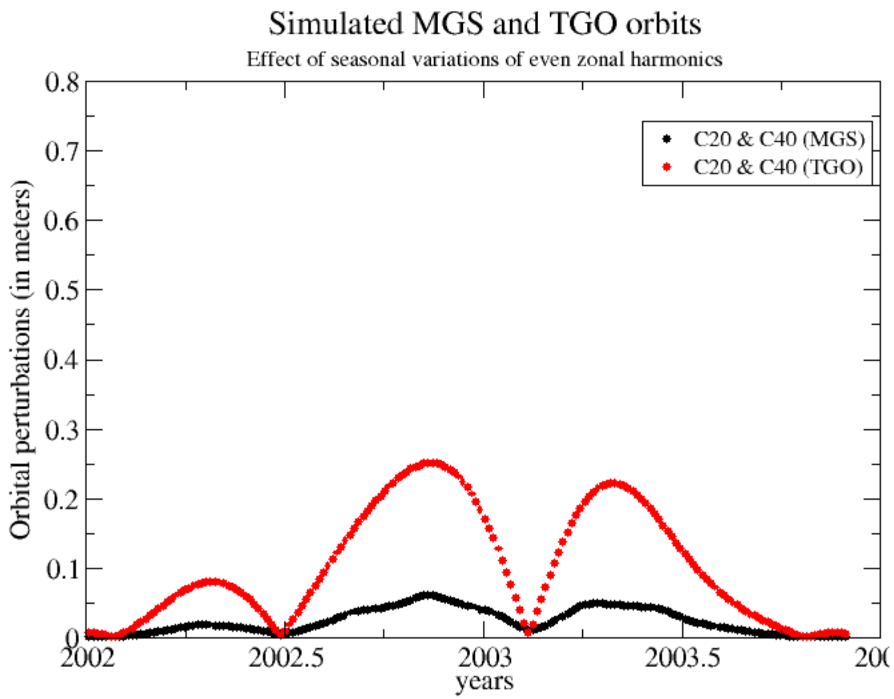
$$\boxed{\frac{dp}{dt} + j\dot{\omega}_S p = -\frac{3n}{2(1-e^2)^3}\sin(i)\left(\frac{a_e}{a}\right)^3\left[\left(\frac{5}{4}\sin^2(i)-1\right)+\left(\frac{1}{2\sin^2(i)}-\frac{31}{8}+\frac{15}{4}\sin^2(i)\right)e^2\right]C_{30}}$$

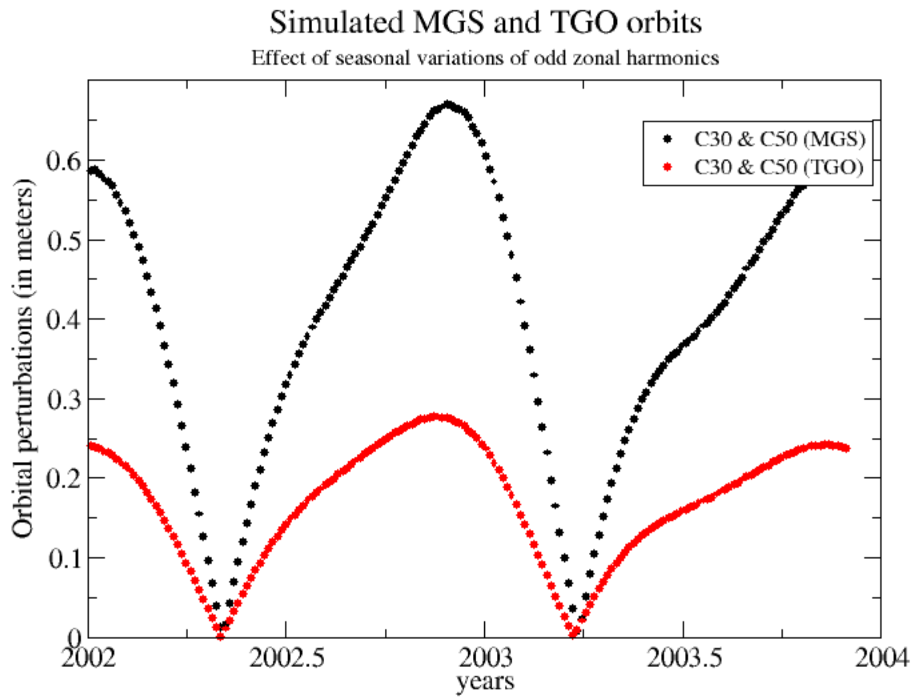
*Even zonal harmonics variations detection favored by non- polar orbit:*

$$\boxed{\frac{d\Omega}{dt} = \frac{3n\cos(i)}{2(1-e^2)^2}\left(\frac{a_e}{a}\right)^2 C_{20}}$$

*New tracking data of orbiter with non-polar orbit are welcome*
*Opportunity: ESA's Exomars spacecraft (**Trace Gas Orbiter**)*
*TGO Non-polar (**74° inclination**) near-circular orbit (400x400 km).*

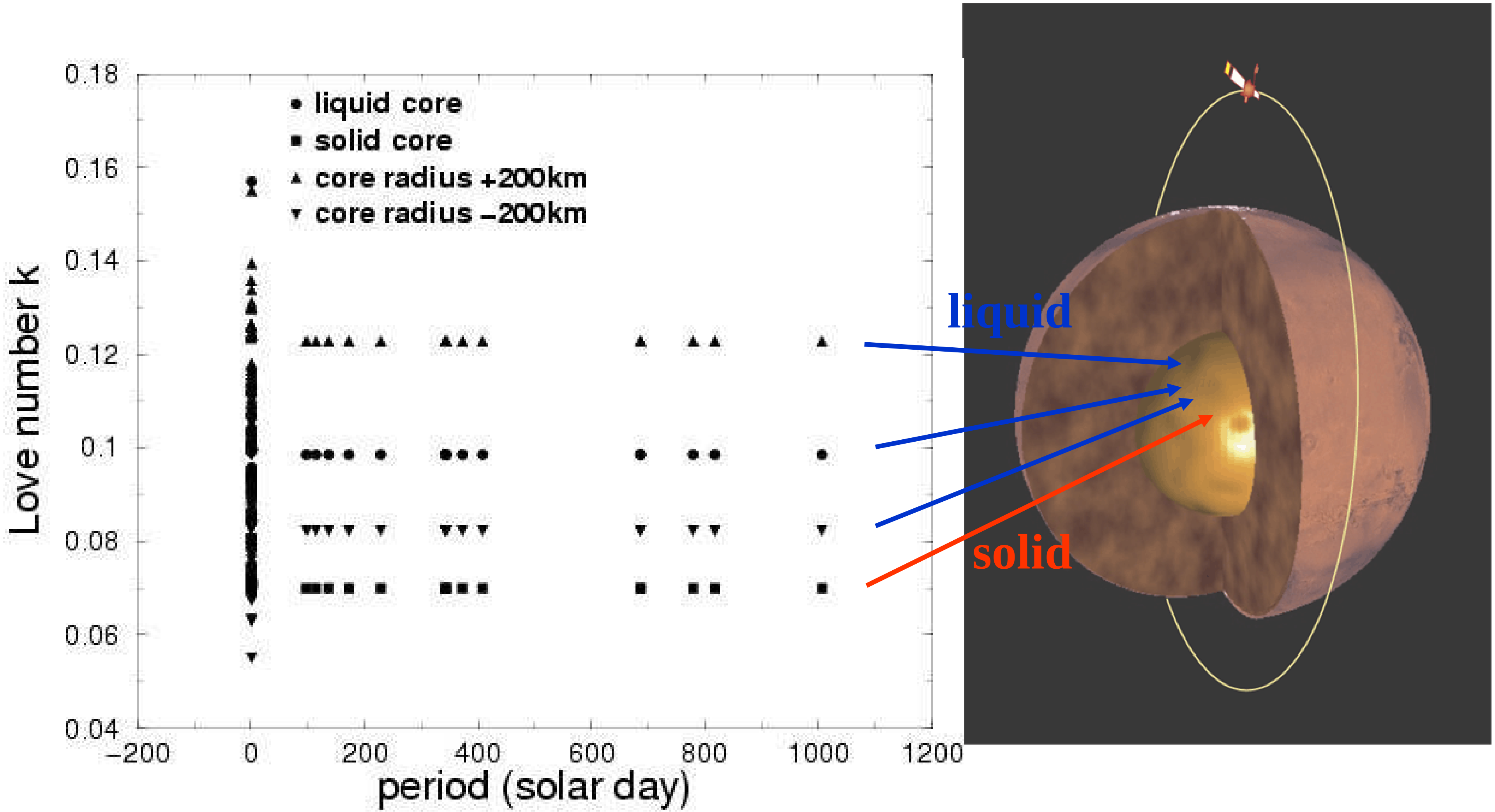
# How to improve Mars seasonal gravity solutions? (2)



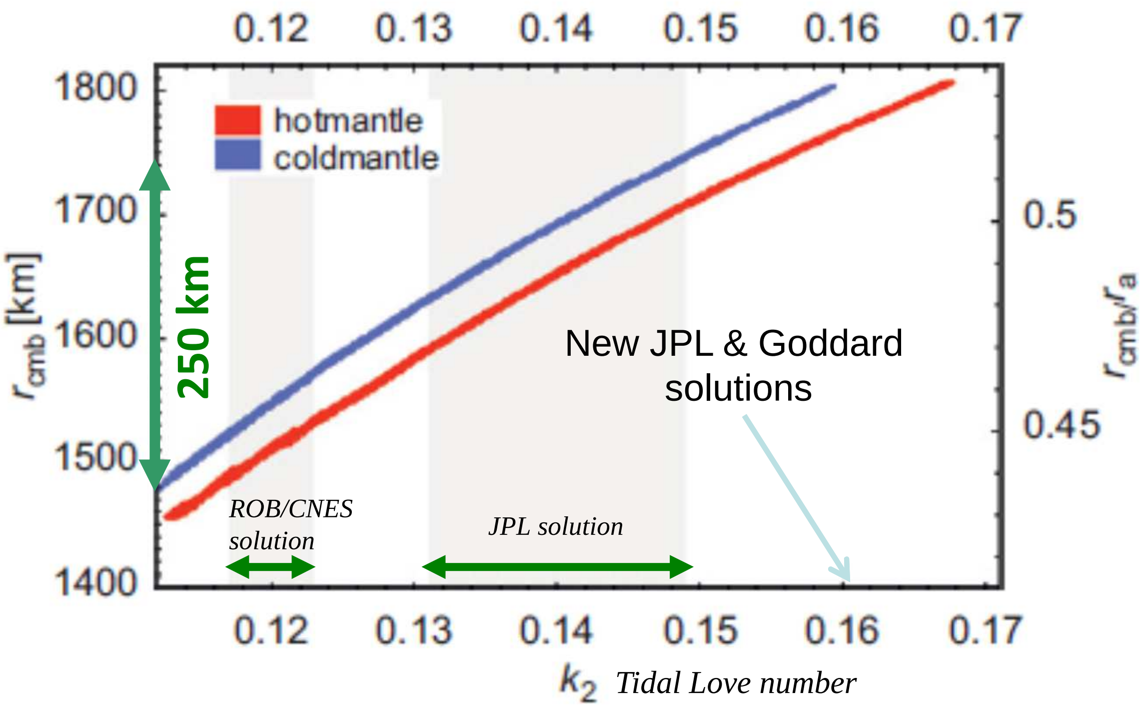

- **Challenging !**

# Tidal potential (*k* Love numbers) and the state and size of Mars’ core



# Current knowledge of the Martian core from geodetic data

(e.g. Yoder et al., 2003; Konopliv et al., 2006; Marty et al., 2009)




*Core radius estimates given possible mantle temperature end-members, mantle rheology, and crust density and thickness range (from A. Rivoldini, ROB).*

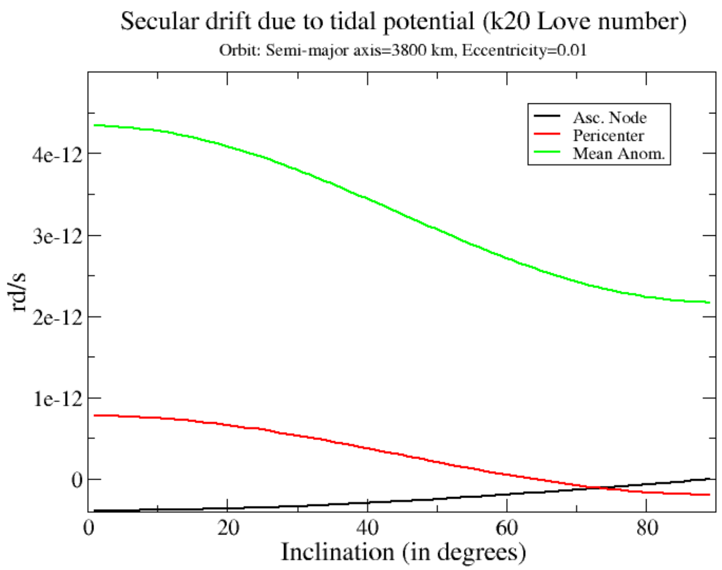
All solutions of $k_2$ indicate a liquid core inside Mars ($k_2 > 0.08$), but discrepancies still remain, thus implying a too large uncertainty for core radius estimate **(+/- 250 km)** to better constrain Mars' deep interior structure.

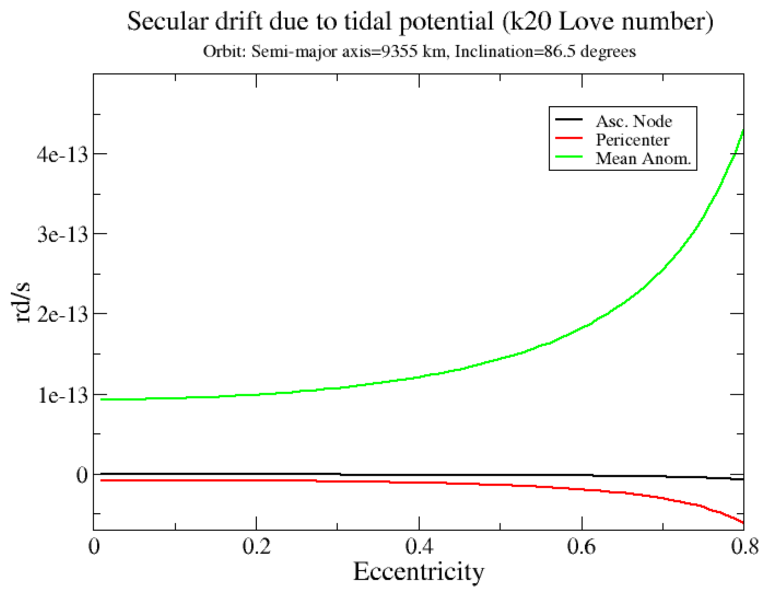
This is due to the difficulty to extract the weak signal of the $k_2$ from the current reconstructed orbits of the Martian spacecraft.

