

## *Fondements des Conventions IERS*

### **Le champ de gravité, *partie statique***

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# Les Conventions IERS

## *Chapitre 6: geopotential*

## 6 Geopotential

Gravitational models commonly used in current (2010) precision orbital analysis represent a significant improvement with respect to geopotential model EGM96, the past conventional model of the IERS Conventions (2003), thanks to the availability of CHAMP <[1]> and, most importantly, GRACE <[2]> data in the 2000s. The IERS, recognizing the recent development of new gravitational models derived from the optimal combination of GRACE data with high resolution gravitational information obtained from surface gravimetry and satellite altimetry data, recommends at this time the EGM2008 model as the conventional model.

The conventional model that is presented in Section 6.1 describes the static part of the field and the underlying background model for the secular variations of some of its coefficients. In addition, other time varying effects should be taken into account: solid Earth tides (Section 6.2), ocean tides (Section 6.3), solid Earth pole tide (Section 6.4), and ocean pole tide (Section 6.5).

The geopotential field  $V$  at the point  $(r, \phi, \lambda)$  is expanded in spherical harmonics up to degree  $N$  as

$$V(r, \phi, \lambda) = \frac{GM}{r} \sum_{n=0}^N \left(\frac{a_e}{r}\right)^n \sum_{m=0}^n [C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)] P_{nm}(\sin \phi) \quad (6.1)$$

(with  $\bar{S}_{n0} = 0$ ), where  $\bar{C}_{nm}$  and  $\bar{S}_{nm}$  are the normalized geopotential coefficients and  $\bar{P}_{nm}$  are the normalized associated Legendre functions. The normalized Legendre function is related to the classical (unnormalized) one by

$$\bar{P}_{nm} = N_{nm} P_{nm}, \quad (6.2a)$$

where

$$N_{nm} = \sqrt{\frac{(n-m)!(2n+1)(2-\delta_{0m})}{(n+m)!}}, \quad \delta_{0m} = \begin{cases} 1 & \text{if } m = 0 \\ 0 & \text{if } m \neq 0 \end{cases} \quad (6.2b)$$

Correspondingly, the normalized geopotential coefficients  $(\bar{C}_{nm}, \bar{S}_{nm})$  are related to the unnormalized coefficients  $(C_{nm}, S_{nm})$  by

$$C_{nm} = N_{nm} \bar{C}_{nm}, \quad S_{nm} = N_{nm} \bar{S}_{nm}. \quad (6.3)$$

The scaling parameters  $(GM, a_e)$  associated with the model are described in Section 6.1. Sections 6.2 to 6.5 provide variations to the normalized coefficients  $(\bar{C}_{nm}, \bar{S}_{nm})$  due to the physical effects described in each section.

## 6.1 Conventional model based on the EGM2008 model

Table 6.1: Suggested truncation levels for use of EGM2008 at different orbits

Orbit radius / km	Example	Truncation level
7331	Starlette	90
12270	Lageos	20
26600	GPS	12

The EGM2008 model (Pavlis *et al.*, 2008) is complete to degree and order 2159, and contains additional spherical harmonic coefficients up to degree 2190 and order 2159. The  $GM_{\oplus}$  and  $a_0$  values reported with EGM2008 ( $398600.4415 \text{ km}^3/\text{s}^2$  and  $6378136.3 \text{ m}$ ) should be used as scaling parameters with its gravitational potential coefficients. They are to be considered as TT-compatible values. The recommended TCG-compatible value,  $GM_{\oplus} = 398600.4418 \text{ km}^3/\text{s}^2$ , should be used with the two-body term when working with Geocentric Coordinate Time (TCG) ( $398600.4415$  or  $398600.4356$  should be used by those still working with Terrestrial Time (TT) or Barycentric Dynamical Time (TDB) respectively). The EGM2008 model (including error estimates) is available at [\[3\]](#).

Although EGM2008 is complete to degree and order 2159, most users in space geodesy will find their needs covered by a truncated version of the model. Suggested truncation levels as a function of the orbit of interest are listed in Table 6.1. It is expected that these truncation levels provide a 3-dimensional orbit accuracy of better than 0.5 mm for the indicated satellites (Ries, 2010).

The EGM2008 model was based on the ITG-GRACE03S GRACE-only gravitational model ([\[4\]](#), see also Mayer-Gurr, 2007) which is available along with its complete error covariance matrix to degree and order 180. Therefore the static gravitational field was developed assuming models complying with the IERS Conventions (2003) and complemented by the following:

- ocean tides: FES2004 (Lyard *et al.*, 2006),
- ocean pole tide: Desai (2003, see Section 6.5),
- atmosphere and ocean de-aliasing: AOD1B RL04 (Flechtner, 2007).

For some of the low-degree coefficients, the conventional geopotential model uses values which are different from the original EGM2008 values. The static field also assumes values for the secular rates of low-degree coefficients. In order to use the static field properly and project it in time, the secular rates should be accounted for. The instantaneous values of coefficients  $\bar{C}_{n0}$  to be used when computing orbits are given by:

$$C_{n0}(t) = C_{n0}(t_0) + dC_{n0}/dt \times (t - t_0) \quad (6.4)$$

where  $t_0$  is the epoch J2000.0 and the values of  $C_{n0}(t_0)$  and  $dC_{n0}/dt$  are given in Table 6.2. Note that the zero-tide  $C_{20}$  coefficient in the conventional geopotential model is obtained from the analysis of 17 years of SLR data approximately centered on year 2000 and has an uncertainty of  $2 \times 10^{-11}$  (Cheng *et al.*, 2010). It differs significantly from the EGM2008 value obtained from 4 years of GRACE data, as it is expected that tide-like aliases will affect GRACE-based  $C_{20}$  values, depending on the averaging interval used. The tide-free value of  $C_{20}$  can be obtained as described in Section 6.2.2.

Table 6.2: Low-degree coefficients of the conventional geopotential model

Coefficient	Value at 2000.0	Reference	Rate / $\text{yr}^{-1}$	Re
$C_{20}$ (zero-tide)	$-0.48416948 \times 10^{-3}$	Cheng <i>et al.</i> , 2010	$11.6 \times 10^{-12}$	Nerem
$C_{30}$	$0.9571612 \times 10^{-6}$	EGM2008	$4.9 \times 10^{-12}$	Cheng
$C_{40}$	$0.5399659 \times 10^{-6}$	EGM2008	$4.7 \times 10^{-12}$	Cheng

<sup>1</sup><http://op.gfz-potsdam.de/champ/>

<sup>2</sup><http://www.csr.utexas.edu/grace/>

<sup>3</sup><http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/>

# Plan

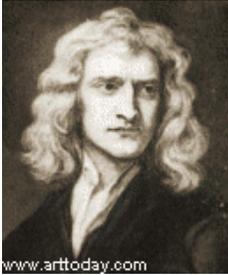
- *La modélisation harmonique*
- *Les modèles globaux de champ de gravité*
- *GM et bas degrés*
- *Modélisations alternatives*
- *Les corrections de gravité*
- *Evolution des Conventions*

# Plan

- *La modélisation harmonique*
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# Le champ de gravité de la Terre

Toute la théorie du champ de gravité repose sur la loi de la gravitation universelle énoncée par I. Newton en 1686 :

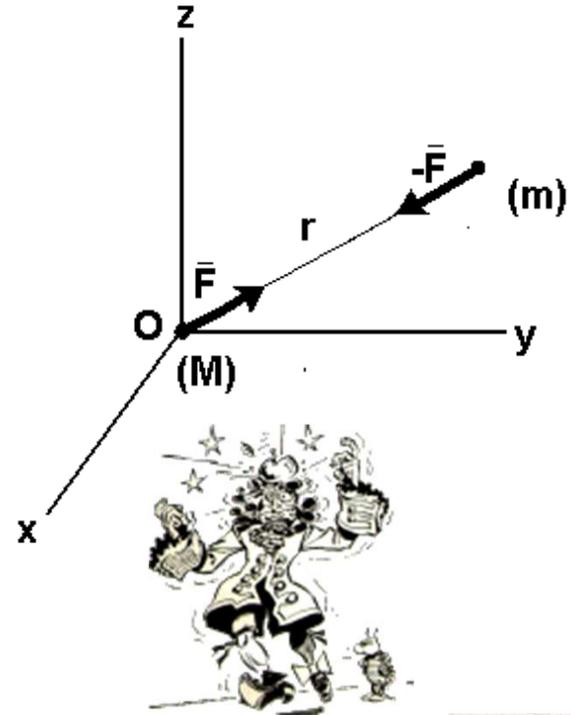


$$F = G \frac{Mm}{r^2}$$

Le point M crée en tout point de l'espace un champ dit newtonien qui dérive du potentiel :

$$U = \frac{GM}{r}$$

tel que :  $\ddot{\vec{r}} = \frac{\partial U}{\partial \vec{x}} = -GM \frac{\vec{r}}{r^3}$



Ce potentiel vérifie l'équation de Laplace (1785):

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$



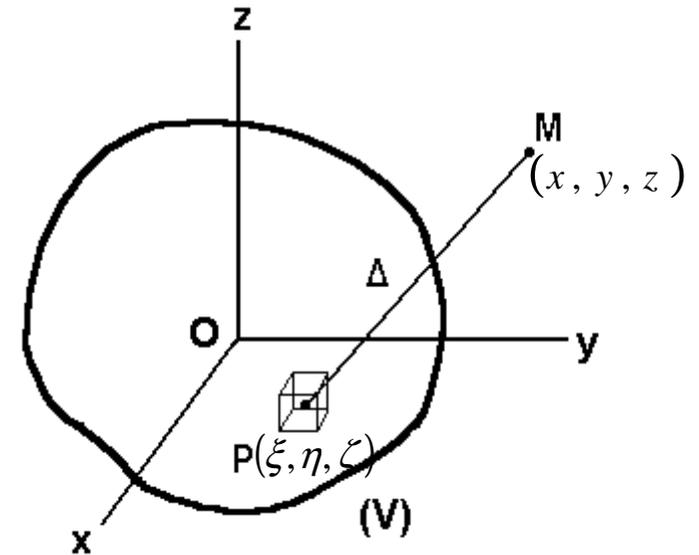
## Cas de la Terre inhomogène

Le potentiel créé en  $M(x,y,z)$  par l'ensemble des points de la Terre  $P(\xi, \eta, \zeta)$  s'exprime :

$$U = \iiint_V \frac{G}{\Delta} dm$$

où  $dm = \delta(\xi, \eta, \zeta) d\xi d\eta d\zeta$

et  $\Delta = PM = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}$



Il satisfait l'équation de Laplace en tout point extérieur au corps :

