Corrections relativistes actuellement appliquées

J.M. Lemoine

Situation actuelle

❖ Corrections sur la dynamique des satellites

➢ Terme de Schwarzschild

$$\vec{a}_{sch} = \frac{\mu}{c^2 r^3}\left[\left[2(\beta+\gamma)\frac{\mu}{r} - \gamma v^2\right]\vec{r} + 2(1+\gamma)(\vec{v}\cdot\vec{r})\vec{v}\right]$$

$\vec{r}$: the position of the satellite in an inertial reference centred on the central body

$\vec{v}$: the speed of the satellite in an inertial reference centred on the central body

β,γ : PPN parameters equaling 1 in the theory of general relativity

- **Terme de Coriolis (ou précession géodésique)**

$$\vec{a}_{body} = 2\vec{\Omega}_{body} \wedge \vec{v}$$

where $$\vec{\Omega}_{body} = \frac{(1+2\gamma)}{2c^2} \sqrt{\frac{\mu_{Sun}^3}{r_{body/Sun}^5}} \vec{u}_{zEcliptic}$$ (standard model)

or $$\vec{\Omega}_{body} = -\frac{(1+2\gamma)}{2} \frac{\mu_{Sun}}{c^2 r_{Sun}^3} \left(\vec{v}_{Sun} \wedge \vec{r}_{Sun}\right)$$ (IERS 2003 standard)

$\vec{r}_{Sun}$: the position of the Sun in an inertial reference centred on the central body

$\vec{v}_{Sun}$: the speed of the Sun in an inertial reference centred on the central body

Situation actuelle

➤ **Terme de Lense-Thirring** (représentant l'enroulement de l'espace dû à la rotation terrestre)

$$\vec{a}_{LT} = (1+\gamma)\frac{\mu}{c^2 r^3}\left[\frac{3}{r^2}\left(\vec{r}\cdot\vec{J}\right)\left(\vec{r}\wedge\vec{v}\right)+\left(\vec{v}\wedge\vec{J}\right)\right] = (1+\gamma)\frac{\mu}{c^2 r^3}\left[\frac{3}{r^2}\left(\vec{r}\cdot\vec{J}\right)\vec{r}-\vec{J}\right]\wedge\vec{v}$$

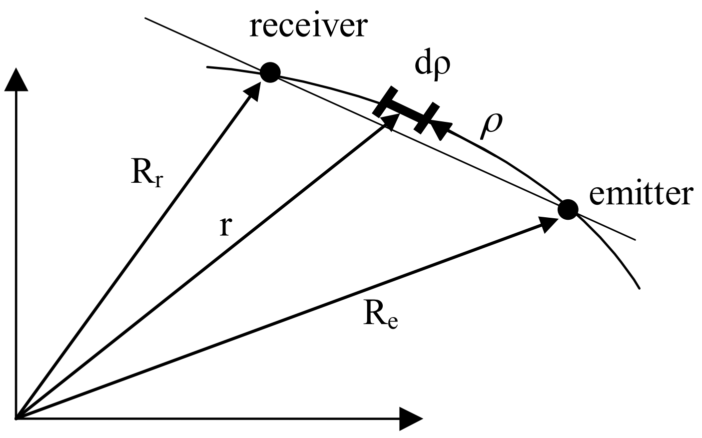
where $\vec{J} = \frac{2}{5}\times R_{Earth}{}^2 \times \Omega_{Earth} \times \vec{u}_{zEarth}$ (standard model)

where $\vec{J} = 9{,}8\times 10^8\,\vec{u}_{zEarth}$ (IERS 2010 standard)

Ω_{Earth} : represents the rotation speed of the central body on itself

$\vec{u}_{zEarth}$: unit vector according to the rotation axis of the central body in the positive direction relative to the rotation.

❖ Corrections sur la propagation



(2) $$\Delta t_{transit} \approx \frac{\rho}{c} + 2\frac{\mu}{c^3}\ln\left(\frac{R_e + R_r + \rho}{R_e + R_r - \rho}\right)$$

Where:

ρ is the curvilinear trajectory of the photon; close at the first order to the geometrical distance between the emitter and the receiver

$\mu = GM$, with G: gravitational constant, M: mass of the Earth

R_e and R_r are the geometrical distance of the emitter (resp. receiver) to the center of the reference frame, coincident with the center of mass of the Earth

Situation actuelle

❖ Corrections sur les horloges

$$d\tau \approx \left[1 - \frac{U}{c^2} - \frac{V^2}{2c^2}\right] dt$$

Where:

U is the gravitational potential at the location of the clock

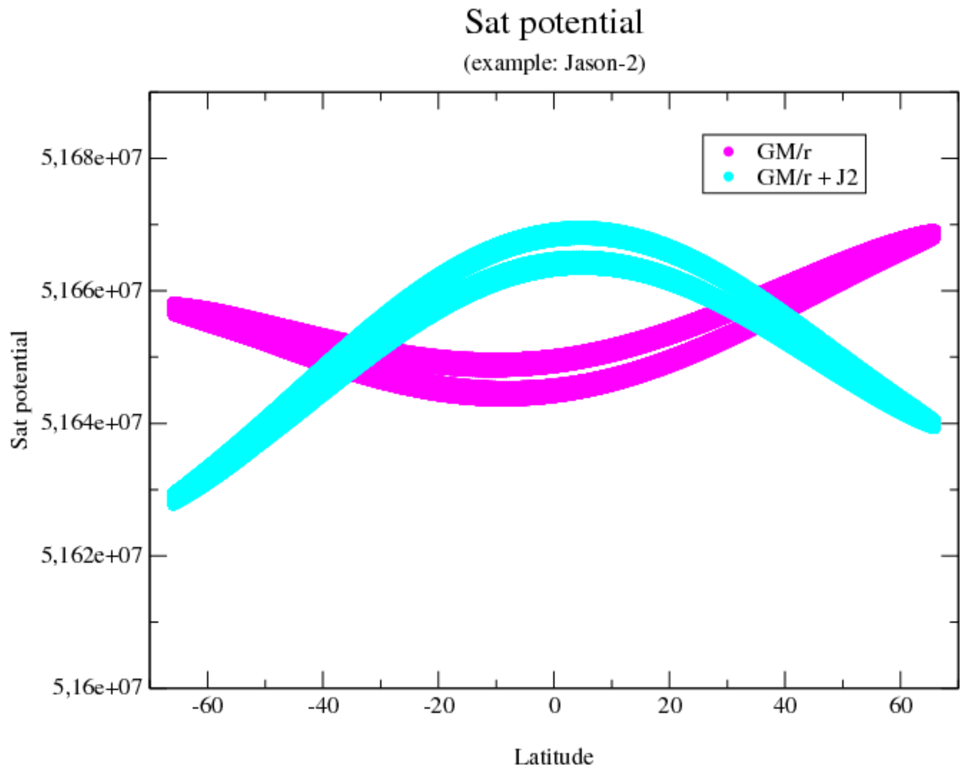
V is the velocity of the clock in the coordinate reference frame

c is the velocity of light in the vacuum

- **ATTENTION** : pour les satellites bas, prendre U=GM/r n'est pas suffisant !!! (cf. planche suivante)

Correction sur les horloges

❖ Jason-2, orbite de 5 jours en 2010



Résidus \ Correction relativiste	U=GM/r	U=GM/r + J2
DORIS (mm/s)	0.3193	0.3192
SLR (cm) (sous-pondéré / DORIS)	2.104	2.088
XOVERS (cm) (sous-pondéré)	4.161	4.159