# Secular effect of the $k_2$ Love number on orbital perturbations

$$U_{n,m,p,p',q,q'} = \frac{GM_P}{a_P}\left(\frac{R_0}{a_P}\right)^n\left(\frac{R_0}{a}\right)^{n+1} k_n\delta_m \frac{(n-m)!}{(n+m)!} F_{n,m,p}(i)F_{n,m,p'}(i_P)$$
$$G_{n,p,q}(e)G_{n,p',q'}(e_P)\cos\psi_{n,m,p,p',q,q'}$$

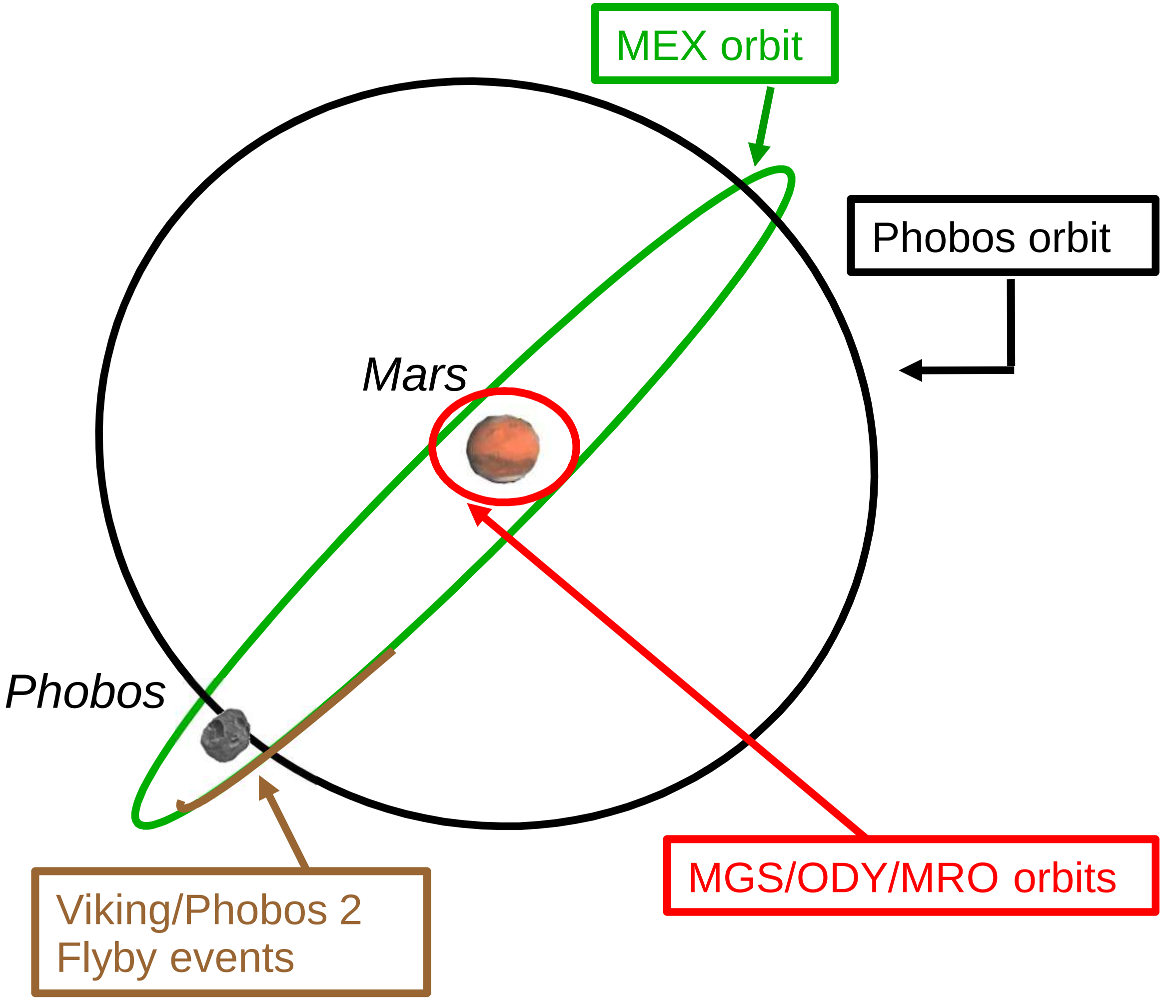
$$\psi_{n,m,p,p',q,q'} = (n-2p+q)M-(n-2p'+q')M_P+(n-2p)\omega-(n-2p)\omega_P+m(\Omega-\Omega_P)$$




- Analytical developments, following Kaula’s approach: near-polar orbits are not the most suitable orbits for $k_2$ determination

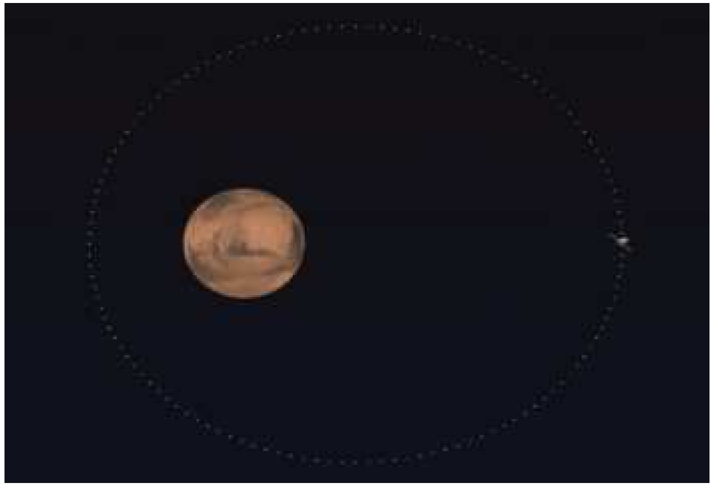
# Improved determination of the mass of the Martian moons with MEX.



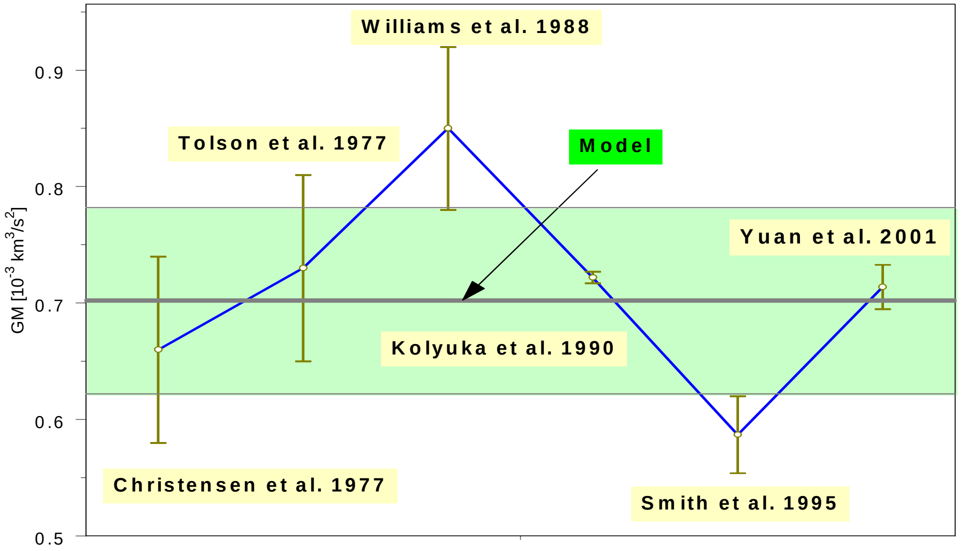

- A small secular drift of spacecraft orbits is caused by the Martian moon masses
- The higher elliptical orbit of MEX makes it more sensitive to the gravitational attraction of the Martian moons.
- Secular solution of the mass of the moons from stacking together all MEX data-arcs.
- Flyby solution with MEX too.

- ***Taking advantage of the higher elliptical orbit of MEX to improve the mass of the Martian moons: secular & flyby (Phobos) solutions.***

# MEX orbital motion perturbations at close flyby to Phobos

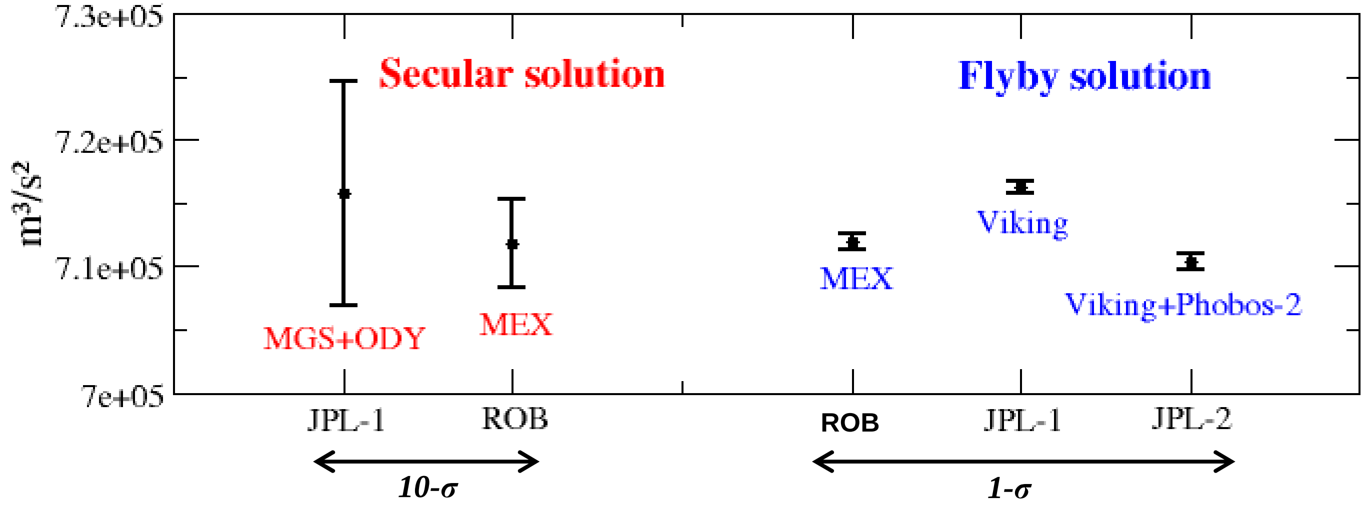


# Much better determination of Phobos mass




Before Mars Express:

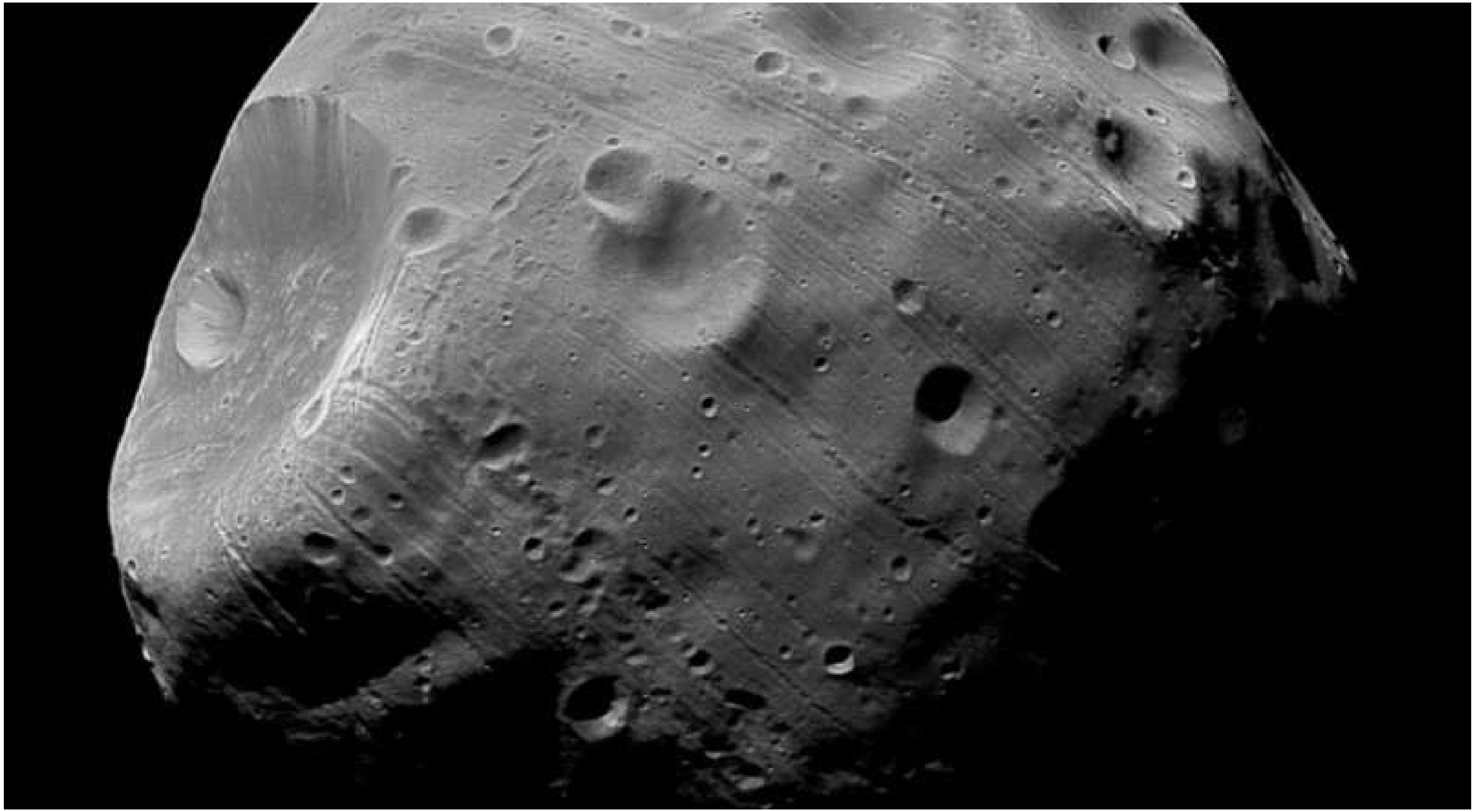
8.8 $10^{15}$ kg - 1.27 $10^{16}$ kg
~40% of error




After Mars Express:

1.06 $10^{16}$ kg
~0.4% of error

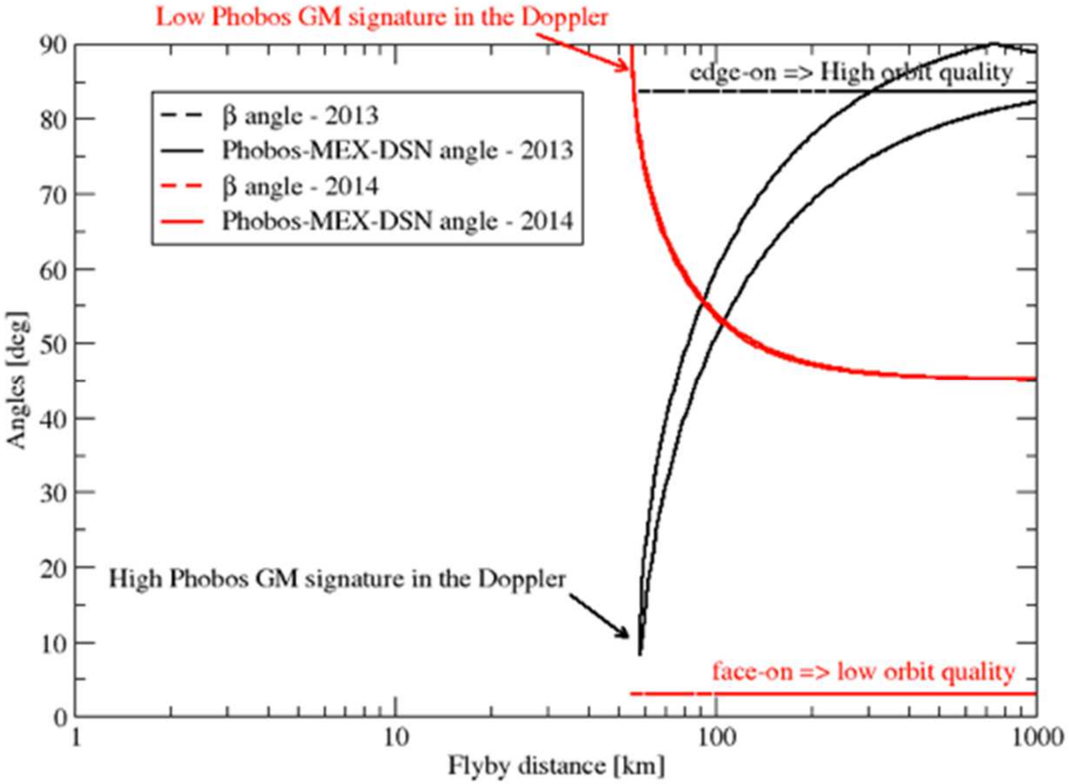
# Determination of the gravity field of Phobos