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$$



A l'intérieur du corps, les dérivées secondes sont liées par l'équation de Poisson :

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = -4\pi G\rho \quad (\rho : \text{densité au point considéré})$$

# Modélisation du potentiel gravitationnel

Soit :  $V = \frac{r}{\Delta}$  et  $\rho = \frac{r'}{r}$

on a dans le triangle MOP:  $\Delta^2 = r^2 - 2rr' \cos \theta + r'^2$

et donc :  $V = \frac{1}{(1 - 2\rho \cos \theta + \rho^2)^{1/2}}$

V est développable en série entière de  $\rho$ , convergente pour  $\rho < 1$ .

$$V = P_0 + \rho P_1 + \rho^2 P_2 + \dots + \rho^n P_n + \dots$$

ou les  $P_n$  sont les **polynômes de Legendre** de degré  $n$  en  $\cos \theta$ :

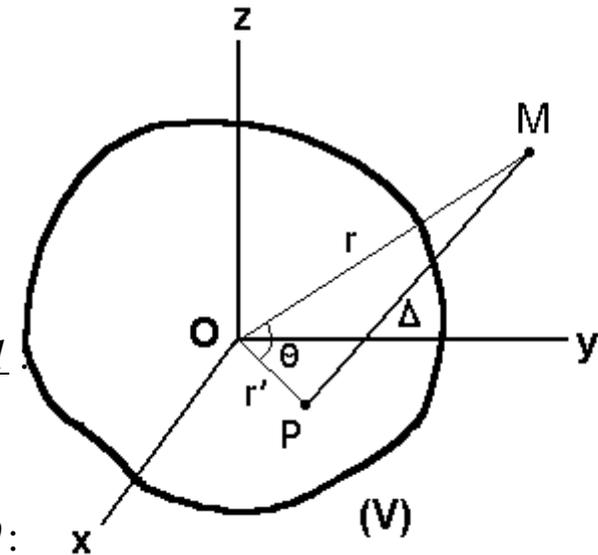
$$P_n = \sum_{k=0}^{n/2} (-1)^k \frac{(2n-2k)!}{2^n k!(n-k)!(n-2k)!} \cos^{n-2k} \theta$$

V est une fonction harmonique à l'extérieur du domaine (V), c'est à dire :

- elle a ses dérivées secondes continues,
- son Laplacien  $\Delta V$  est nul.

$P_n$  est un polynôme harmonique de degré  $n$  qui vérifie :

- l'équation différentielle :  $P_n(x) = \frac{1}{2^n n!} \frac{d^n (x^2 - 1)^n}{dx^n}$
- la propriété d'orthogonalité :  $\int_{-1}^{+1} P_n(x) P_m(x) dx = \frac{2\delta_{n,m}}{2n+1}$



**Expressions littérales jusqu'au degré 6 :**

$$P_0(x) = 1$$

$$P_1(x) = x = \cos \theta$$

$$P_2(x) = (3x^2 - 1)/2 = (3 \cos 2\theta + 1)/4$$

$$P_3(x) = (5x^3 - 3x)/2 = (5 \cos 3\theta + 3 \cos \theta)/8$$

$$P_4(x) = (35x^4 - 30x^2 + 3)/8 = (35 \cos 4\theta + 20 \cos 2\theta + 9)/64$$

$$P_5(x) = (63x^5 - 70x^3 + 15x)/8 = (63 \cos 5\theta + 35 \cos 3\theta + 30 \cos \theta)/128$$

$$P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16 = (231 \cos 6\theta + 126 \cos 4\theta + 105 \cos 2\theta + 50)/512$$

**Polynômes bornés entre [-1,+1] :**

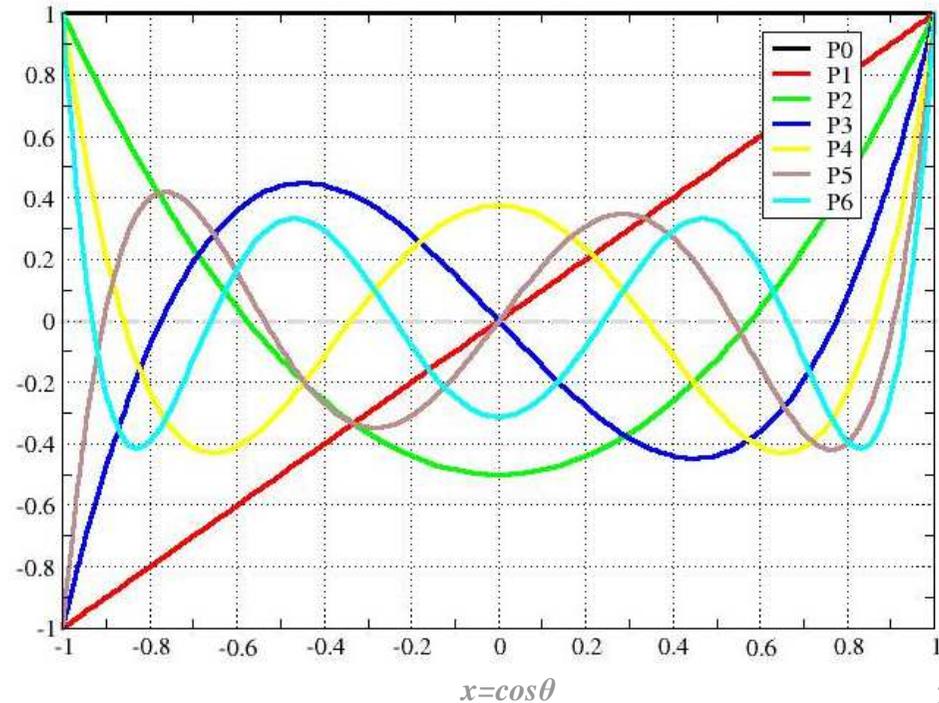
$$P_n(0) = 0 \quad \text{pour } n \text{ impair}$$

$$P_n(0) = (-1)^{n/2} \frac{1.3.5 \dots (n-1)}{2.4 \dots n}$$

$$= (-1)^{n/2} \frac{n!}{2^n (n/2)!^2} \quad \text{pour } n \text{ pair}$$

$$P_n(1) = 1$$

$$P_n(-1) = (-1)^n$$



## Les polynômes de Legendre (1783)

<b>Equation différentielle de Legendre :</b>	$(1-x^2)\frac{d^2 P_n}{dx^2} - 2x\frac{dP_n}{dx} + n(n+1)P_n = 0$
<b>Formules de récurrence (par degré) :</b>	$nP_n - (2n-1)xP_{n-1} + (n-1)P_{n-2} = 0$
	$(n+1)(P_{n+1} - xP_n) - n(xP_n - P_{n-1}) = 0$
<b>Orthogonalité :</b>	$\int_{-1}^{+1} P_n(x)P_m(x)dx = \frac{2\delta_{n,m}}{2n+1}$
<b>Formule de Rodrigues :</b>	$P_n(x) = \frac{1}{2^n n!} \frac{d^n (x^2-1)^n}{dx^n}$
<b>Dérivation :</b>	$(x^2-1)\frac{dP_n}{dx} = n(xP_n - P_{n-1})$
	$(2n+1)(x^2-1)\frac{dP_n}{dx} = n(n+1)(P_{n+1} - P_{n-1})$
<b>Fonction/série génératrice :</b>	$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)t^n$

## Passage en coordonnées sphériques

Par trigonométrie sphérique entre  $P(r', \varphi', \lambda')$  et  $M(r, \varphi, \lambda)$ , on établit la relation :

$$\cos \theta = \sin \varphi \sin \varphi' + \cos \varphi \cos \varphi' \cos(\lambda - \lambda')$$

Chaque polynôme harmonique  $P_n(\cos \theta)$  vérifie l'équation de Laplace qui, exprimée en coordonnées polaires, vaut:

$$r^2 \Delta P_n = \frac{\partial}{\partial r} \left( r^2 \frac{\partial P_n}{\partial r} \right) + \frac{1}{\cos \varphi} \frac{\partial}{\partial \varphi} \left( \cos \varphi \frac{\partial P_n}{\partial \varphi} \right) + \frac{1}{\cos^2 \varphi} \frac{\partial^2 P_n}{\partial \lambda^2} = 0$$

dont une solution de type  $f(r)g(\varphi)h(\lambda)$  fait intervenir les polynômes de Legendre:

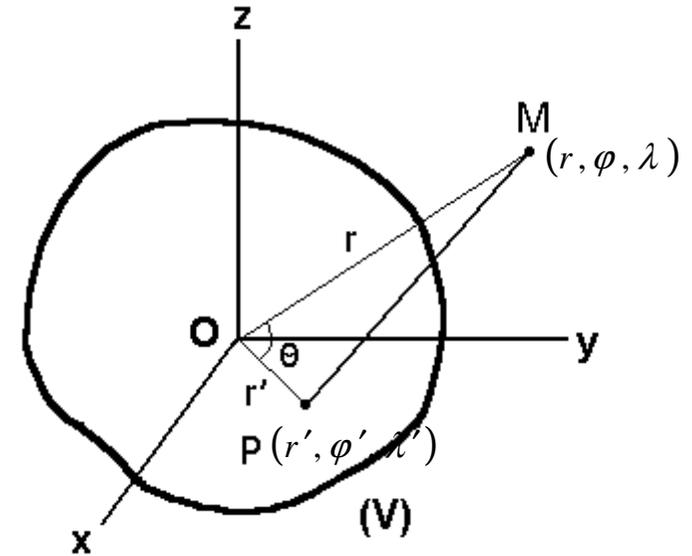
$$P_n(\cos \theta) = P_n(\sin \varphi)P_n(\sin \varphi') + 2 \sum_{m=1}^n \frac{(n-m)!}{(n+m)!} P_{n,m}(\sin \varphi)P_{n,m}(\sin \varphi') \cos(m(\lambda - \lambda'))$$

$P_{n,m} \cos m\lambda$  et  $P_{n,m} \sin m\lambda$  sont des fonctions harmoniques sphériques de degré  $n$  et d'ordre  $m$ .