MEX/HRSC image

***Results using flyby with Mars Express***

# Flyby distance and Doppler geometry:



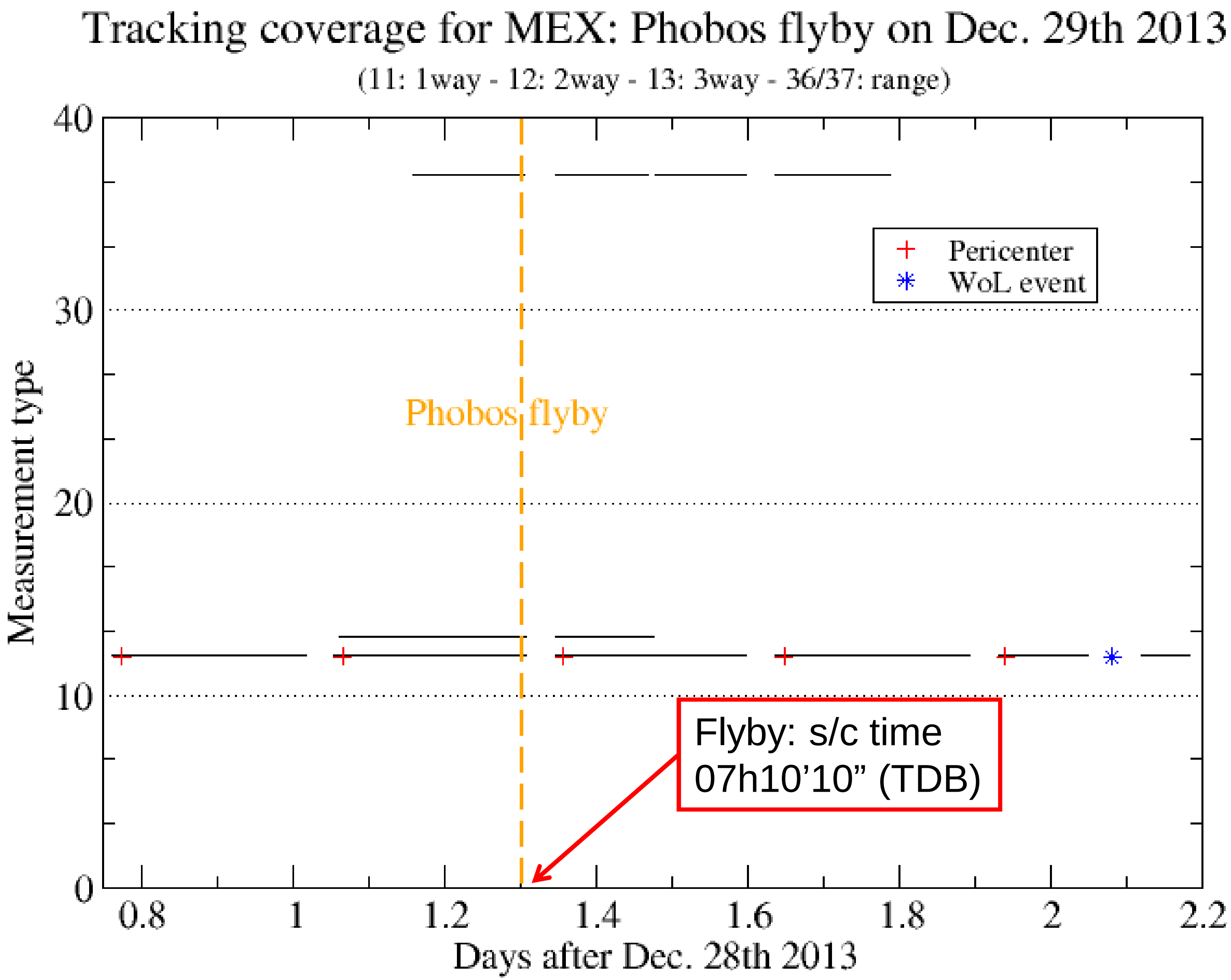

- Good Doppler geometry for sensitivity to MEX orbital perturbations and especially Phobos GM perturbations:

  → Phobos-MEX-Earth almost aligned (10° ) at the closest approach.

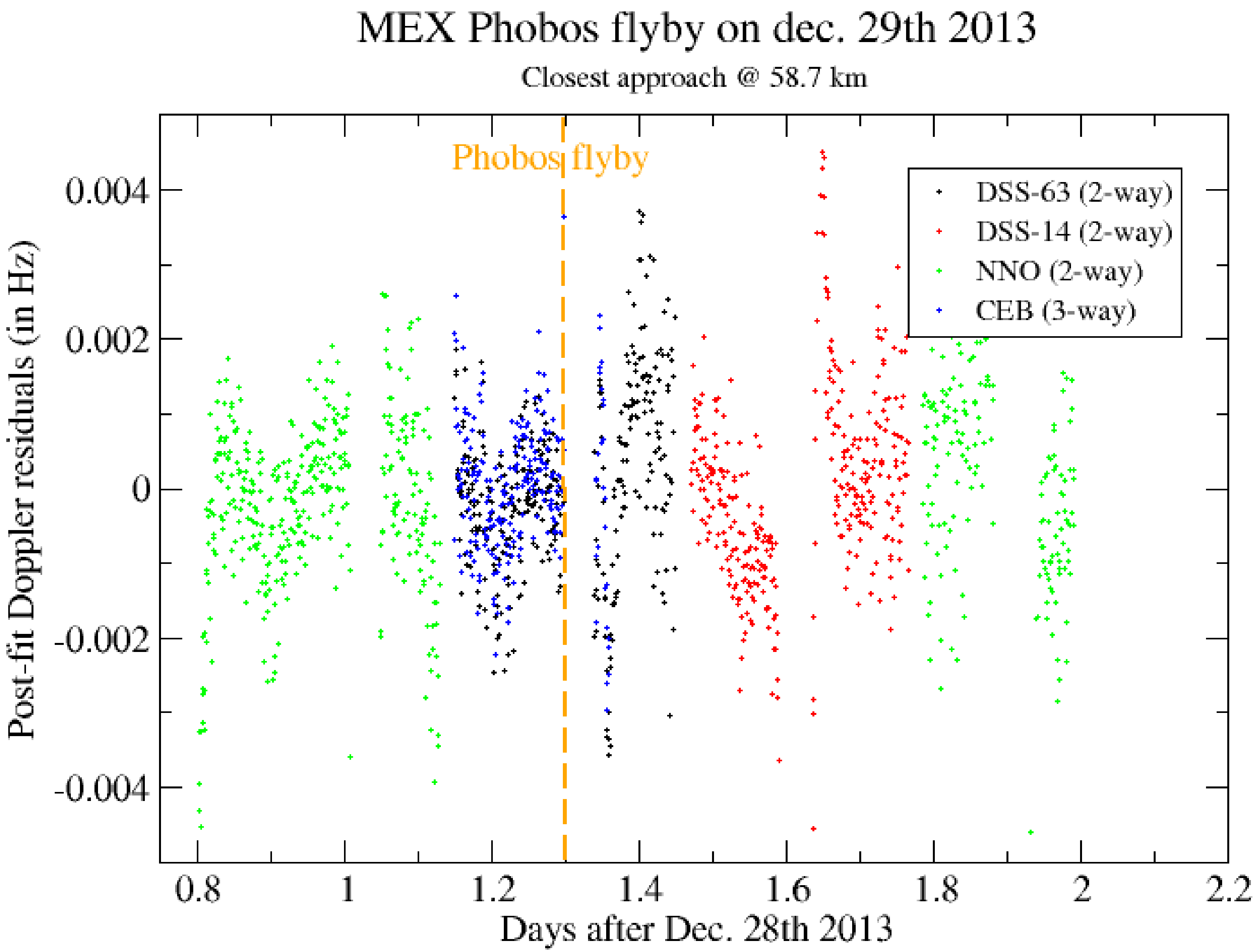
- Closest approach ever:

  58.71 km (from mar097 ephemeris)
  58.61 km (from New IMCCE ephemeris)

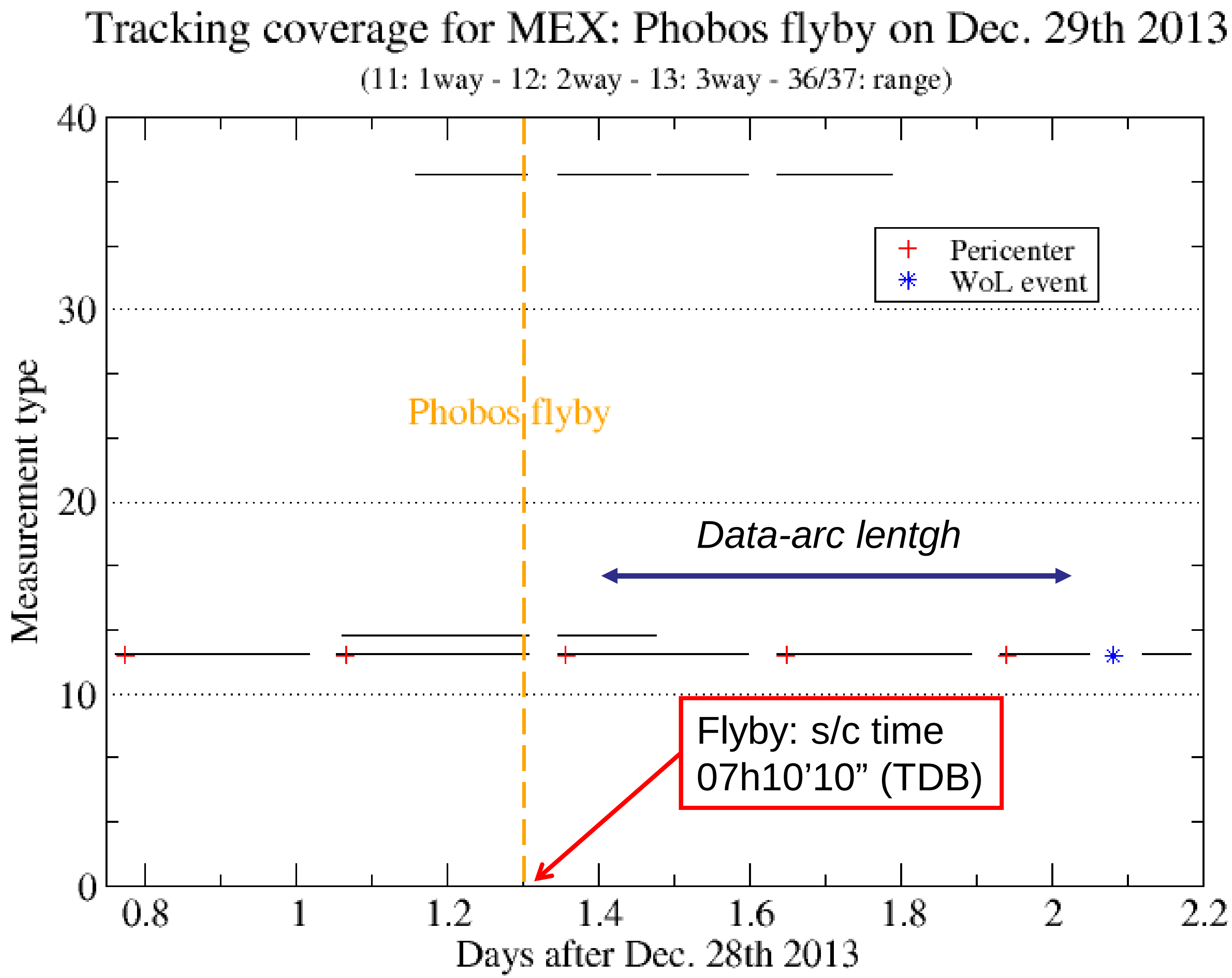
  58.77 km (from mar085 ephemeris)
  59.21 km (from Lainey et al. (2007) epehemris)




- ✓ Quasi-continuous tracking up to 2 orbital revolutions before and after the flyby.
- ✓ POD fit using all this tracking data: long data-arc of about 1.2 days

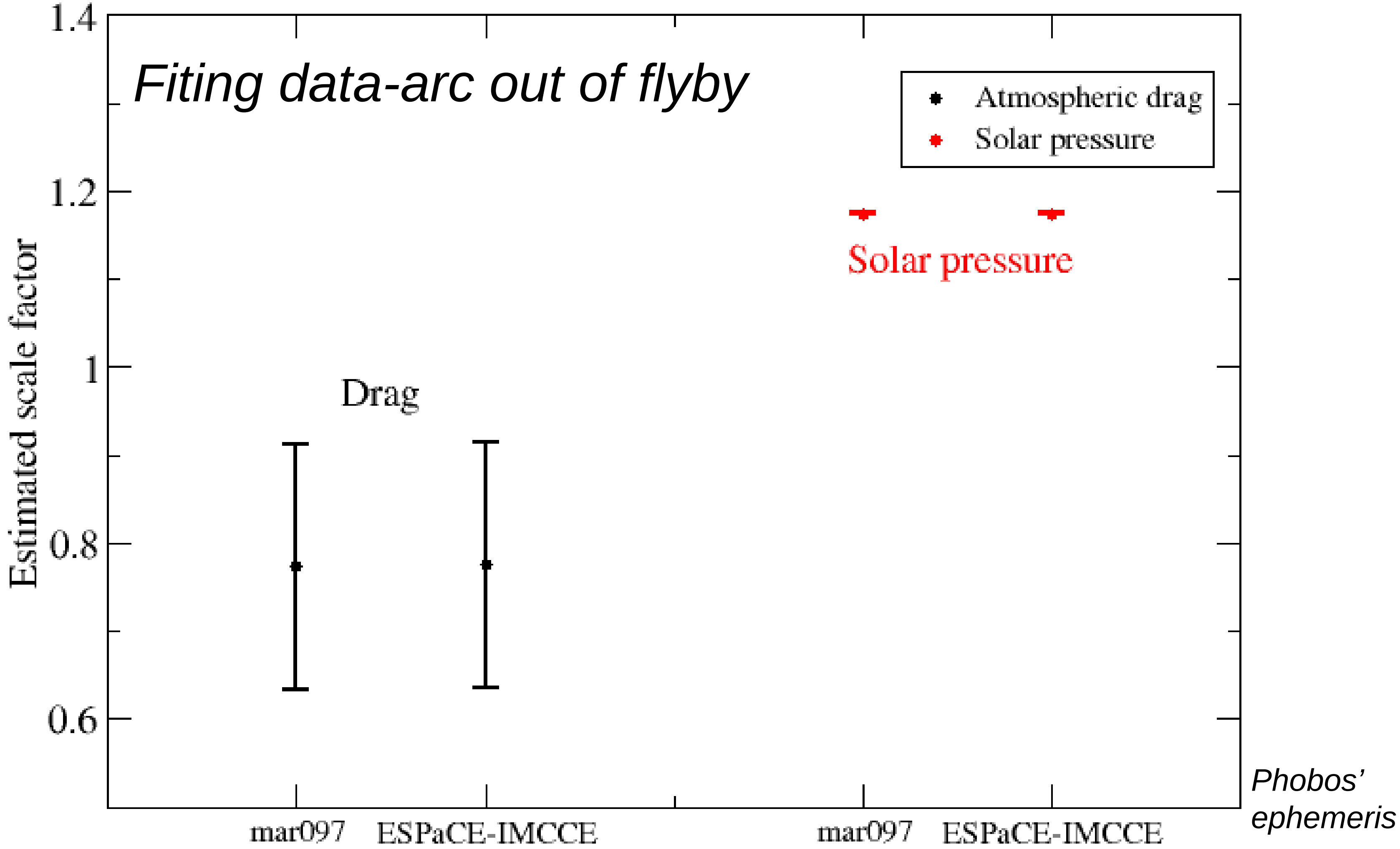



- ✓ Doppler post-fit residuals: RMS value ~ 1 mHz
- ✓ But irrealistic scale factor (drag, solar pressure) *Not shown here*

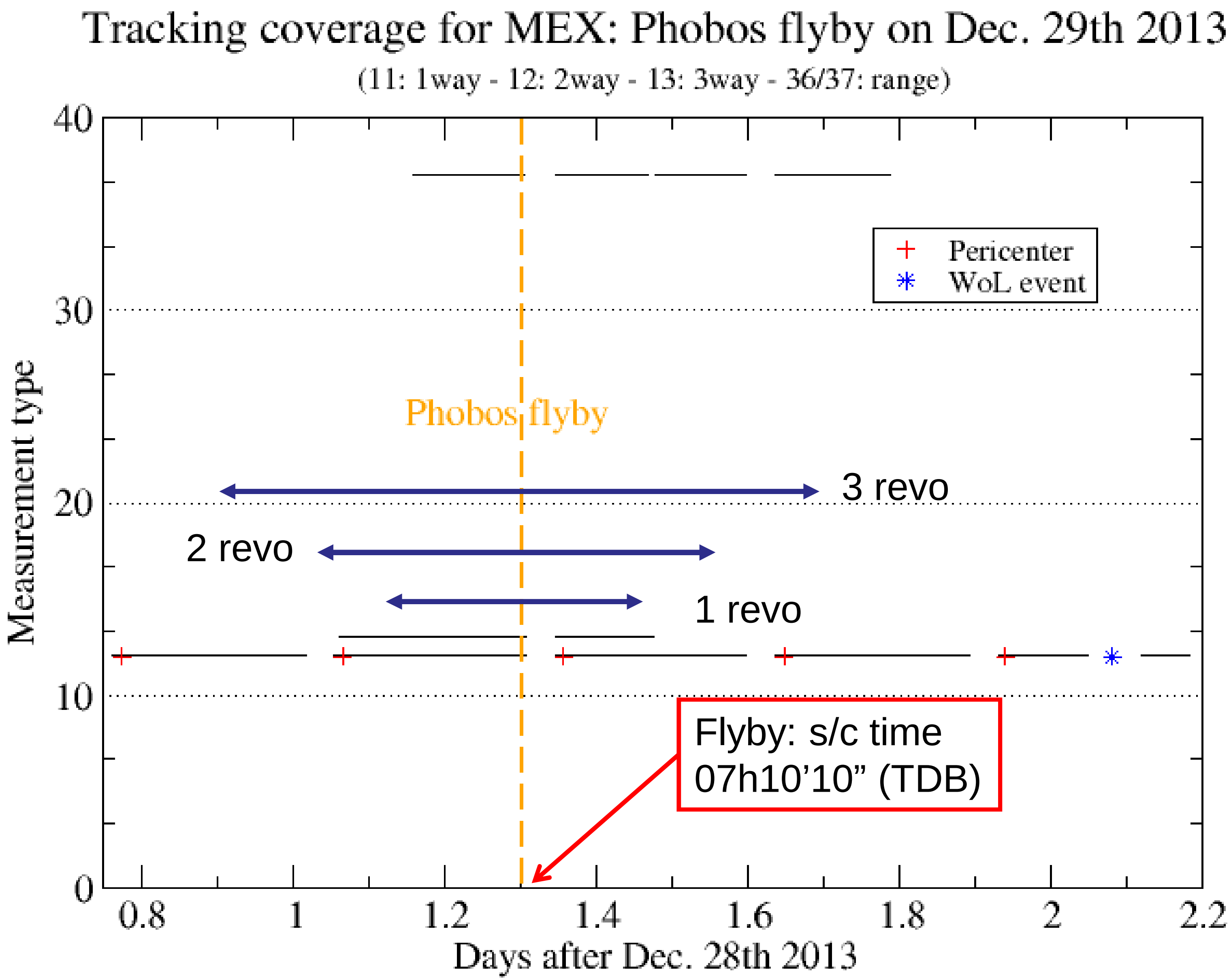



✓ Quasi-continuous tracking up to 2 orbital revolutions before and after the flyby.

# Estimating dynamical parameters others than Phobos' gravity field

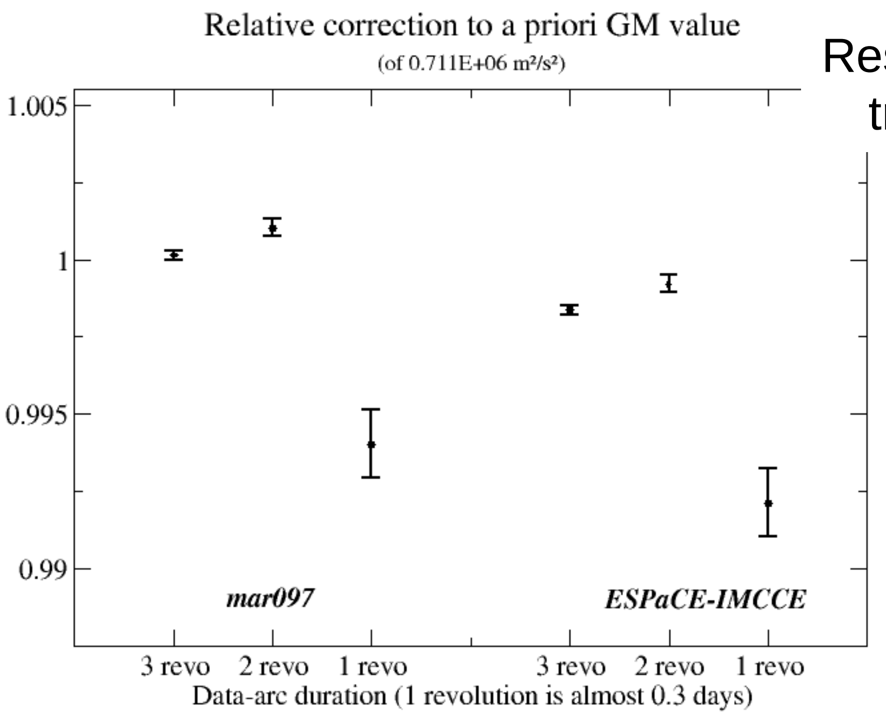



- ✓ Drag scale factor: 0.77 +/- 0.14 & Solar pressure scale factor: 1.1743 /- 0.0016
- ✓ Independent of Phobos' ephemeris.
- ✓ Taken as a priori value and constraints for GM/C20 fit with data-arcs including flyby

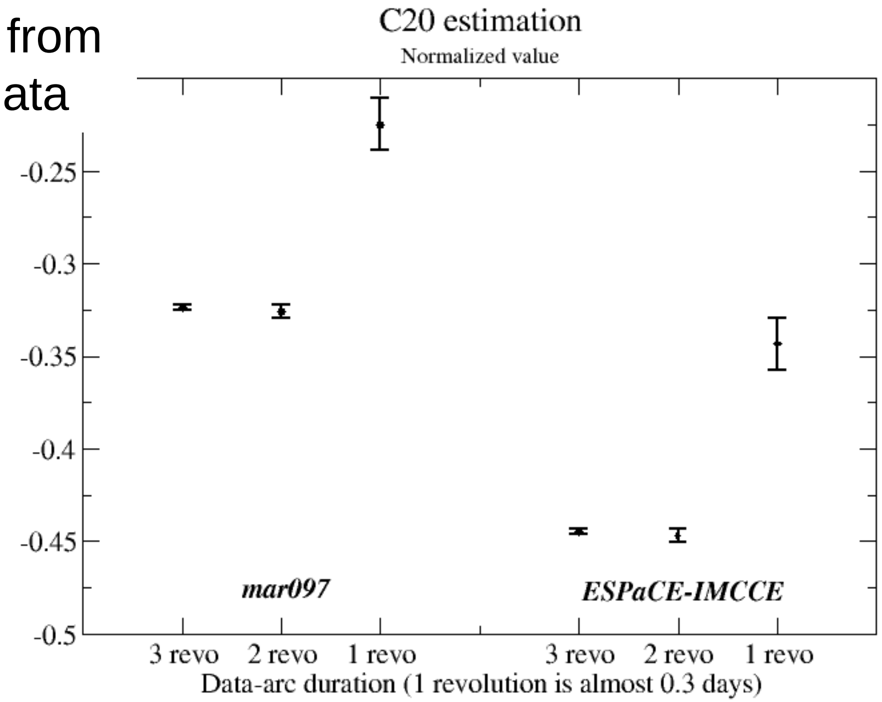



✓ Data-arc length: 3, 2 and 1 orbital revolutions centered on flyby

# Estimating dynamical Phobos' gravity field from flyby




Results from true data




- ✓ GM estimated close to initial value of 0.711 E+06 m3/s2 with formal error of about 0.02%.
- ✓ $C_{20}$ estimated close to about -0.32 with formal error of about 0.002 (0.6%).

- ✓ Precise solution of GM & $C_{20}$ but large biais to $C_{20}$ solution (physically unplausible)
- ✓ 'Slight' biais between solutions using mar097 vs ESPaCE-IMCCE ephemeris

Possible explanation → biased solutions due to error on ephemeris?

# Simulation process

Simulation of Doppler tracking data:

Data-arc duration : 1 revo., 2 revo., 3 revo., around flyby
Ground station as for MEX tracking of December 29th 2013
Initial state vector (position/velocity) from FD MEX orbit
***Initial GM=0.711 $10^6$ $m^3/s^2$ and $C_{20}$=-0.1***

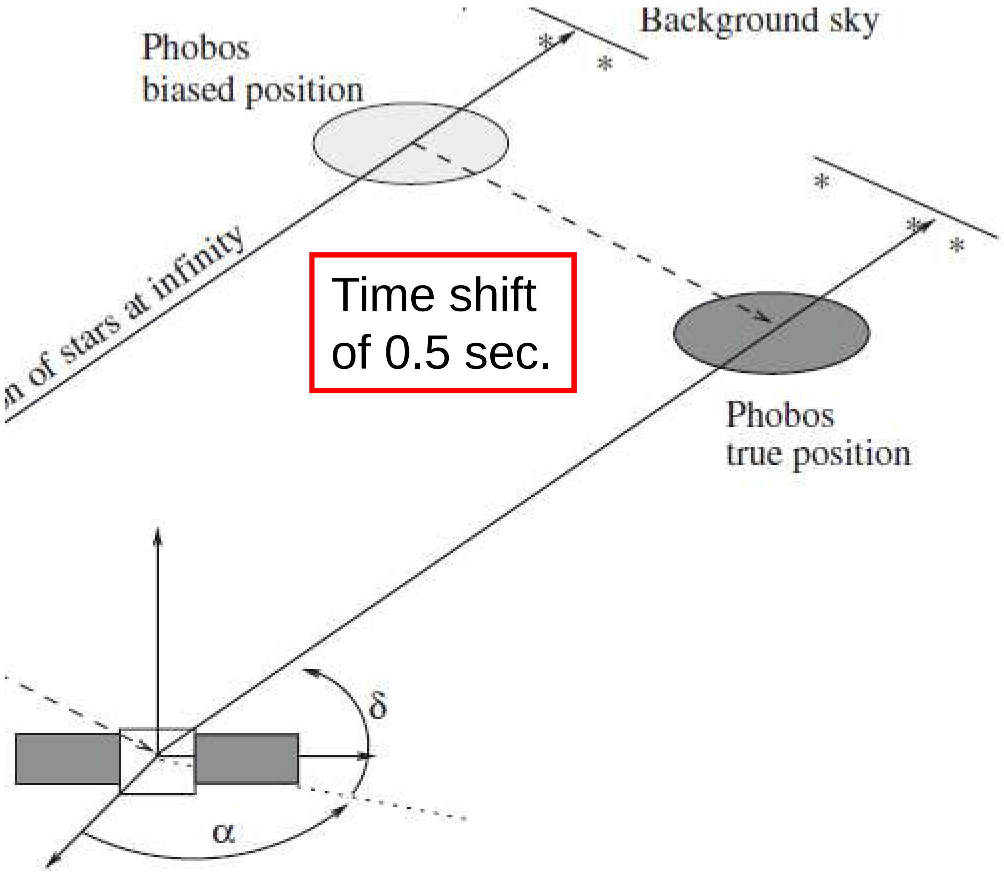
Simulated Doppler data (60sec sampling time) with white noise at 0.02 mm/s.

Modified parameter value as *a priori* value: ***GM = 0.709 $10^6$ $m^3/s^2$*** and ***C20 = 0.0***
and perturbed ephemeris at ***1000 meters, 100 meters and 10 meters level.***

*Fitted parameter:*
Initial state vector (position/velocity) of MEX at the beginning of the data-arc and ***GM and $C_{20}$.***

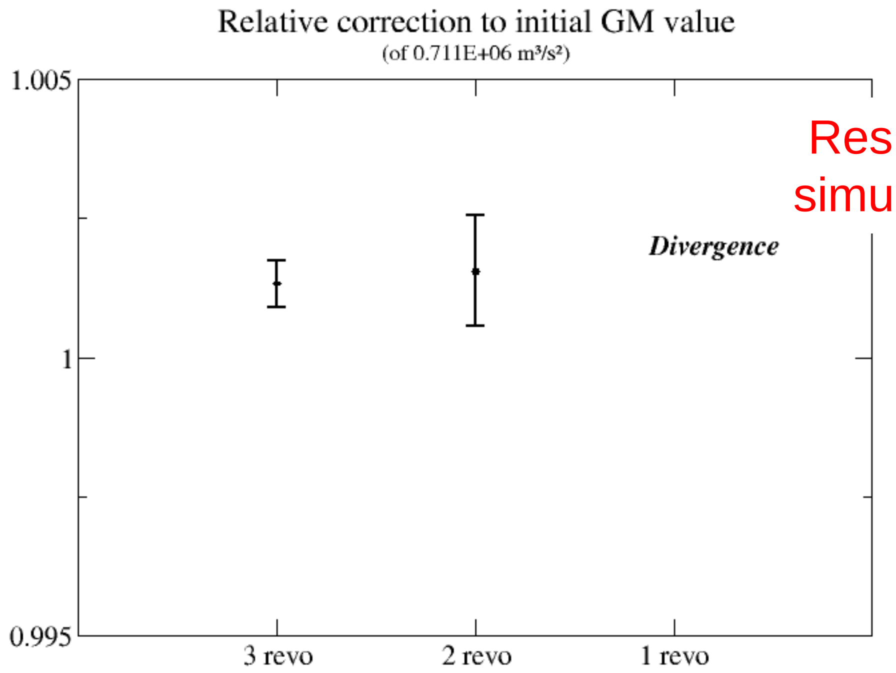
*Results:*
Solution of *GM* and C20 (adjusted value and formal error).
Impact of ephemeris error

# Simulation of Phobos’ ephemeris error

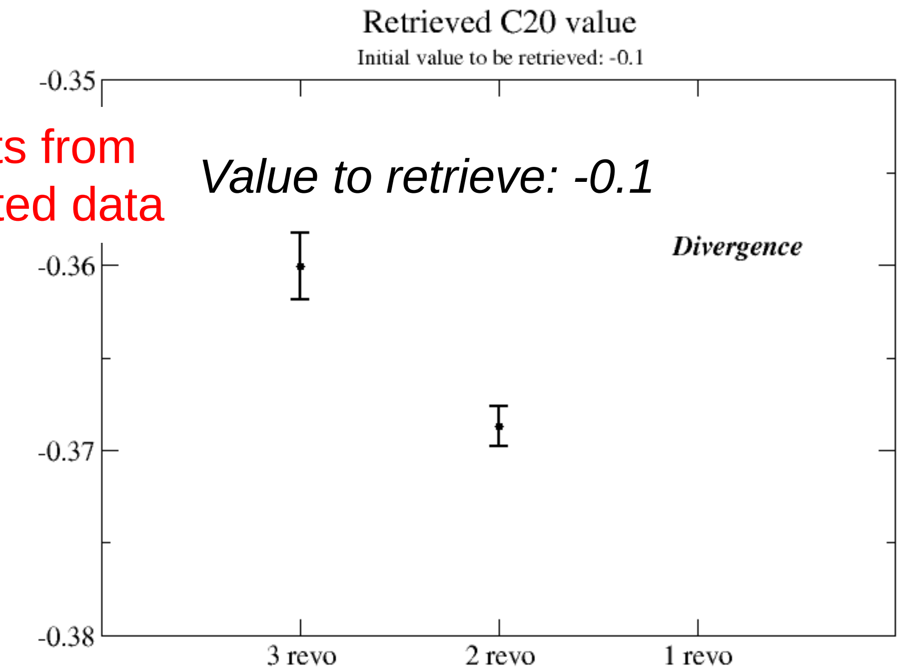



✓ Simulation of Phobos’ ephemeris error: Shifting Phobos position with a constant biais of 1km.

# Result of simulations: effect of Phobos' ephemeris error




Results from simulated data




✓ Slight bias on GM retrieval: 0.1%

✓ Large bias on $C_{20}$ retrieval

✓ Phobos ' ephemeris error of 1 km mimics the bias observed on true data for the $C_{20}$ solution (*better simulation adding the $C_{22}$ to be done*)

✓ 100 meters on ephemeris bias → about 10% of biais on $C_{20}$
10 meters on ephemeris bias → about 1% of bias on $C_{20}$

# Summary for Mars

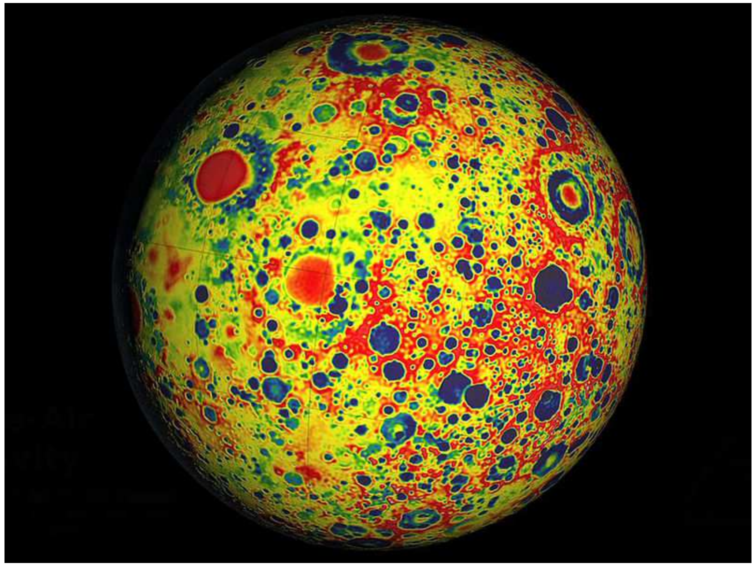
- ✓ Reconstruction of spacecraft orbiting motion is currently the single way to determine the gravity field of planets (solution as, $C_{lm}$, $S_{lm}$)

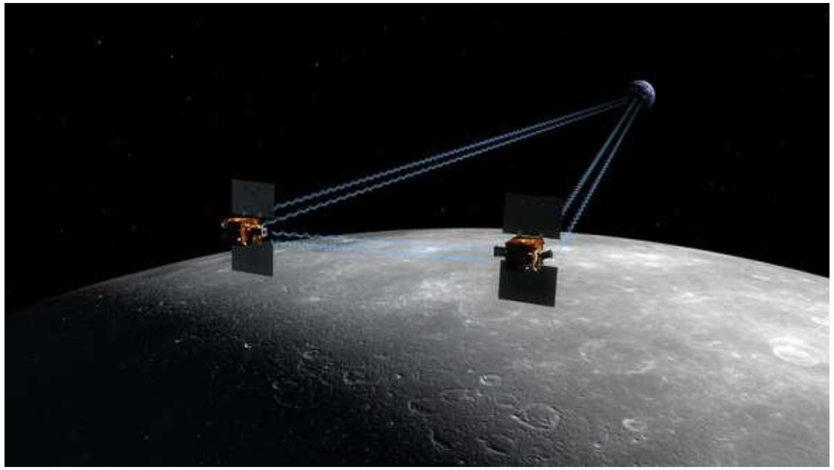
- ✓ Each orbit provides its own sensitivity to each $C_{lm}$, $S_{lm}$ (spatial resolution, “degree strength’).
  → Combination of spacecraft with different orbits to improve gravity field solution.