Les fonctions de Legendre  $P_{n,m}$  vérifient :

- l'équation différentielle : 
$$P_{n,m}(x) = \frac{1}{2^n n!} (1-x^2)^{m/2} \frac{d^{n+m}(x^2-1)^n}{dx^{n+m}}$$

- la propriété d'orthogonalité : 
$$\int_{-1}^{+1} P_{n,m}(x) P_{k,m}(x) dx = \frac{2 \delta_{n,k}}{2n+1} \frac{(n+m)!}{(n-m)!}$$



## Expressions littérales des premiers degrés et ordres (jusqu'aux degré/ordre 4)

$$P_{1,1}(x) = (1 - x^2)^{1/2}$$

$$P_{2,1}(x) = 3x(1 - x^2)^{1/2}$$

$$P_{2,2}(x) = 3(1 - x^2)$$

$$P_{3,1}(x) = \frac{15}{2}(1 - x^2)^{1/2} \left( x^2 - \frac{1}{5} \right)$$

$$P_{3,2}(x) = 15x(1 - x^2)$$

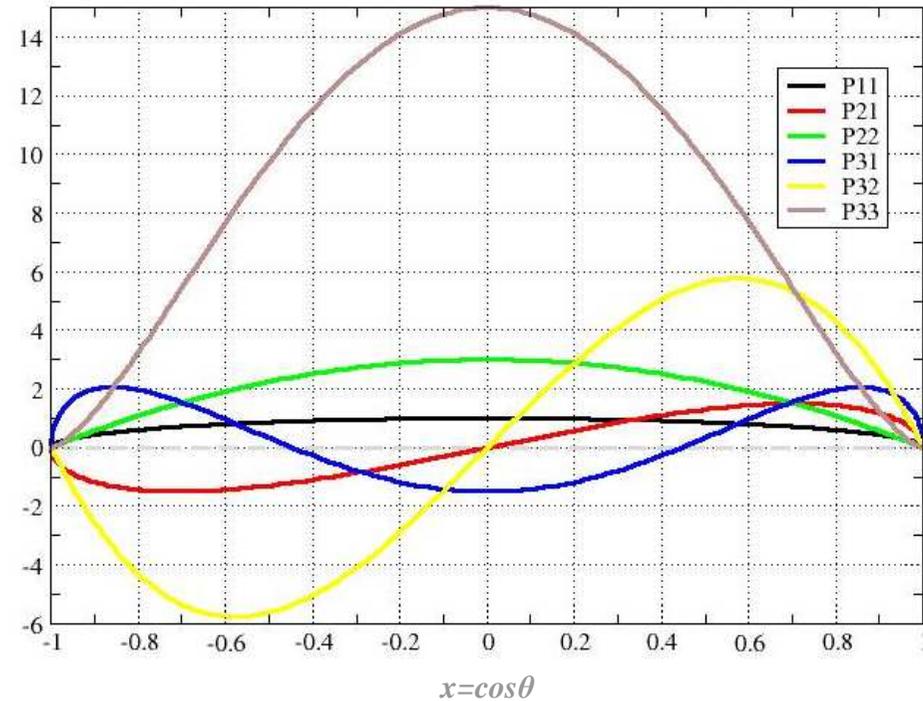
$$P_{3,3}(x) = 15(1 - x^2)^{3/2}$$

$$P_{4,1}(x) = \frac{35}{2}(1 - x^2)^{1/2} \left( x^3 - \frac{3}{7}x \right)$$

$$P_{4,2}(x) = \frac{105}{2}(1 - x^2) \left( x^2 - \frac{1}{7} \right)$$

$$P_{4,3}(x) = 105x(1 - x^2)^{3/2}$$

$$P_{4,4}(x) = 105(1 - x^2)^2$$



**Fonctions non bornées convergentes vers 0 à -1 et +1**

**Valeurs aux bornes et au centre:**

$$P_{n,m}(1) = 0$$

$$P_{n,m}(-1) = 0$$

$$P_{n,m}(0) = 0 \quad \text{pour } n - m \text{ impair}$$

$$P_{n,m}(0) = (-1)^{(n-m)/2} \frac{(n+m)!}{2^n \left( \frac{(n+m)}{2}! \right) \left( \frac{(n-m)}{2}! \right)} \quad \text{pour } n - m \text{ pair}$$

## Les fonctions associées de Legendre (de premier type)

<b>Equation différentielle :</b>	$(1-x^2)\frac{d^2 P_{n,m}}{dx^2} - 2(m+1)x\frac{dP_{n,m}}{dx} + (n-m)(n+m+1)P_{n,m} = 0$
<b>Formules de récurrence (par degré) :</b>	$(n-m+2)P_{n+2,m} - (2n+3)xP_{n+1,m} + (n+m+1)P_{n,m} = 0$
	$(n-m+2)P'_{n+2,m} - (2n+3)(xP'_{n+1,m} + P_{n+1,m}) + (n+m+1)P'_{n,m} = 0$
<b>Formule de récurrence (par ordre) :</b>	$P_{n,m+2} - (2m+1)\frac{x}{(1-x^2)^{1/2}}P_{n,m+1} + (n-m)(n+m+1)P_{n,m} = 0$
<b>Formule différentielle :</b>	$P_{n,m}(x) = (-1)^m \frac{(1-x^2)^{m/2}}{2^n n!} \frac{d^{n+m}(x^2-1)^n}{dx^{n+m}}$

Relation polynôme / fonction associée de Legendre: 
$$P_{n,m}(x) = \underbrace{(1-x^2)^{m/2}}_{\sin^2\theta} \underbrace{\frac{d^m}{dx^m} [P_n(x)]}_{\text{Polynôme d'Helmoltz}}$$

avec: 
$$P_n(x) = \frac{1}{2^n n!} \frac{d^n (x^2-1)^n}{dx^n}$$

# Développement du potentiel terrestre en harmoniques sphériques

$$U = \frac{G}{r} \iiint_V V \, dm$$

$V$  étant développable en harmoniques sphériques, on a :

$$V = \sum_{n=0}^{\infty} \rho^n P_n \quad \text{avec : } \rho = \frac{r'}{r} \quad \text{et} \quad P_n(\cos \theta) = \sum_{m=0}^n (2 - \delta_{m0}) \frac{(n-m)!}{(n+m)!} P_{n,m}(\sin \varphi) P_{n,m}(\sin \varphi') \cos(m(\lambda - \lambda'))$$

d'où :

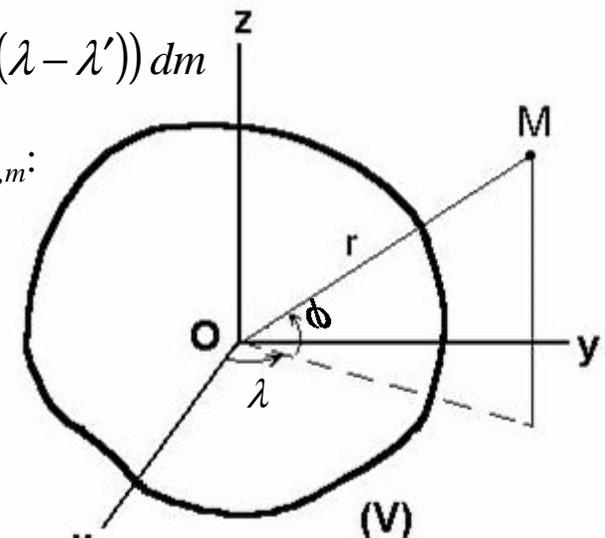
$$U = \frac{G}{r} \iiint_V \sum_{n=0}^{\infty} \sum_{m=0}^n (2 - \delta_{m0}) \frac{(n-m)!}{(n+m)!} \left(\frac{r'}{r}\right)^n P_{n,m}(\sin \varphi) P_{n,m}(\sin \varphi') \cos(m(\lambda - \lambda')) \, dm$$

Soit en isolant la partie intégrale sous la forme de coefficients  $C_{n,m}$  et  $S_{n,m}$ :

$$U = \frac{GM}{a_e} \sum_{n=0}^{\infty} \left(\frac{a_e}{r}\right)^{n+1} \sum_{m=0}^n P_{n,m}(\sin \varphi) (C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda)$$

où :

$$Ma_e^n \begin{Bmatrix} C_{n,m} \\ S_{n,m} \end{Bmatrix} = \iiint_V (2 - \delta_{m0}) \frac{(n-m)!}{(n+m)!} r'^n P_{n,m}(\sin \varphi') \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} dm$$



$$GM = 398600.4415 \text{ km}^3/\text{s}^2$$

$$a_e = 6378136.46 \text{ m}$$

Les termes  $C_{n,0}$  ( $= -J_n$ ) sont les coefficients de Stokes des harmoniques zonales ;  
 les termes  $C_{n,m}$ ,  $S_{n,m}$  sont les coefficients de Stokes des harmoniques tessérales ;  
 les termes  $C_{n,n}$ ,  $S_{n,n}$  sont les coefficients de Stokes des harmoniques sectorielles.