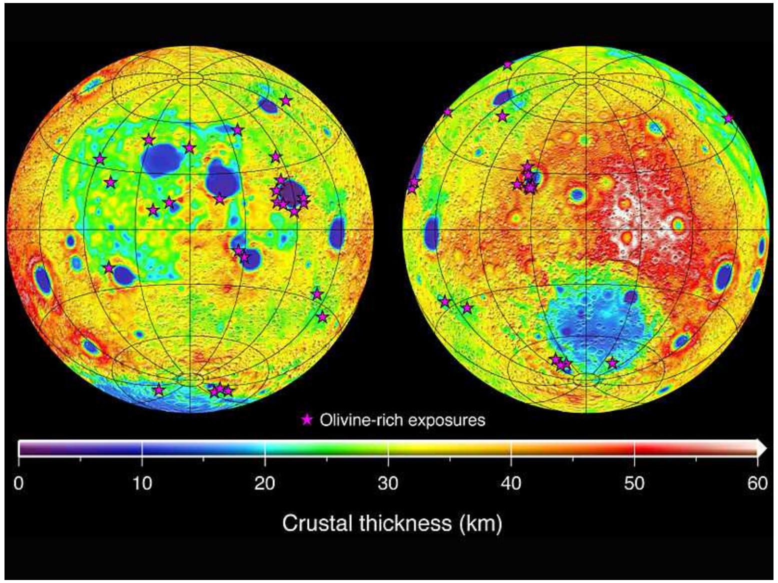
- ✓ The precision on gravity field solution directly depends on precision on orbit reconstruction (→ Mars seasonal gravity variations, $k_2$ and Phobos gravity field)

- ✓ Single straightforward observation of liquid core inside Mars & Venus → deep interior structure and evolution.

# The state-of-the-art: The GRAIL mission





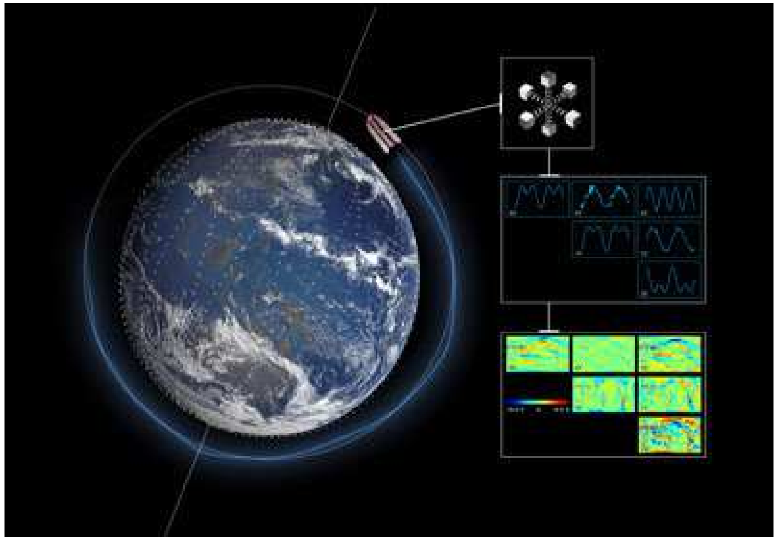


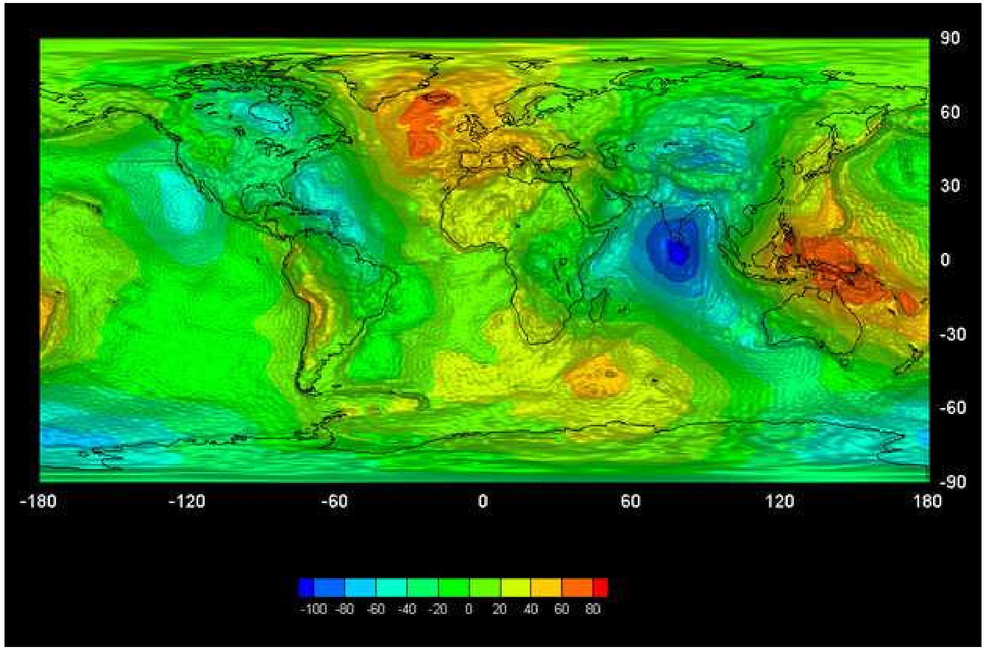

NASA's mission GRAIL:
Two spacecraft orbiting the Moon.
Radio-link between both.

Best gravity field of the Moon ever obtained
And the best gravity field of the Solar system
to date: Spatial resolution ~ 25 km
*Four times better than GOCE Earth's solution*

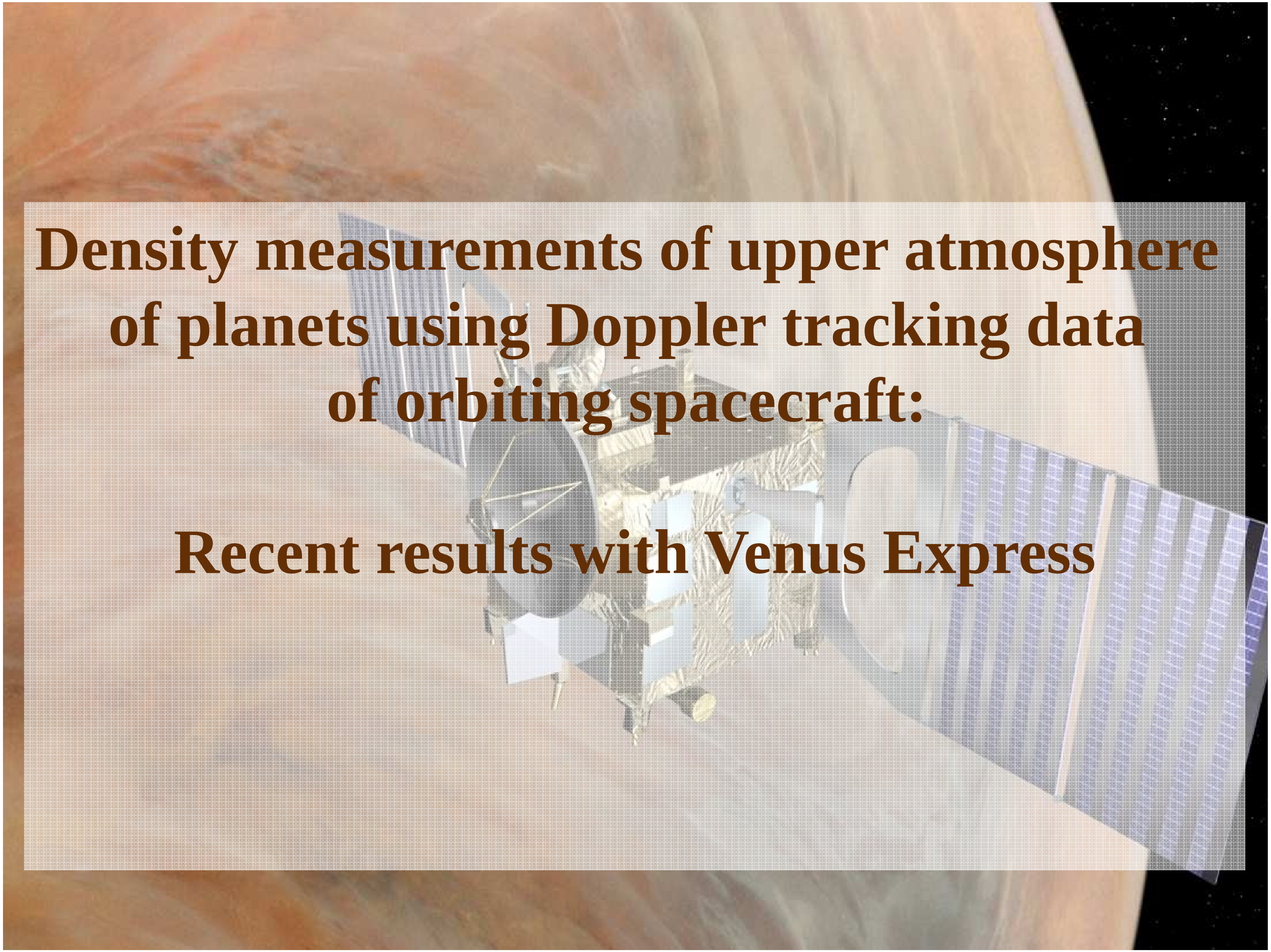
*… for Mars ? GRAIL-like, GOCE-like experiment?*

# The state-of-the-art: The GOCE mission






ESA's mission GOCE:
Best gravity field of the Earth ever obtained
Spatial resolution 100x100 km
Precision on the Geoid: 1 cm
GINS used to produce this new solution



# Density measurements of upper atmosphere of planets using Doppler tracking data of orbiting spacecraft:

## Recent results with Venus Express

# Some basic physics: '*The drag paradox*'

- Drag force is opposite to velocity
- We may guess a deceleration of the spacecraft

$$\delta W = \vec{F}_D . \vec{V} \; dt = -F_D \, V \; dt$$

- Actually it corresponds to an acceleration !

- Drag force is a dissipative force that decreases the mechanical energy → <u>$dE < 0$</u>:
- How does it affect a circular orbit?

(1) <u>Decrease of semi-major axis "*a*":</u>

$$\delta W = dE = d\left(-\frac{GM}{2a}\right) = \frac{GM}{a^2} da$$

$\boxed{\text{➢ } dE < 0\text{, hence } da < 0}$

(2) <u>Acceleration of orbital speed "*V*":</u>

$$dV = d\left(\sqrt{\frac{GM}{a}}\right) = -\frac{1}{2}\sqrt{\frac{GM}{a^3}} da$$

$\boxed{\text{➢ } da < 0\text{, hence } dV > 0}$

(3) <u>Shortening of orbital period "*T*":</u>

$$dT = d\left(\frac{2\pi}{\sqrt{GM}} a^{3/2}\right) = 3\sqrt{\frac{a}{GM}} da$$

$\boxed{\text{➢ } da < 0 \text{ hence } dT < 0}$

# Motivation for VExADE

- Existing density models built with orbital period approach and also mass spectrometers, probes, ground-based observations …):

  VTS3 (Hedin et al., 1983) *fortran-routine*
  VIRA (Keating et al., 1985) *look-up table*
  Venus Gram (recently released) *fortran-routines*

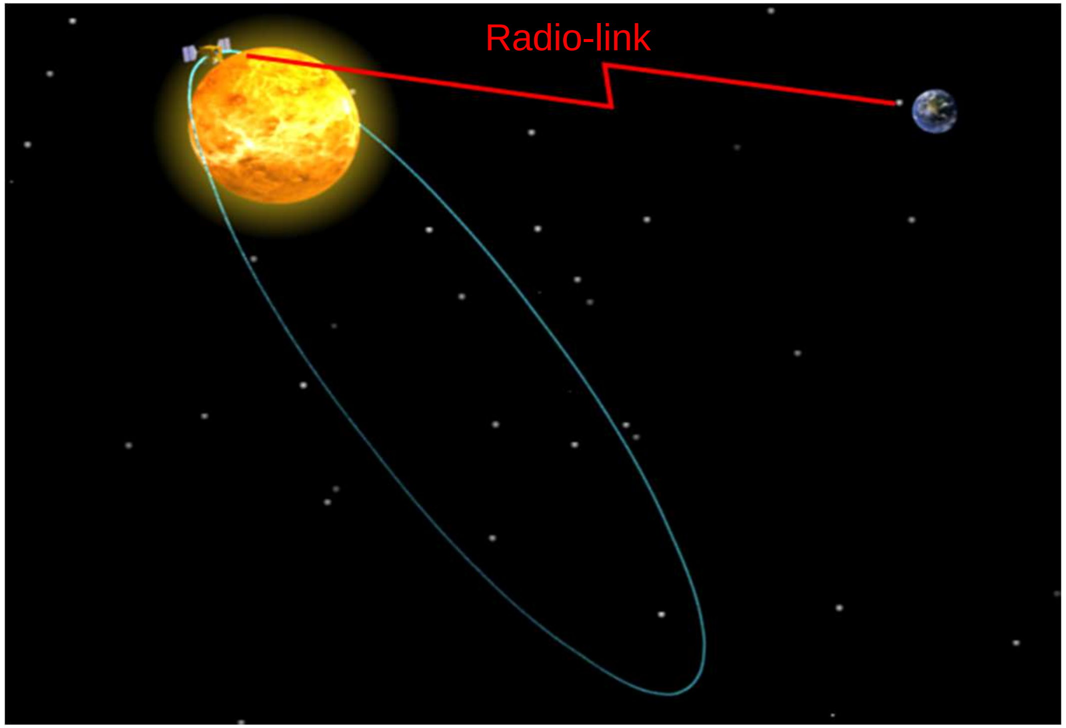
- **These models are constrained for equatorial latitudes**

  **→ Needs of new data for higher latitudes**

- **Venus Express Atmospheric Drag Experiment: VExADE**

  To probe *in situ* the polar thermosphere at low solar activity

# The VExADE campaigns




- 10 campaigns have been performed, each over a few successive days from 2008 to 2012.
  Back to nominal orbit (peri at 250 km) between each campaign.

- Derivation of Venus upper atmosphere neutral density using Doppler tracking data → Precise Orbit Determination (POD).