# Normalité et orthogonalité

Harmonique de surface de Laplace :

$$H_{lm}(\varphi, \lambda) = P_{lm}(\sin \varphi) e^{im\lambda}$$

Orthogonalité sur la sphère de rayon unité :

$$\int_S H_{lm}(\sin \varphi) H_{l'm'}(\sin \varphi) \cos \varphi d\lambda d\varphi = 0 \quad \text{si } l \neq l' \text{ ou } m \neq m'$$

Normalité sur la sphère de rayon unité :

$$\int_S [H_{lm}(\sin \varphi)]^2 \cos \varphi d\lambda d\varphi = \frac{4\pi}{\Pi^2}$$

Facteur de normalisation  $\Pi$  tel que :  $\Pi^2 = (2 - \delta_{0m})(2l+1) \frac{(l-m)!}{(l+m)!}$

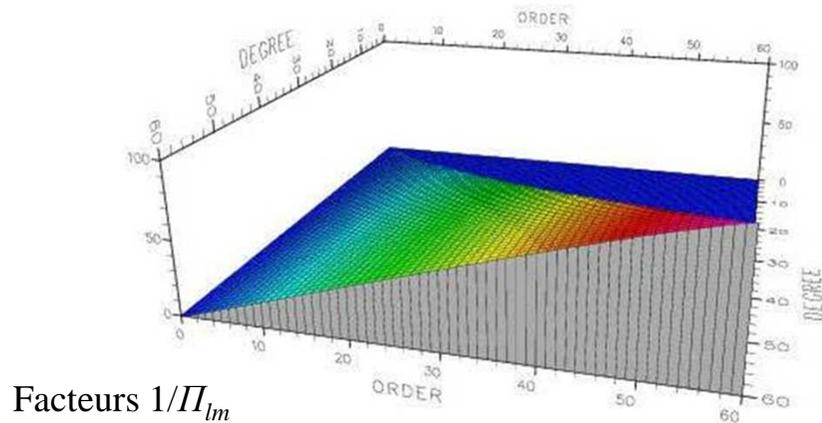
Harmonique de Laplace normalisé :  $\bar{H}_{lm} = \Pi_{lm} H_{lm}$

Coefficient de Stokes normalisé  $\cdot \bar{C}_{lm} = \frac{C_{lm}}{\Pi_{lm}}$

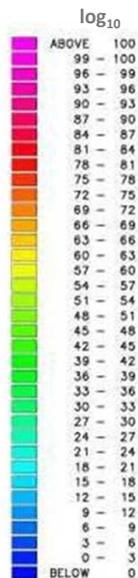
Soit :  $\int_S \bar{P}_{lm}(\sin \varphi) \bar{P}_{l'm'}(\sin \varphi) e^{im\lambda} e^{im'\lambda} ds = 4\pi \delta_{ll'} \delta_{mm'}$

	Ordre m						
Degré l	0	1	2	3	4	5	6
0	1						
1	$\sqrt{3}$	$\sqrt{3}$					
2	$\sqrt{5}$	$\sqrt{\frac{5}{3}}$	$\sqrt{\frac{5}{12}}$				
3	$\sqrt{7}$	$\sqrt{\frac{7}{6}}$	$\sqrt{\frac{7}{80}}$	$\sqrt{\frac{7}{360}}$			
4	$\sqrt{9}$	$\sqrt{\frac{9}{10}}$	$\sqrt{\frac{9}{180}}$	$\sqrt{\frac{9}{2520}}$	$\sqrt{\frac{9}{20160}}$		
5	$\sqrt{11}$	$\sqrt{\frac{11}{15}}$	$\sqrt{\frac{11}{420}}$	$\sqrt{\frac{11}{10080}}$	$\sqrt{\frac{11}{181440}}$	$\sqrt{\frac{11}{1814400}}$	
6	$\sqrt{13}$	$\sqrt{\frac{13}{15}}$	$\sqrt{\frac{13}{840}}$	$\sqrt{\frac{13}{30240}}$	$\sqrt{\frac{13}{907200}}$	$\sqrt{\frac{13}{19958400}}$	$\sqrt{\frac{13}{239500800}}$

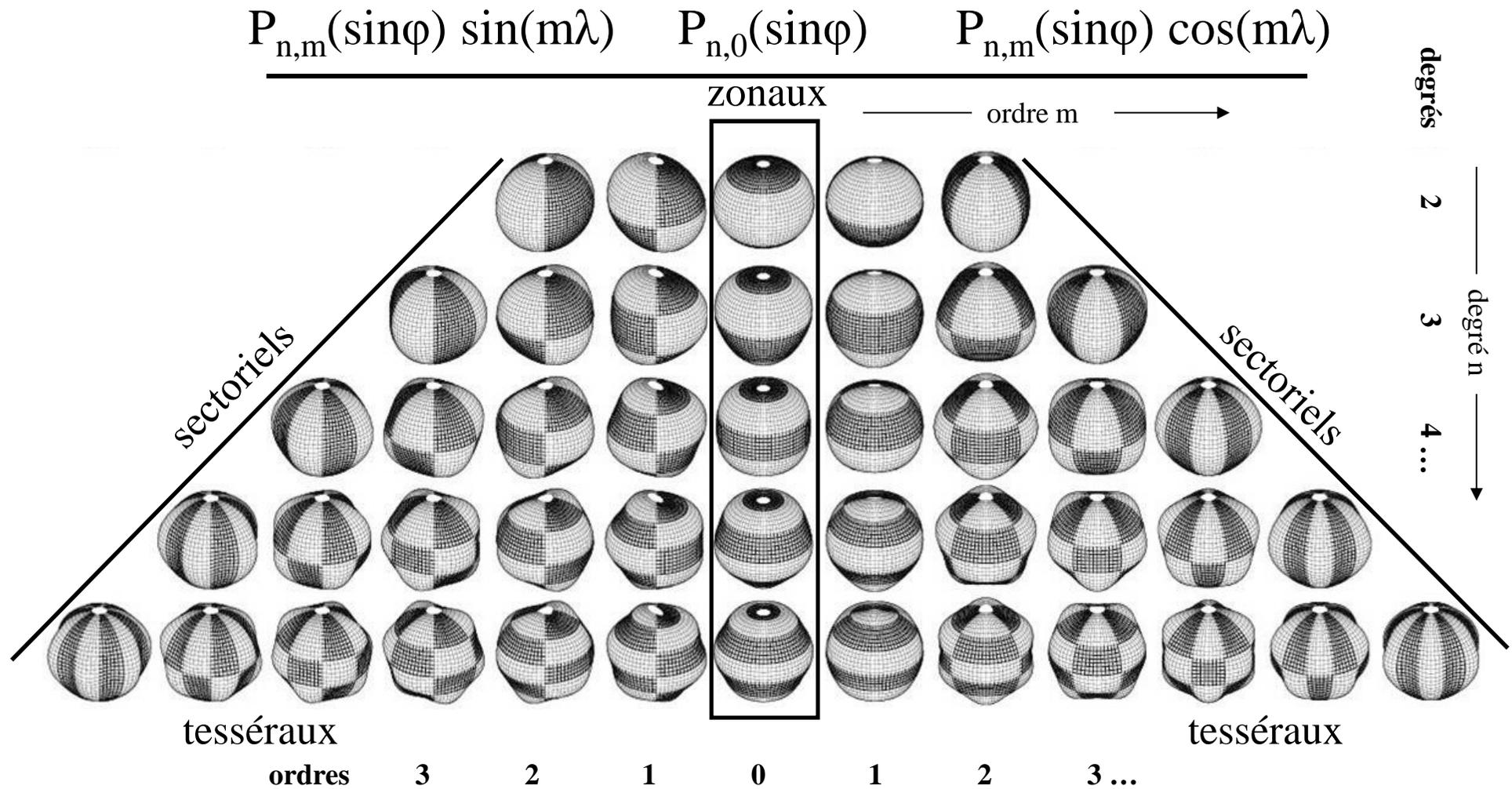
Facteurs  $\Pi_{lm}$



Facteurs  $1/\Pi_{lm}$



# Les fonctions harmoniques sphériques

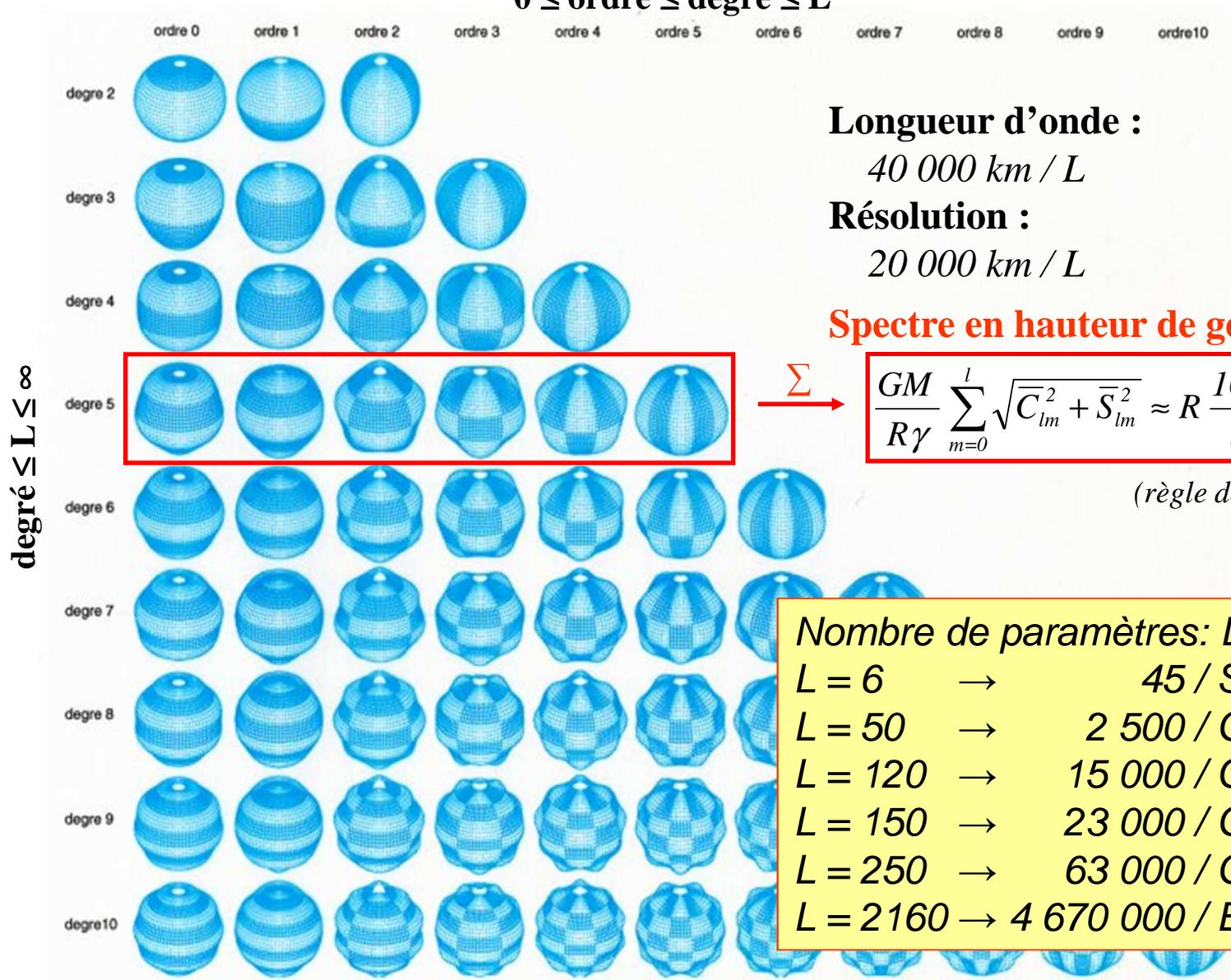


$$U = \frac{GM}{a_e} \sum_{n=0}^{\infty} \left(\frac{a_e}{r}\right)^{n+1} \sum_{m=0}^n P_{n,m}(\sin\varphi) (C_{n,m} \cos m\lambda + S_{n,m} \sin m\lambda)$$