# POD process

<u>Input :</u>

Spacecraft dynamical model
*Tracking data* : Doppler + Range from ESTRACK/DSN

Least Squares fit of spacecraft dynamical model to tracking data on ***successive data-arcs of a given duration*** using ***GINS*** software

<u>Output :</u>

Precise Reconstructed Orbit
***Upper atmosphere density, and others parameters***

*GINS: Géodésie par Intégrations Numériques Simultanées, developed by CNES and applied by ROB for planetary geodesy applications*

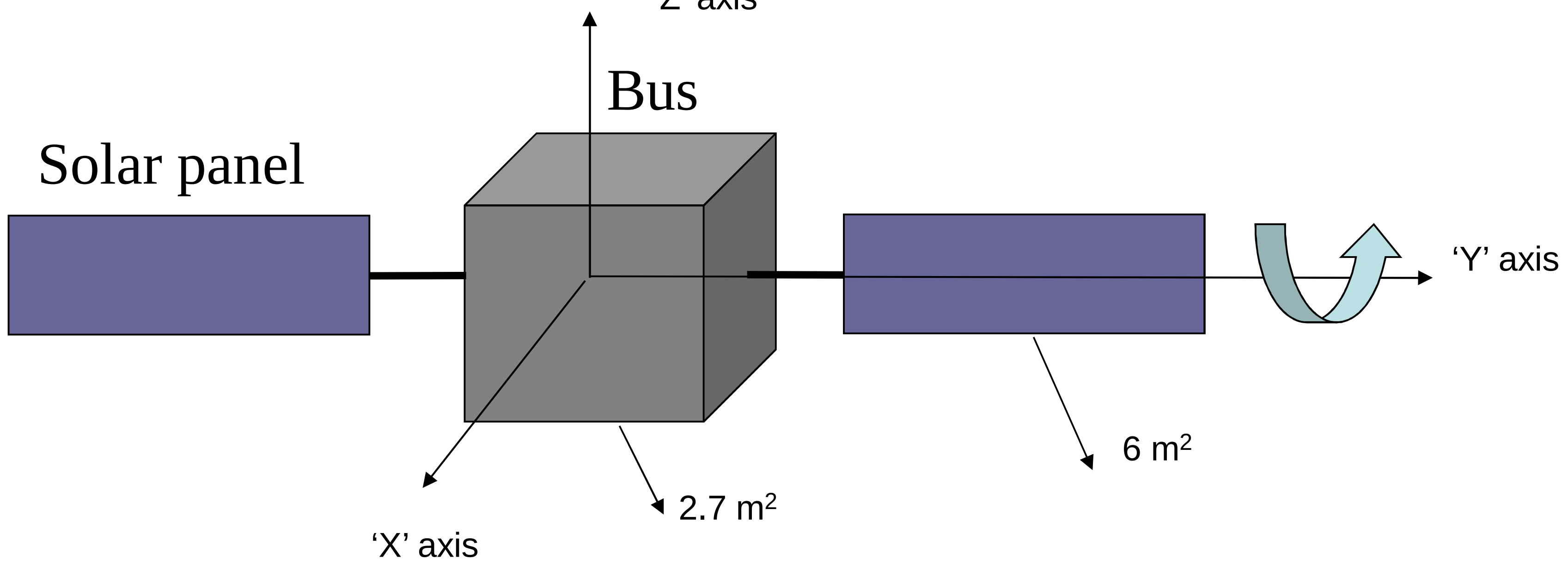
# Venus Express dynamical model

- Model of all the forces acting on the spacecraft

- Gravitational forces:
  - Venus' static gravity field (JPL 180x180)
  - Point mass representation of other solar system bodies using updated JPL planetary ephemeris.

- Non-gravitational forces :
  - Atmospheric drag (atm. Density model): scale factor FD
  - solar pressure radiation.
  - albedo & IR radiations (mean value).
  - Residual accelerations induced by each unbalanced inertial Wheel-Off-Loading (WoL) event

# Venus Express “Box-and-Wings” model




✓ “Box-and-Wings” model or “Macro-Model” (from navigation team):

1) Shape of spacecraft:

flat plates (6 for the bus & 4 for solar panels) with known area

2) Optical properties:

known reflectivity and absorptivity coefficients

3) Orientation of this Box-and-Wings model in space (quaternions)

*The shift between the s/c center of mass and the HGA1 or HGA2 center of phase*

# Precise modeling of the drag acceleration

$$A_D = -\frac{1}{2}\rho F_D \sum_{i=1}^{i=N} C_{Di} \frac{S_i}{m}(V_r.n_i)V_r$$

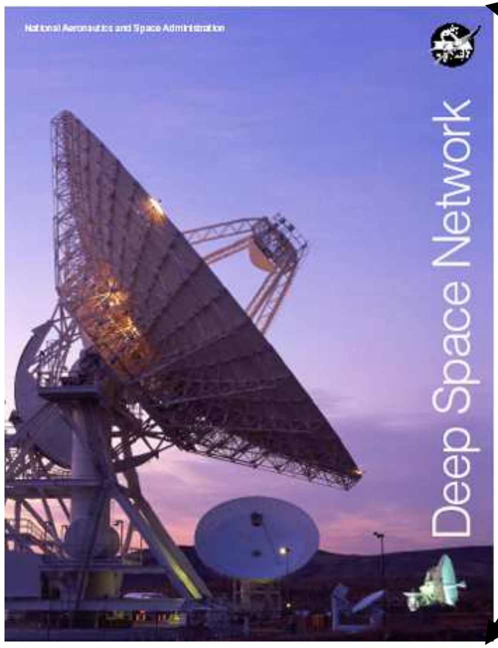
Where:
$A_d$ = *Drag acceleration*
***$\rho$ = Predicted atmospheric density (Hedin-VTS3 model or Venus-Gram)***
$S_i$ = *surface area of the plate « i »* $C_{Di}$ = *Drag coefficient of the plate « i »*
$V_r$ = *velocity vector of spacecraft w.r.t. atmosphere*
$n_i$ = *unit vector normal to the face « i » (orientation of the plate)*
$m$ = *spacecraft mass*
***$F_D$ = estimated drag scale factor*** *(a priori = 1, no a priori constraint)*
$N$ = *number of flat plates of the macro-model*

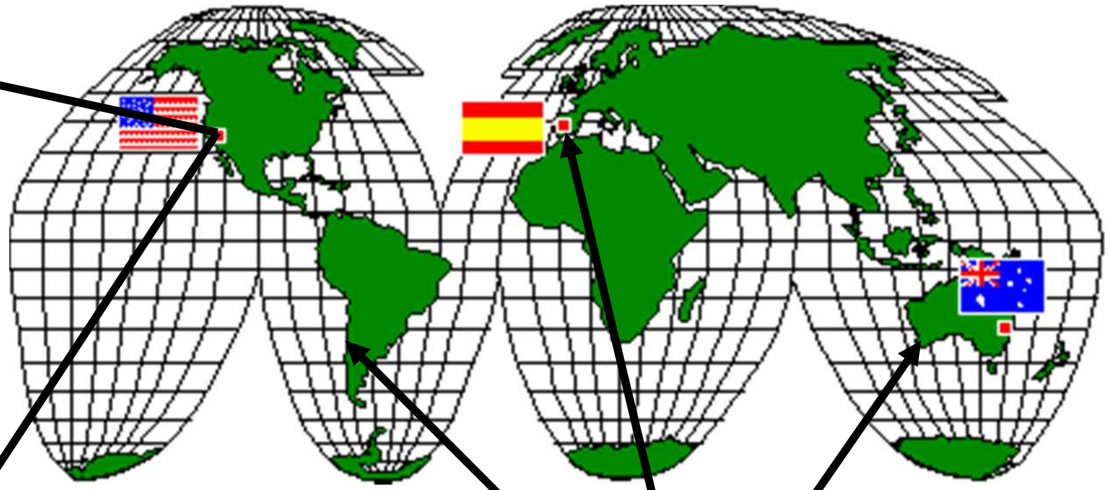
$$\boxed{\rho_{measured} = F_D.\rho_{predicted}}$$

✓ The fit of the force model to the tracking data provides a drag scale factor of the predicted (modeled) atmospheric density model.

# Ground segment: Tracking stations

➢To track any spacecraft or Lander in the solar system



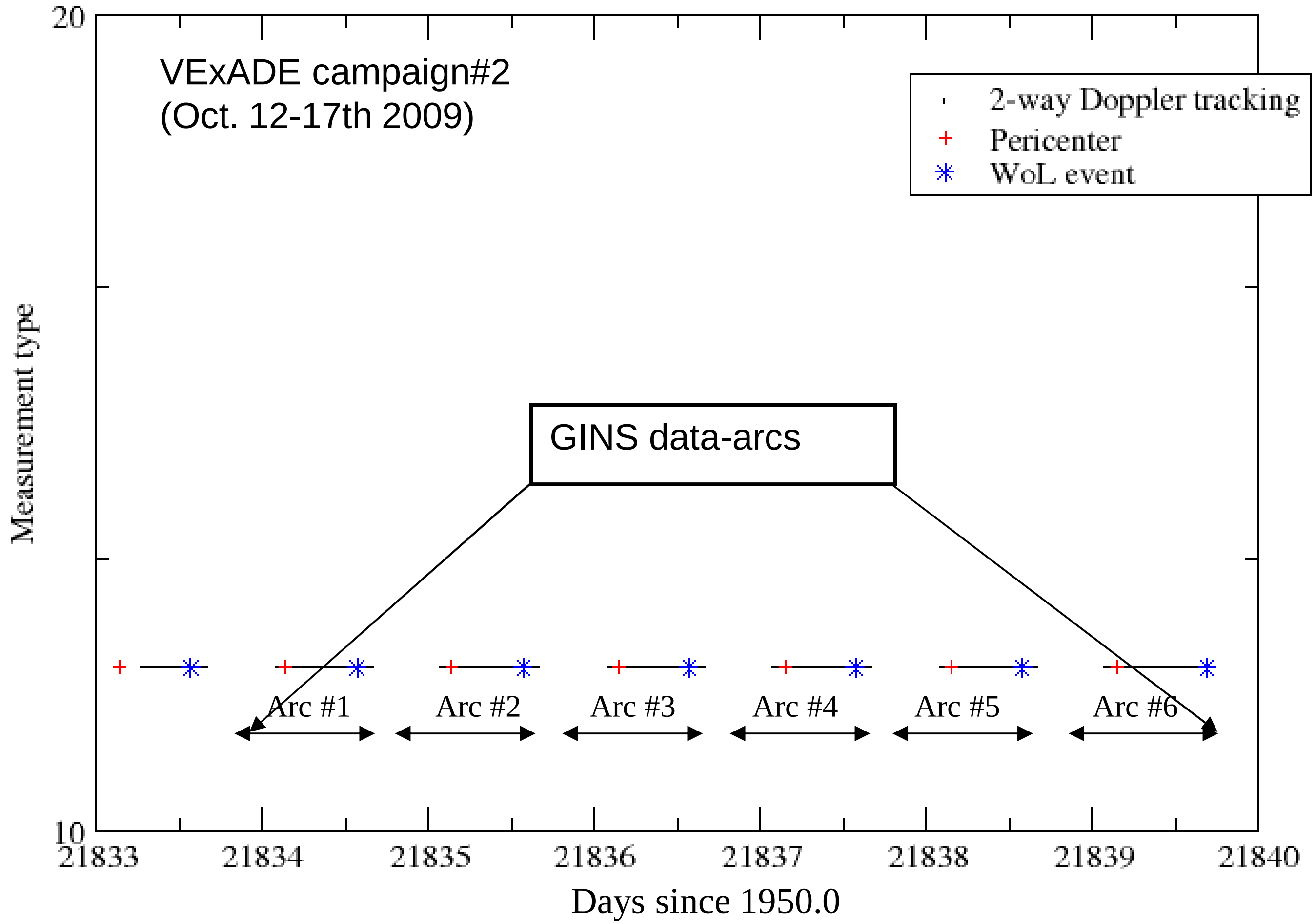



ESTRACK's ESA 34 m. antenna at New Norcia, Cebreros, Malargüe (Mars Express, Venus Expre

Diameter of 25 to 70 meters.
Precision:
0.02-0.05 mm/s (1-3 mHz)
< ~1 m. But bias of ~3-4 m.
Dating: Maser clocks

- Radio-link for data & telemetry
- Doppler & range radio-tracking mainly at frequencies of 8.4 Ghz (X band) (sometimes dual-frequency X/S bands to correct ionospheric and interplanetary plasma perturbations)

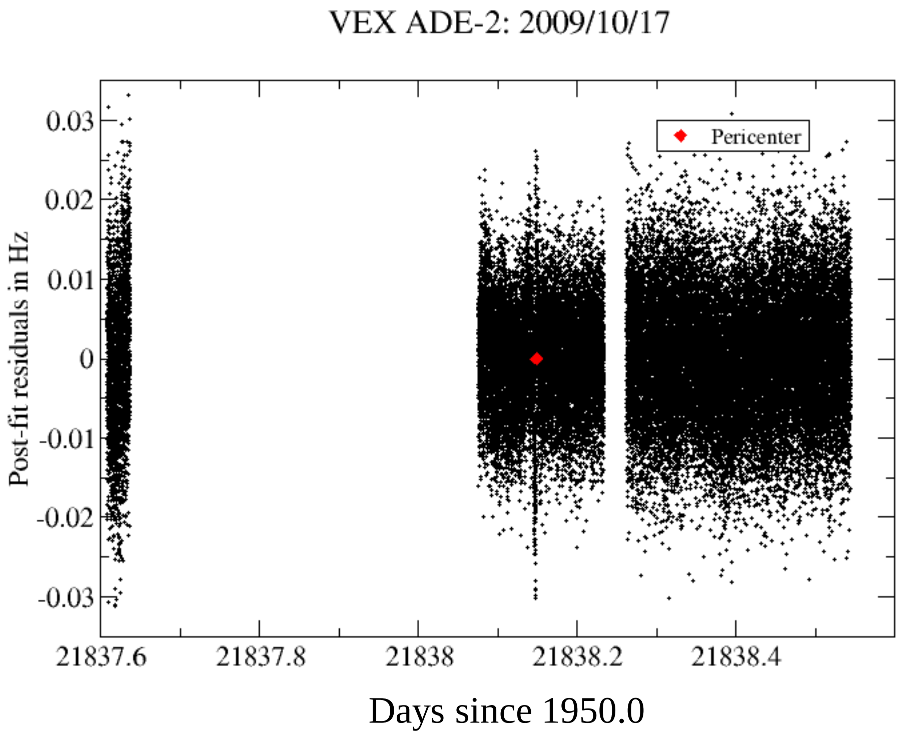
# Example of VExADE good tracking data coverage

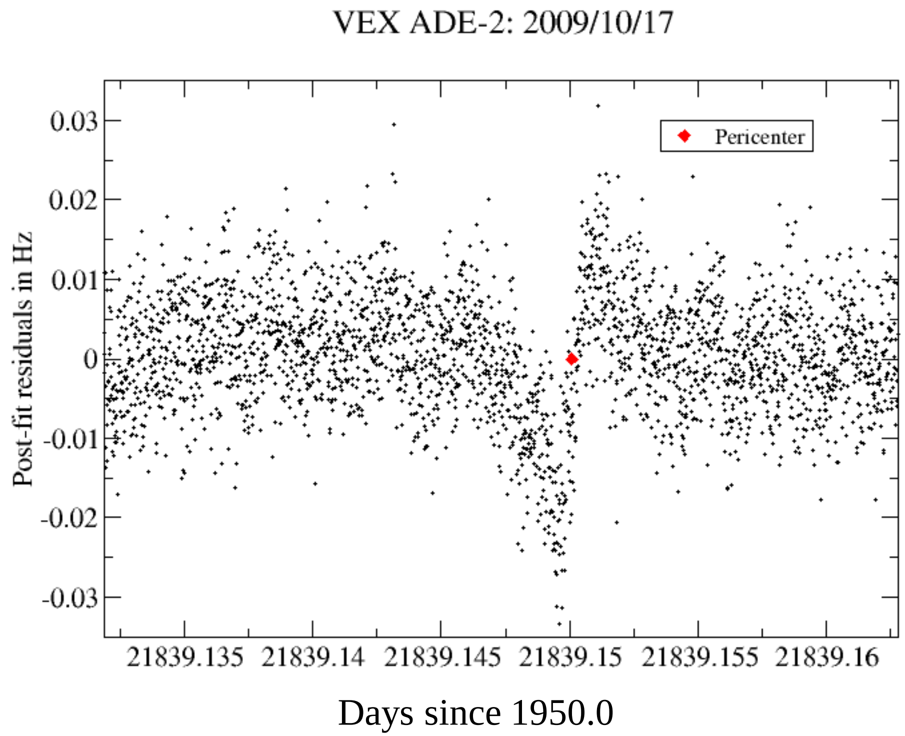



- 4 hours of tracking performed by ESTRACK and DSN around each pericenter pass and 8 hours at high altitude performed by ESTRACK.

- Data-arcs for POD are about 1 day of duration: From one near-apocenter to the next to avoid perturbing WoL maneuvers.