# Harmoniques sphériques de Laplace : $P_{lm}(\sin \varphi) \cos m\lambda$ , $P_{lm}(\sin \varphi) \sin m\lambda$

( $\varphi$  : latitude,  $\lambda$  : longitude)

$$0 \leq \text{ordre} \leq \text{degré} \leq L$$



**Longueur d'onde :**

$$40\,000 \text{ km} / L$$

**Résolution :**

$$20\,000 \text{ km} / L$$

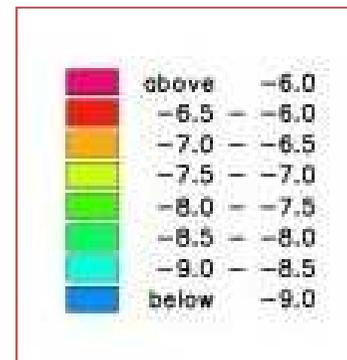
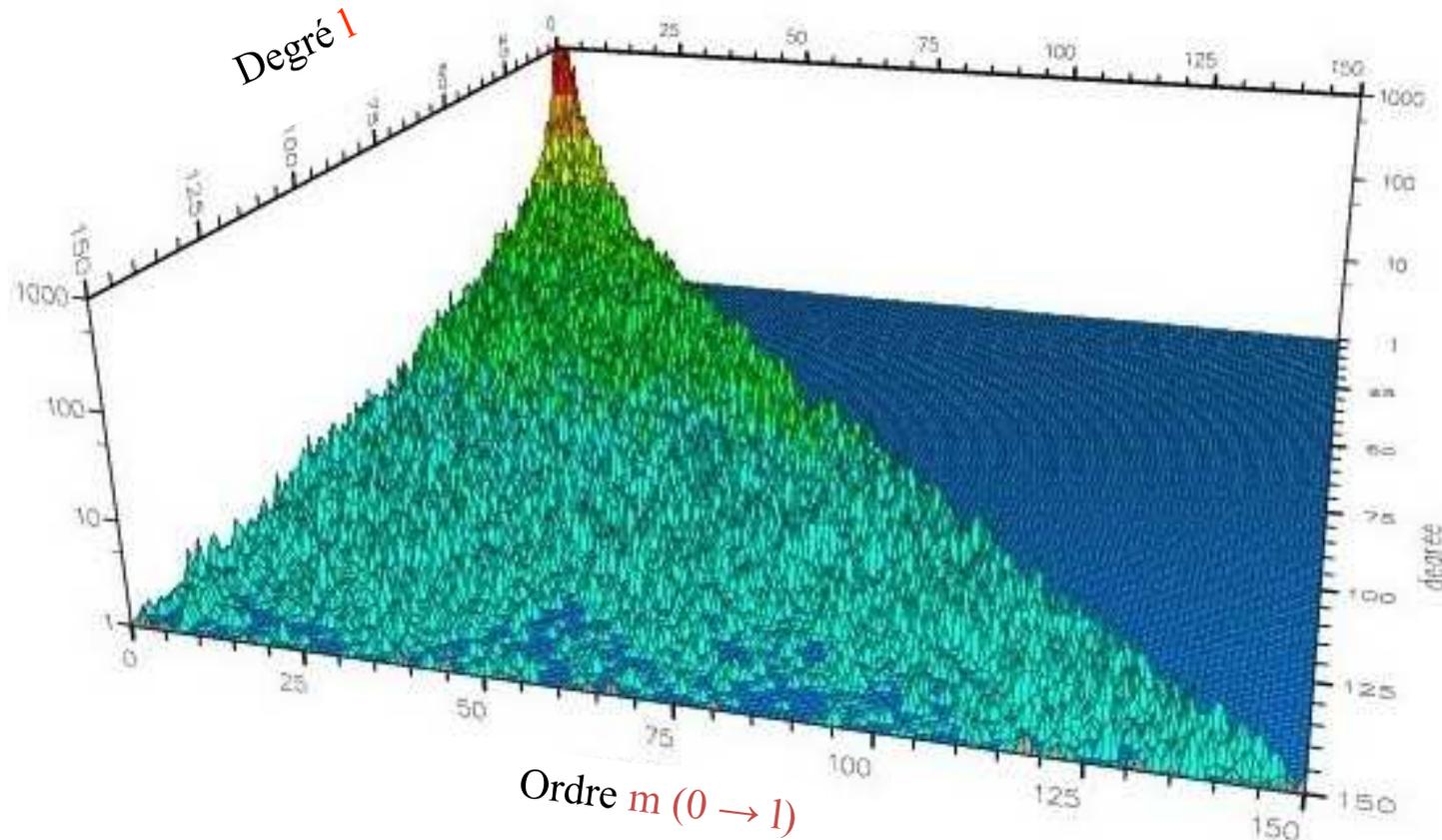
**Spectre en hauteur de géoïde (/ellipsoïde) :**

$$\Sigma \rightarrow \frac{GM}{R\gamma} \sum_{m=0}^l \sqrt{C_{lm}^2 + S_{lm}^2} \approx R \frac{10^{-5}}{l^2} \sqrt{2l+1}$$

(règle de Kaula)

Nombre de paramètres: $L(L+2) \approx L^2$	
$L = 6$	→ 45 / SE-I (1965)
$L = 50$	→ 2 500 / GRIM4 (1991)
$L = 120$	→ 15 000 / CHAMP (2000)
$L = 150$	→ 23 000 / GRACE (2002)
$L = 250$	→ 63 000 / GOCE (2009)
$L = 2160$	→ 4 670 000 / EGM2008

# Amplitudes des coefficients de Stokes normalisés (potentiel terrestre)

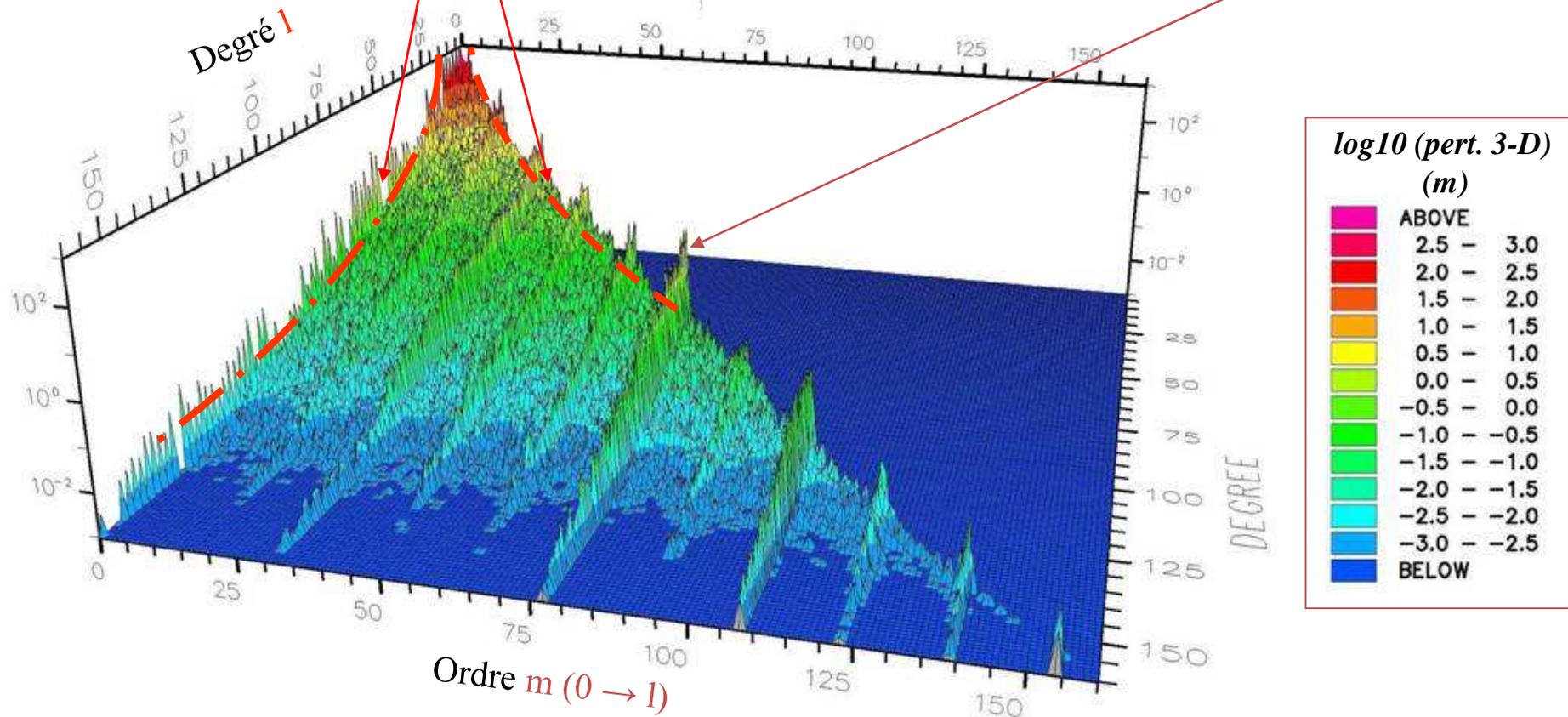


# Amplitude des perturbations gravitationnelles une orbite filtre et résonne

$$\left[ \frac{R}{R+h} \right]^l \approx e^{-2\pi h / \lambda}$$

( $\lambda = 2\pi R / l$ )

$$k n_{orb} \approx m \dot{\theta}_{Terre}$$



CHAMP orbit : 450 km, 87°

# La troncature en degré en fonction du type d'orbite

Table 6.1: Suggested truncation levels for use of EGM2008 at different orbits

Orbit radius / km	Example	Truncation level
7331	Starlette	90
12270	Lageos	20
26600	GPS	12

Tests d'estimation de l'impact de la troncature du champ de gravité sur l'orbite

Satellite/troncature	rms	Radial (mm)	Tangentiel (mm)	Normal (mm)	3D (mm)
GPS	12	.001	.011	.001	.012
Lageos	20	.04	.7	.07	.7
Starlette	90	.39	1.46	.21	1.46
GRACE Range Range-rate Acceleration	150	.25	.17	.06	.31 12 $\mu$ m 50 nm/s .9 nm/s <sup>2</sup>

**Recommandation:** donner le seuil de précision lié à la troncature

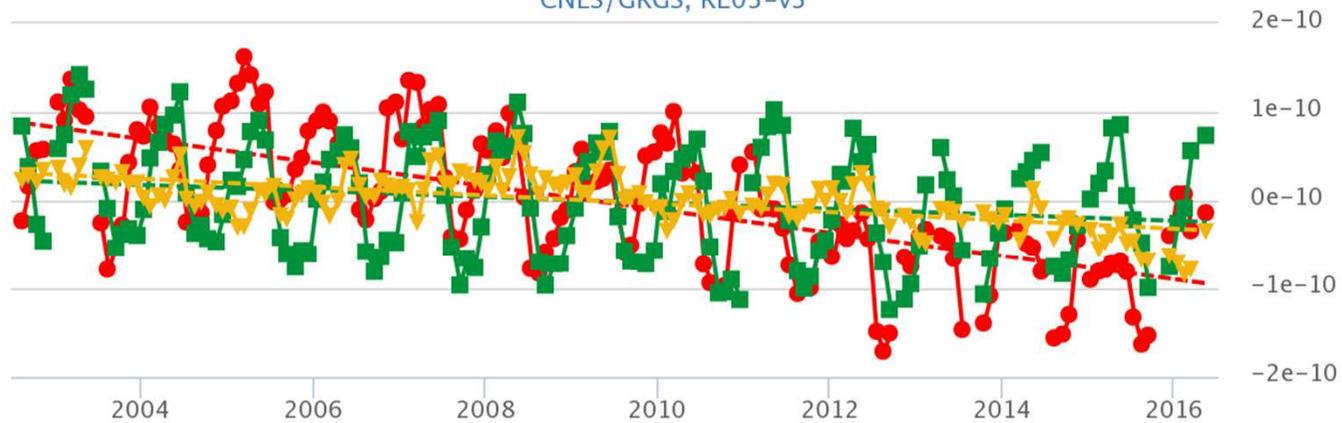
# Les dérives des bas degrés

Table 6.2: Low-degree coefficients of the conventional geopotential model

Coefficient	Value at 2000.0	Reference	Rate / yr <sup>-1</sup>	Reference
$C_{20}$ (zero-tide)	$-0.48416948 \times 10^{-3}$	Cheng <i>et al.</i> , 2010	$11.6 \times 10^{-12}$	Nerem <i>et al.</i> , 1993
$C_{30}$	$0.9571612 \times 10^{-6}$	EGM2008	$4.9 \times 10^{-12}$	Cheng <i>et al.</i> , 1997
$C_{40}$	$0.5399659 \times 10^{-6}$	EGM2008	$4.7 \times 10^{-12}$	Cheng <i>et al.</i> , 1997

## GRACE satellite gravity data

Spherical Harmonics  
CNES/GRGS, RL03-v3

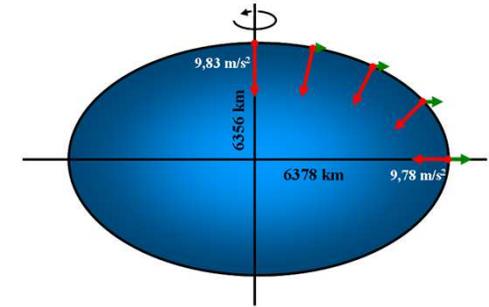


Référence au  
1<sup>er</sup> avril 2009

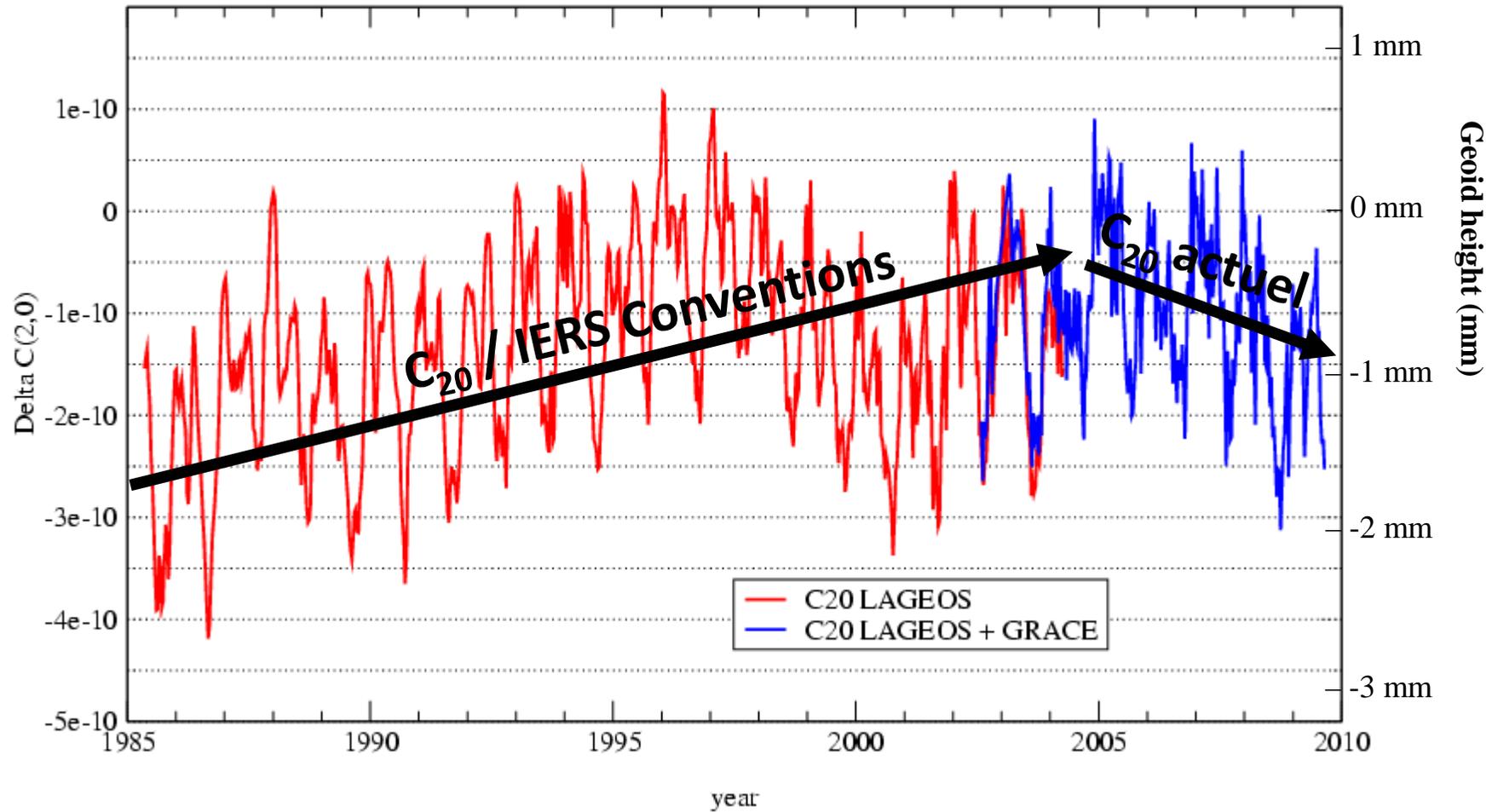
- C(2,0) minus  $-4.84165292963e-4$  --- Trend  $-0.13 \text{ e-}10/\text{year}$
- C(3,0) minus  $9.57173822827e-7$  --- Trend  $-0.03 \text{ e-}10/\text{year}$
- ▲ C(4,0) minus  $5.39982845563e-7$  --- Trend  $-0.05 \text{ e-}10/\text{year}$

www.thegraceplotter.com, by CNES/GRGS

# Aplatissement dynamique (C20) de la Terre à partir des données LAGEOS et GRACE



C(2,0) time series  
(difference to  $-.484165198e-3$ )



# Modélisations et mesures du champ de gravité

Potential of gravitation:

$\pm 1000 \text{ m}^2 \text{ s}^{-2}$

Clock

$$U = \frac{GM}{a_e} \sum_{l=0}^L \left(\frac{a_e}{r}\right)^{l+1} \sum_{m=0}^l \bar{P}_{l,m}(\sin \varphi) (\bar{C}_{l,m} \cos m\lambda + \bar{S}_{l,m} \sin m\lambda)$$

$$U_0 = 62\,494\,812.4 \text{ m}^2 \text{ s}^{-2}$$

Orbital perturbation

$\pm 1000 \text{ m}$

(10 km /  $C_{20}$ )

GRACE

$$\ddot{\vec{r}} = \frac{\partial U}{\partial \vec{x}} \quad \text{Numerical or analytical integration (Lagrange's equations)}$$

$$\frac{\partial U}{\partial r} = -\frac{GM}{a_e^2} \sum_{l=0}^L (l+1) \left(\frac{a_e}{r}\right)^{l+2} \sum_{m=0}^l \bar{P}_{l,m}(\sin \varphi) (\bar{C}_{l,m} \cos m\lambda + \bar{S}_{l,m} \sin m\lambda)$$

Geoid height:

$\pm 100 \text{ m}$

Altimeter

$$N = \frac{T}{\gamma}, \quad T = U - V, \quad V : \text{ellipsoid potential}$$

$$N = a_e \sum_{l=0}^L \sum_{m=0}^n \bar{P}_{l,m}(\sin \varphi) (\bar{C}_{l,m}^* \cos m\lambda + \bar{S}_{l,m}^* \sin m\lambda)$$

Gravity anomaly:

$\pm 500 \text{ mGal}$

( $\text{mGal} = 10^{-5} \text{ m s}^{-2}$ )