# Example of Doppler post-fit residuals (VExADE campaign #2 – Pericenter altitude 177 km)

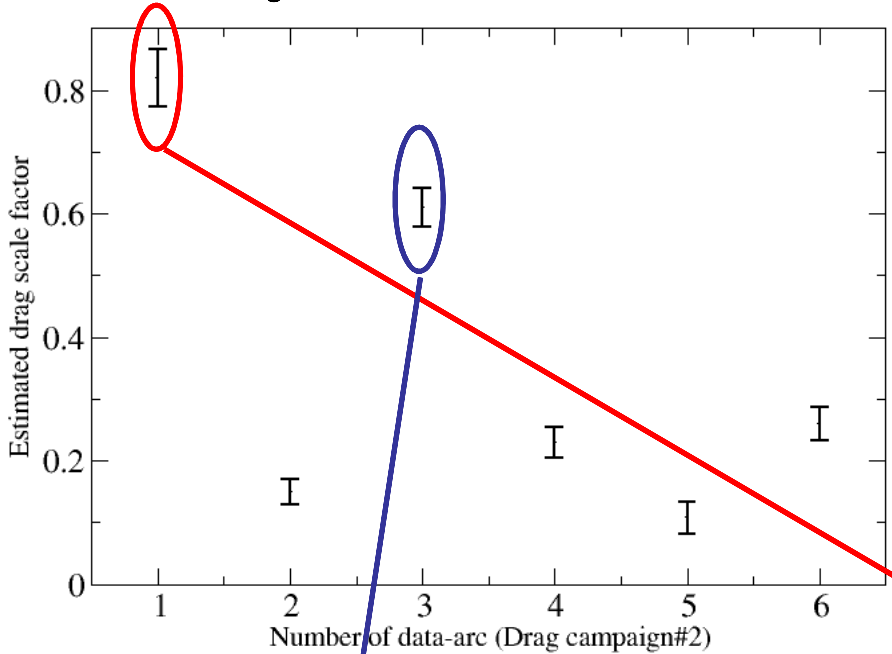


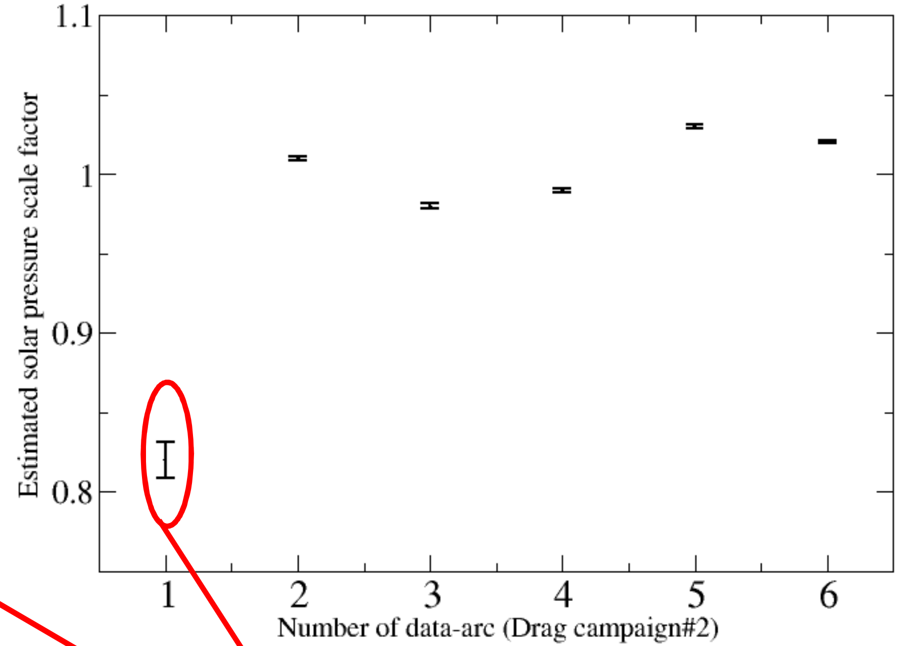

- Good post-fit residuals: *rms* value of ~ 8.3 mHz (or 0.3 mm/s).
- It does not necessarily mean good estimates of drag sale factor.

# Example of POD results: VExADE campaign#2

Drag scale factor estimates




Solar pressure scale factor estimates

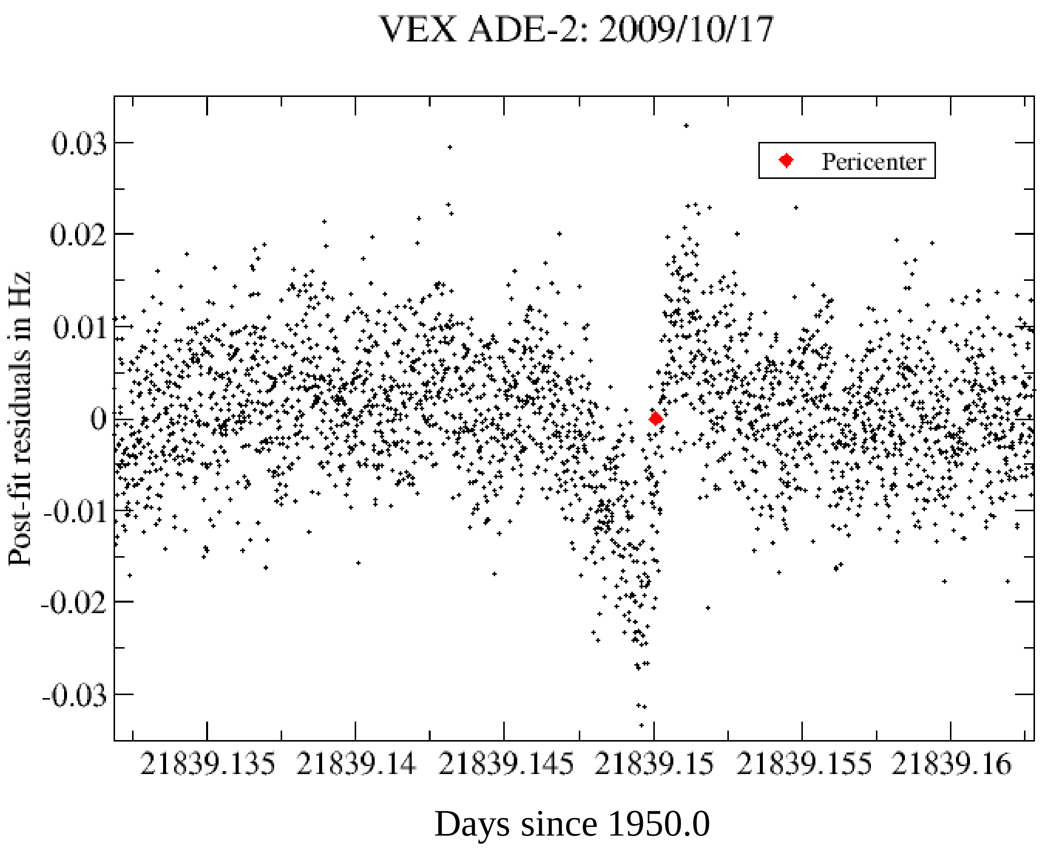



✓ Arc#3: Anomalous drag scale factor value?

✓ Arc#1: Bad solar pressure scale factor value → bad data-arc

- ➢ Other parameters than drag scale factor must have correct values
- ➢ Significant variations drag scale factor can occur (true signal?)

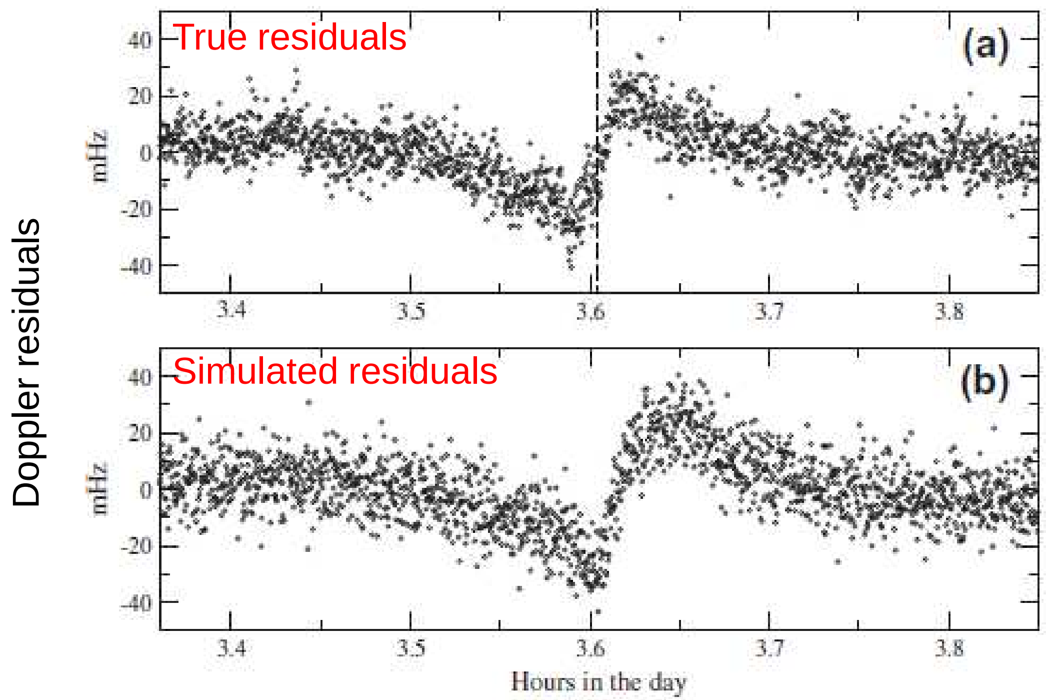
# Example of Doppler post-fit residuals
# (VExADE campaign #2 – Pericenter altitude 177 km)




- Doppler signatures at pericenter pass due to error on other forces: The gravity field ?.

# Realistic error on density estimate from POD

*Rosenblatt et al., Icarus, 217, p. 831, 2012.*



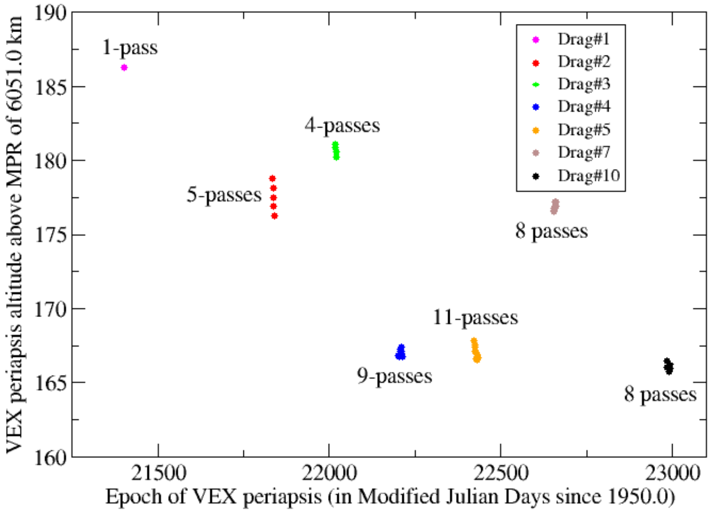

- Observed Doppler signature at pericenter can be reproduced by simulating the effect of long wavelength gravity field error.
- The drag scale factor estimate has a bias at around 6 times its formal error.
- Density estimate error had to be scaled by a factor of 6 (5% → 30%).

# Compiling VExADE

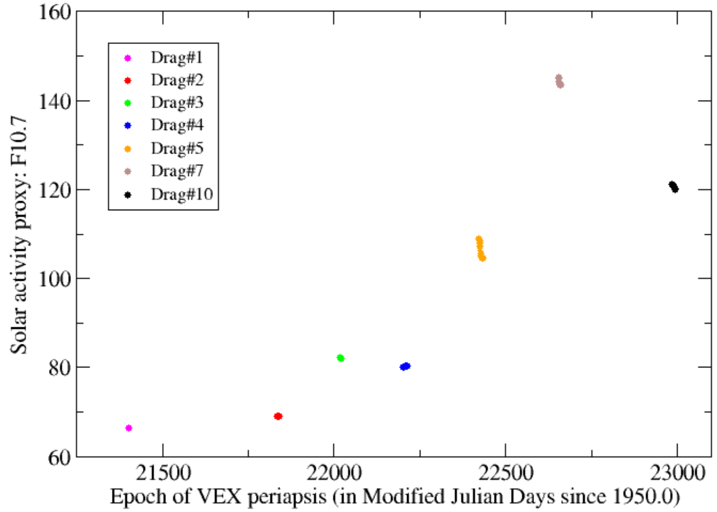
# campaign#1 to #10

# VExADE drag campaign#1 to #10: Dataset

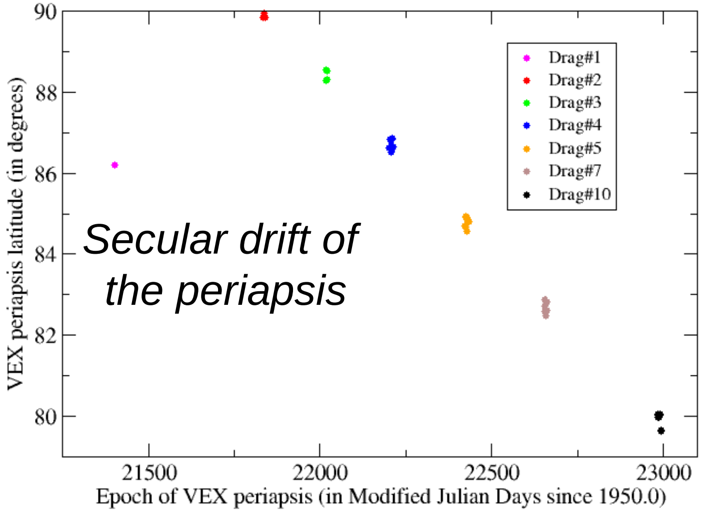
Altitude sampling



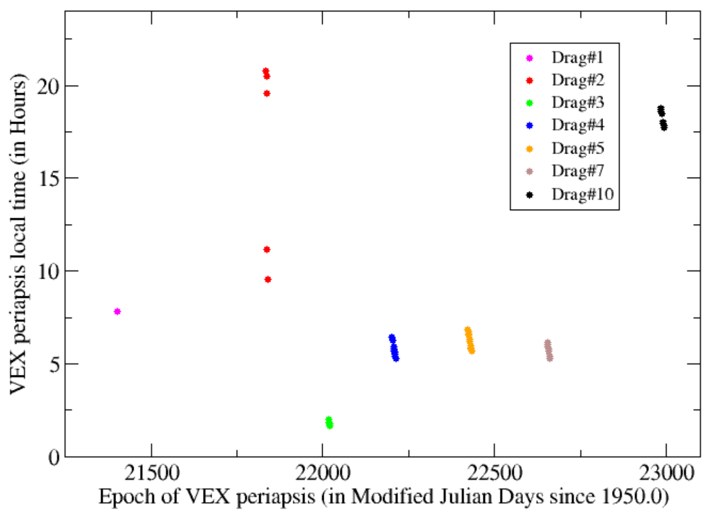

Solar activity sampling




Latitude sampling

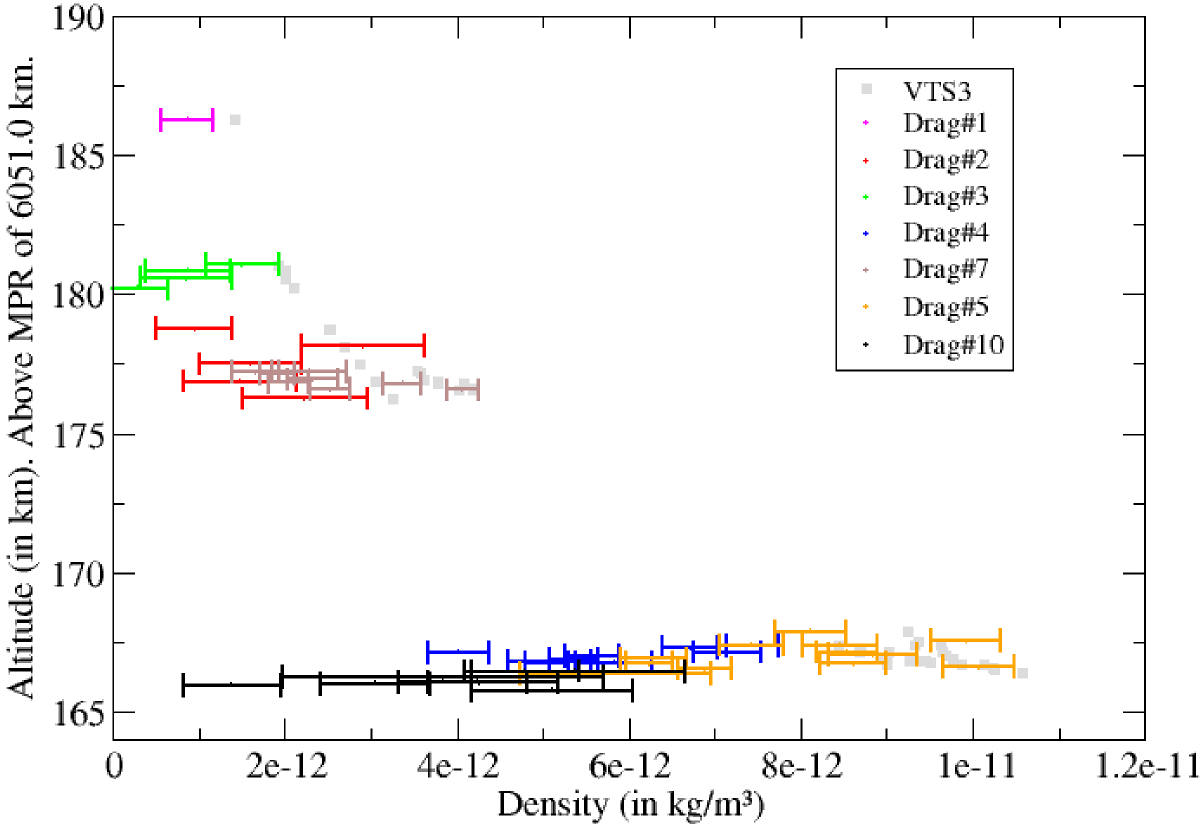



Local time sampling




✓ **46 reliable density estimates**, but limited sampling. Altitude range: ~165-185 km. Latitudes: 80° -90° , local time: 1h, 7h, 18h Solar activity: F10.7 ~66-145.

# VExADE campaigns#1 to #10: Results




$$\rho_{VExADE} = F_D . \rho_{mod}$$

<u>*Where:*</u>

*$\rho_{VExADE}$ is the VExADE density (data-points with error bars as calibrated formal errors)*

*$F_D$ is the drag scale factor*

*$\rho_{mod}$ is the HEDIN-VTS3 density*

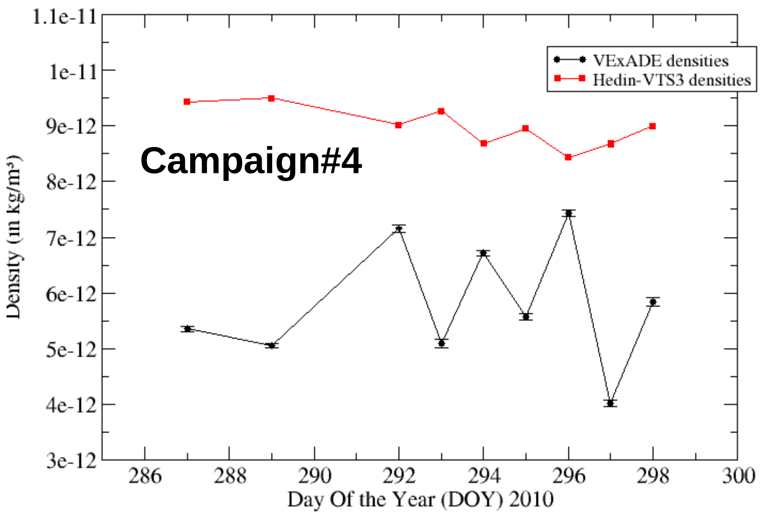
***Rosenblatt et al., Icarus, vol. 217, pp. 831-838, 2012***

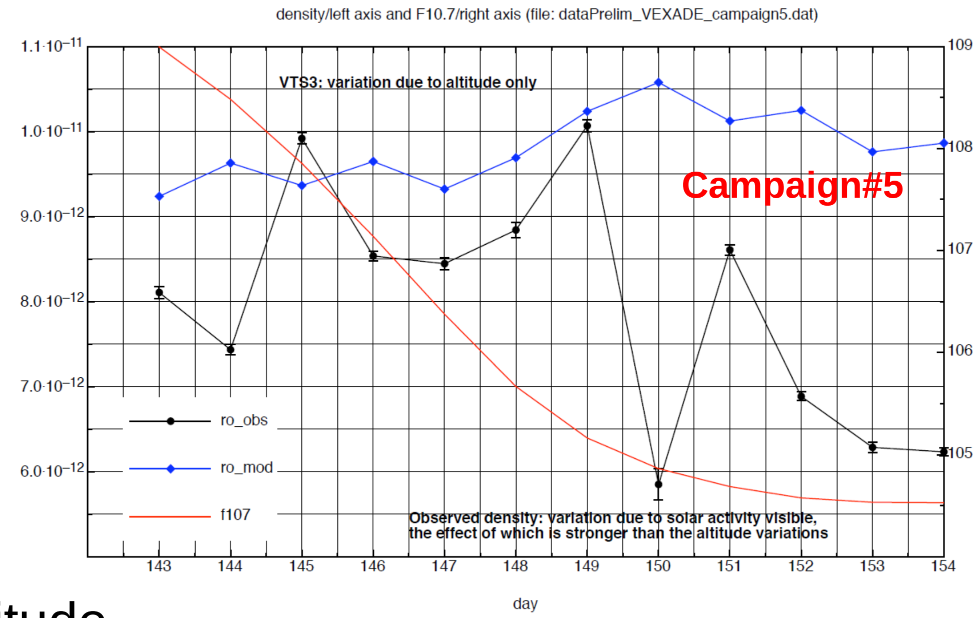
*(method & result for campaign#1 to #3)*

- Venus' upper atmosphere density at polar areas is about half the predicted density by model (Hedin-VTS3).

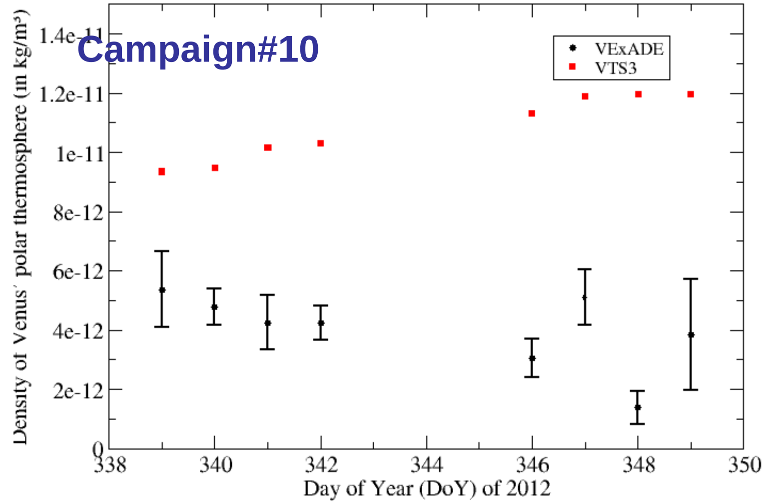
- Checked by independent method using the inertial wheels of the spacecraft.

# Day-to-day variability of observed densities



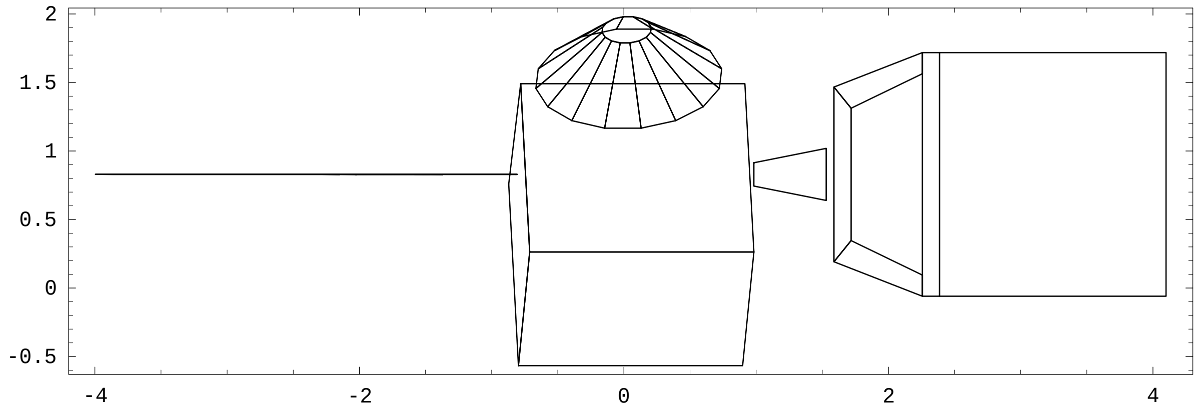




- Day-to-day variability in 1 km altitude range (at low altitude) not accounted for by the VTS3 model.
- Thermospheric temperature variations? Dynamics effects? Others?
- Torque measurements may help to decipher that observation.



# Torque measurements

$$T = \frac{1}{2} c_D \rho A_{eff} V^2 (r_{SA-cop} - r_{SC-com})$$




*From Svedhem et al., 2011*

- ✓ Special attitude mode of spacecraft at VExADE pericenter pass.
- ✓ Torque induced by air pressure measured by rotation rate variati of inertial wheels.
- ✓ Independent measurement of density.

# Profile of density along the pericenter pass

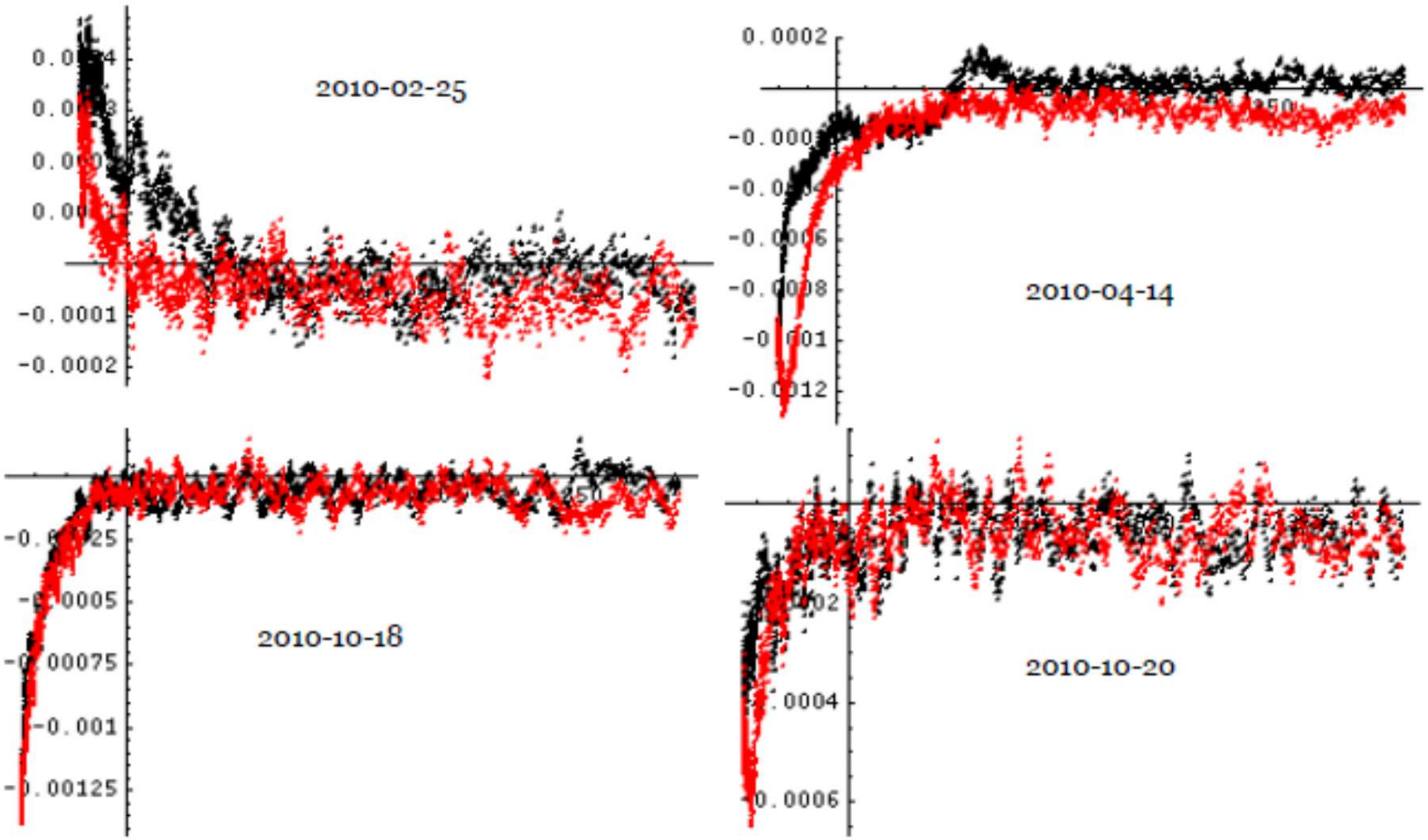
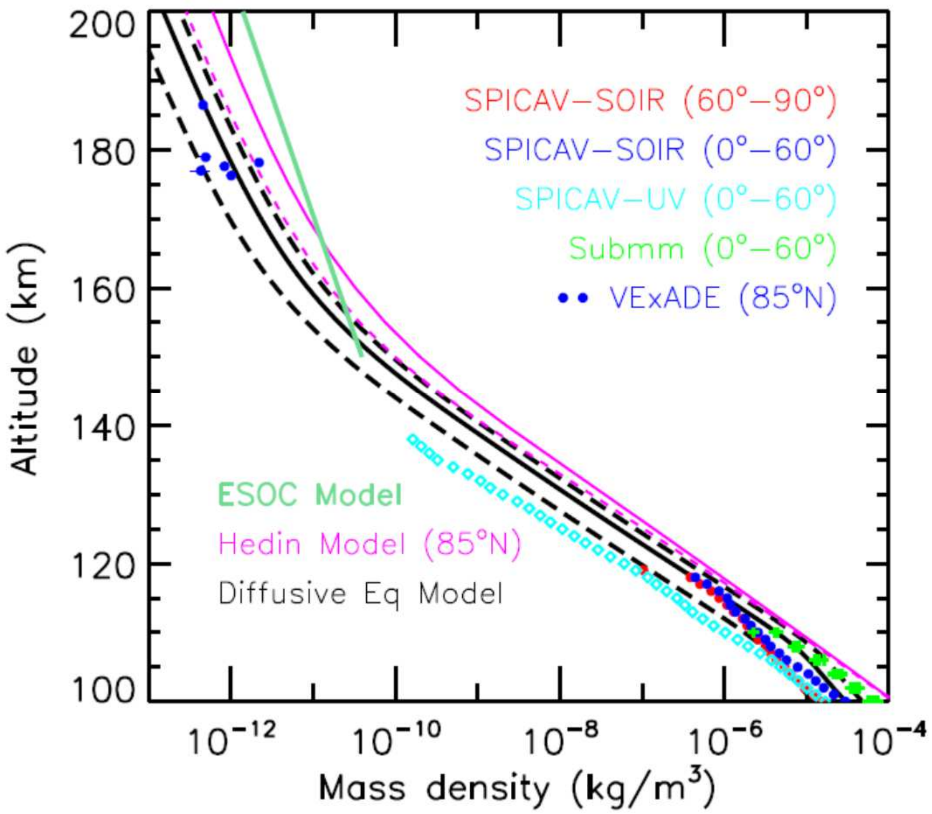



Figure 3: Torque around SC x-axis plot versus altitude. The pre-pericentre and post-pericentre is shown in black and red respectively. On 2010-02-25 the SC was flying from night to day, on 2010-04-14 the SC was flying from day-to-night. For the 2010-10-18 and 2010-10-20 plots the SC is flying almost along the terminator.

# A new model of Venus' polar upper atmosphere density.




- The VExADE estimated densities have contributed for altitudes above 165 km
- Additional remote sensing data from VEX SPICAV-SOIR instrument and from Earth's based JCMT sub-millimeter telescope have contributed at lower altitude range of 100-130 km.

- This new model has been used by the ESOC/ESA teams for the preparation of the July 2014 aero-braking phase of the VEX mission.
- The spacecraft system accelerometer has been used to probe density, still at polar latitudes but at altitudes as low as 140 km (*Nature Physics, 2016*)

# Martian spacecraft tracking data


Fro

MAGE

om NASA

Near-circular


From NASA


From NASA

t at
10400 km

Estimated thermosphere densities over almost 2 solar cycles (1997-2016) using same POD technics

From ESA

# Summary for drag

- ✓ Precise orbit reconstruction allows probing *in-situ* the density of the planetary atmosphere at altitude of pericenter.
- ✓ Accuracy limited by accuracy on gravity field. Limited sampling (VEX)
- ✓ **Application to Mars: Circular orbits (MGS, ODY, update Bruinsma et al., 2014, MRO) and elliptical orbits (MEX, MAVEN)**