Gravimeter

$$\Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r}$$

$$\Delta g = \frac{GM}{a_e^2} \sum_{l=0}^L (l-1) \sum_{m=0}^n \bar{P}_{l,m}(\sin \varphi) (\bar{C}_{l,m}^* \cos m\lambda + \bar{S}_{l,m}^* \sin m\lambda)$$

Vertical deflection

$\pm 50''$

Tiltmeter

$$\xi = -\frac{1}{r\gamma} \frac{\partial T}{\partial \theta}; \quad \eta = -\frac{1}{r\gamma \cos \varphi} \frac{\partial T}{\partial \lambda}$$

Gravity gradient:

$\pm 50 \text{ Eötvös}$

( $\text{Eötvös} = 10^{-9} \text{ s}^{-2}$ )

GOCE

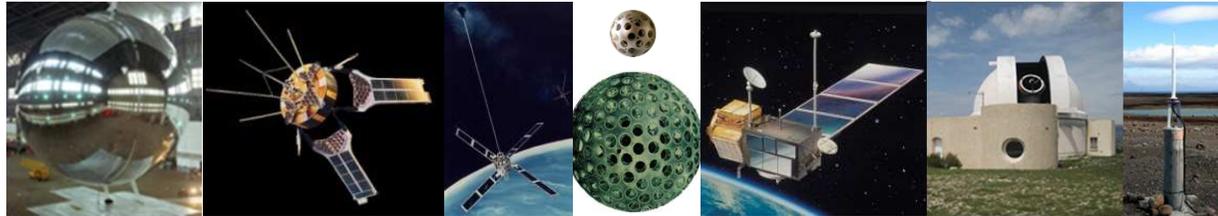
$$\frac{\partial^2 U}{\partial r^2} = \frac{GM}{a_e^3} \sum_{l=0}^L (l+1)(l+2) \left(\frac{a_e}{r}\right)^{l+3} \sum_{m=0}^l \bar{P}_{l,m}(\sin \varphi) (\bar{C}_{l,m} \cos m\lambda + \bar{S}_{l,m} \sin m\lambda)$$

# Plan

- *La modélisation harmonique*
- *Les modèles globaux de champ de gravité*
- *GM et bas degrés*
- *Modélisations alternatives*
- *Les corrections de gravité*
- *Evolution des Conventions*

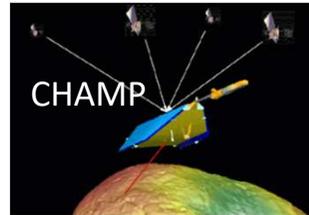
# Missions satellitaires utilisées pour la détermination du champ de gravité

1961 – 2000  
Earth tracking  
100 m – 1 cm



camera  
TRANSIT  
SLR  
PRARE  
DORIS

2000 – 2010  
High-low tracking  
1 cm



*GPS + SLR + accelerometer*

2002 – 2017  
Low-low tracking  
 $\sim 1 \mu\text{m}$ ,  $\sim 0.1 \mu\text{m/s}$

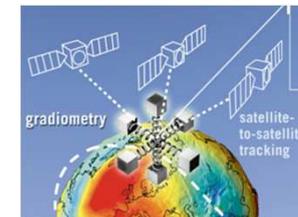


*GPS + SLR + KBR + accelerometer*

2009 – 2013  
In situ measurement  
 $< 10 \text{ mE}$  ( $E=10^{-9} \text{ s}^{-2}$ )



*GPS + SLR + gradiometer*

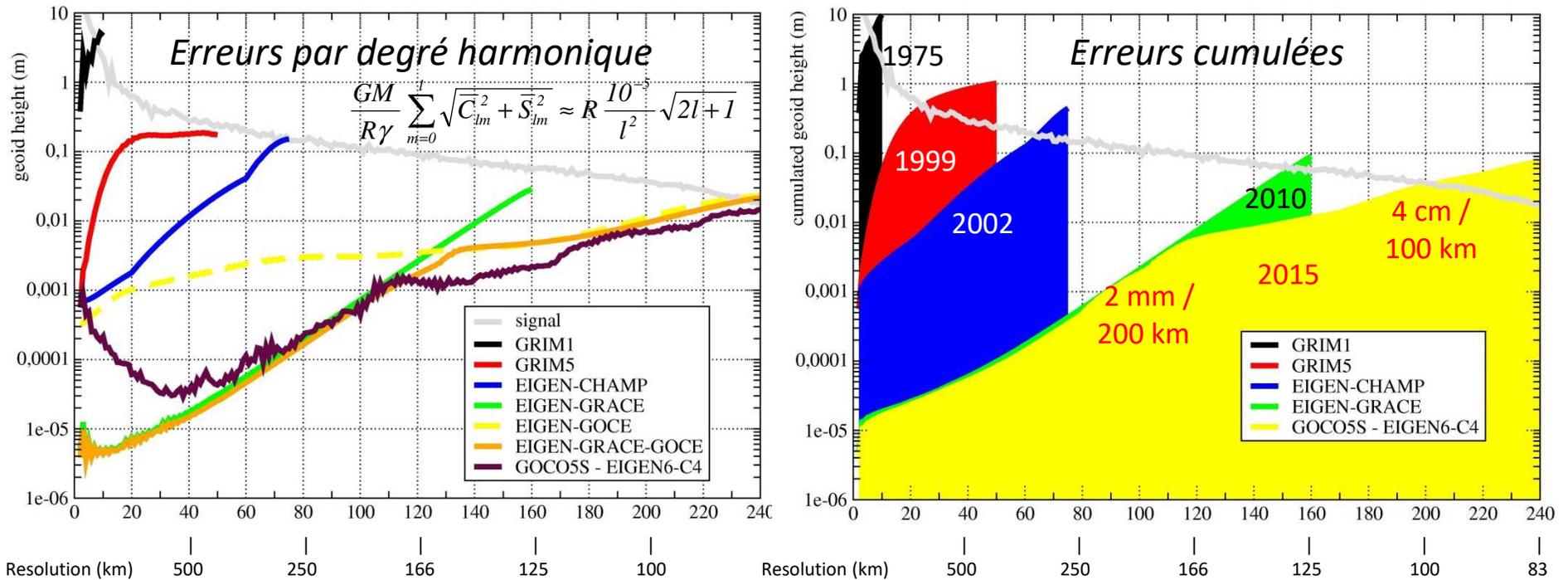


2018 – ...  
Low-low tracking  
50 nm?

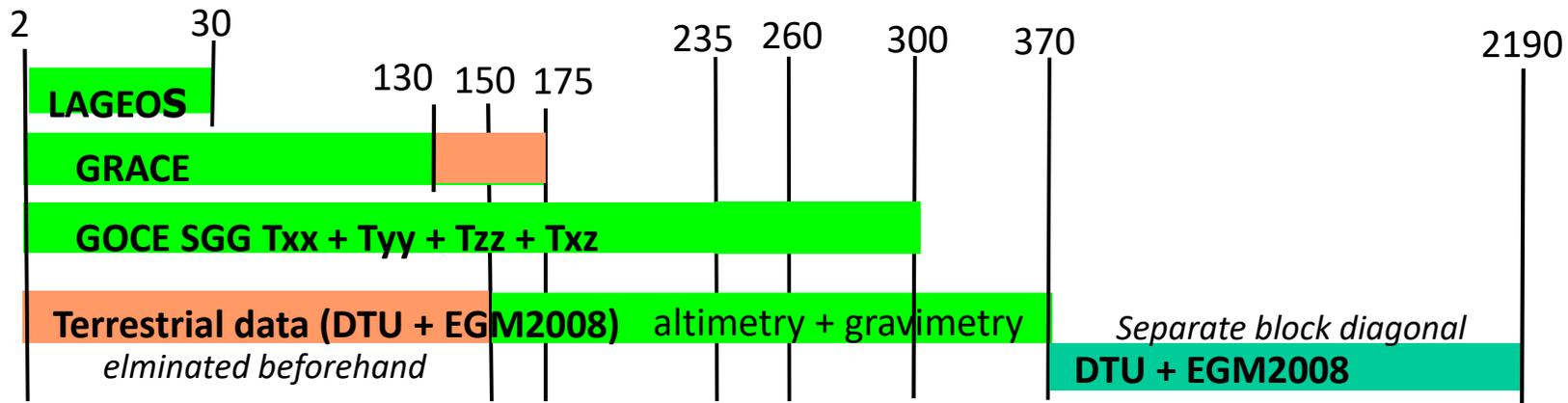


*GPS + SLR + KBR + LRI + accelerometer*

# Puissance spectrale des modèles projetée en hauteur de géoïde

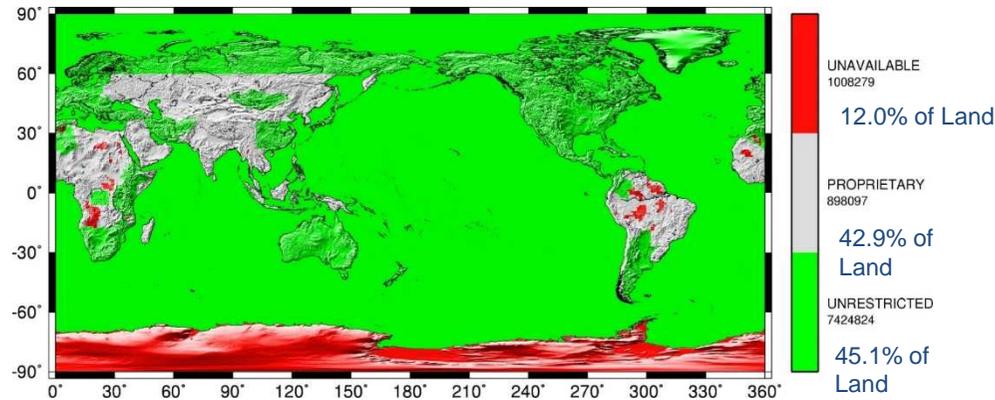


## EIGEN6-C4 (2014)



# An Earth Gravitational Model to Degree 2160: EGM2008

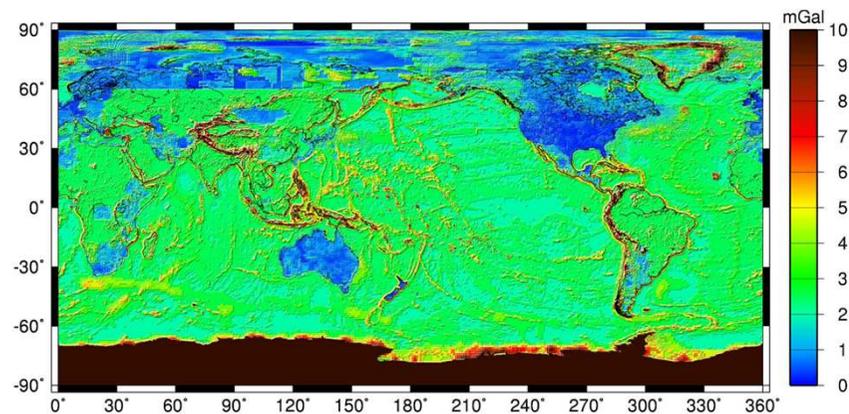
Nikolaos K. Pavlis, Simon A. Holmes, Steve C. Kenyon, and John K. Factor



5'x5' Δg Data Availability

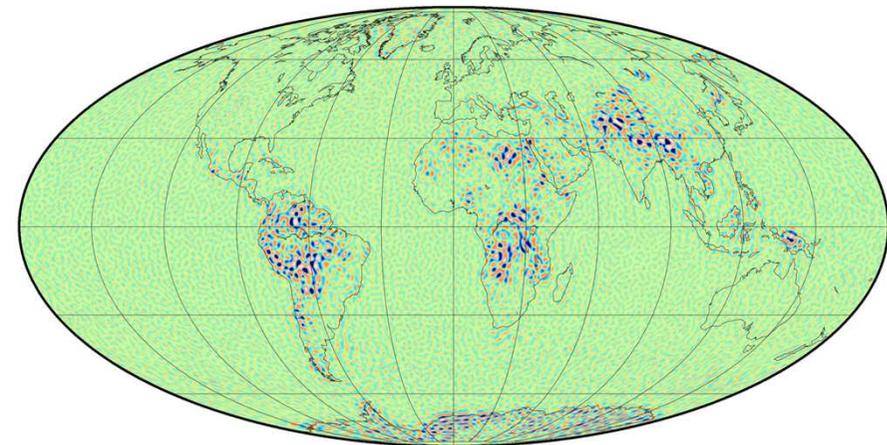
## Problèmes:

- Les sources de données sont de différents types: altimétrie sur les océans, gravimétrie sur les continents;
- Il n'y a pas de données gravimétriques terrestres sur la totalité de la surface de la Terre ou elles ne sont pas disponibles;
- La qualité des données gravimétriques et leur résolution spatiale sont disparates



5'x5' Δg Data Standard Deviations

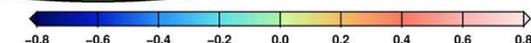
## Comparaison à EIGEN6-C4 (2014)



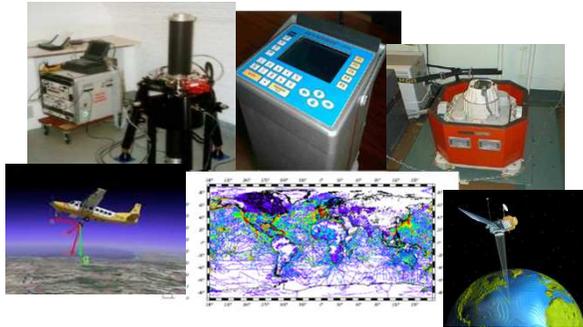
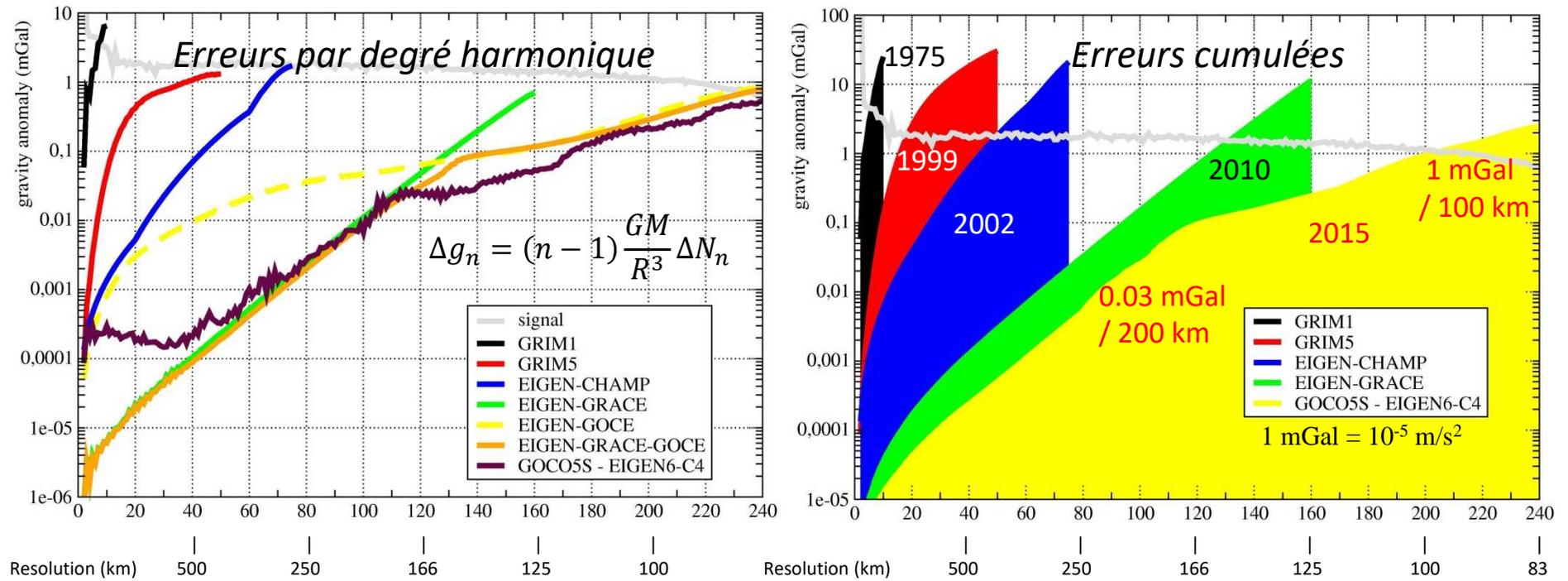
EGM2008 vs. EIGEN-6C max 1400

$\zeta$ ,  $0.2^\circ \times 0.2^\circ$

wrms about mean / min / max = 0.1146 / -2.729 / 2.553 meter



# Puissance spectrale des modèles projetée en anomalie de gravité



## Les résolutions:

- >1 mGal / 10 km: gravimétrie aéroportée
- >1 mGal / 5 km: altimétrie spatiale
- >1 mGal / 100 m: gravimétrie marine
- 1  $\mu$ Gal ponctuellement: gravimétrie terrestre

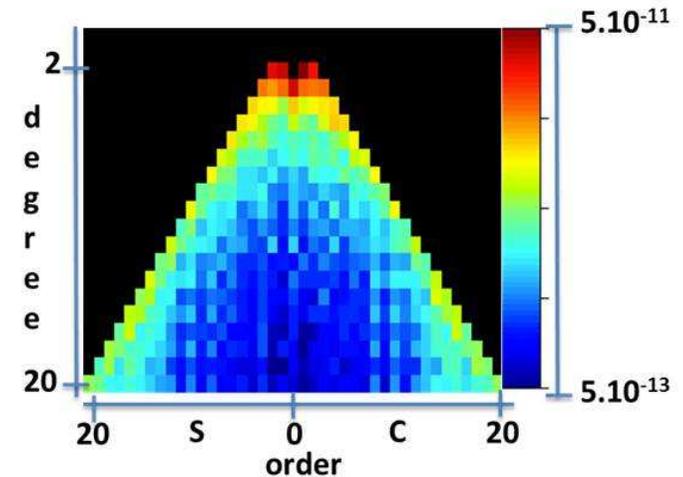
# Suivi géodésique terrestre

Utilisation des données de suivi Laser, DORIS ou GNSS:

- En complément pour les bas degrés (0, 1, 2...)
  - SLR sur Lageos, Starlette, Stella...
  - ...

Differences between the low degree and order terms (< 20) of GRACE monthly gravity field solutions from four processing centers (CSR, GFZ, JPL and GRGS) gauging their “internal” errors.

- En palliatif
  - GPS GRACE-A (mono satellite)
  - SWARM
  - DORIS
  - ...



SWARM / ESA  
2013 - ...  
460 / 530 km, 88 deg.



# Missions de type GRACE (SST)

## GRACE

- Lancé de Vandenberg le 17 mars 2002
- Fin de mission en octobre 2017
- 159 mois de données sur 15,25 ans

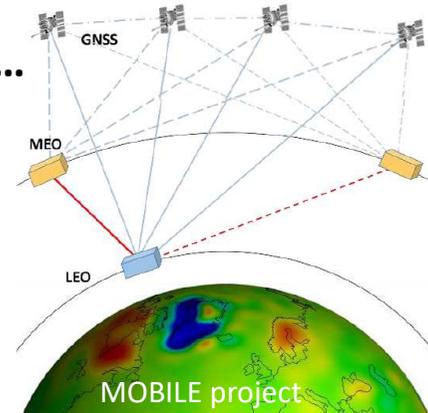
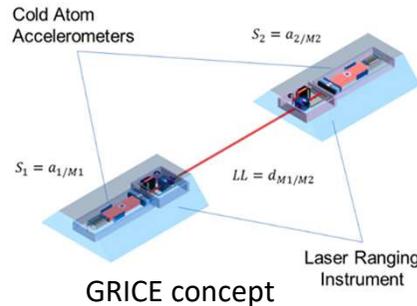
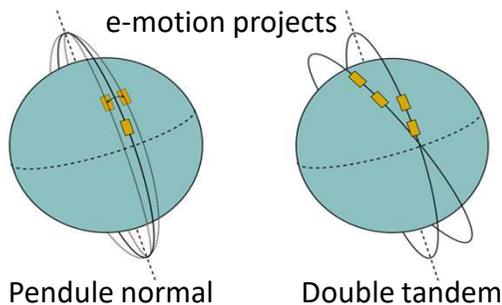
## GRACE-FO

- Lancé de Vandenberg le 22 mai 2018
- Délais de délivrance des données : 2019 (3 mois IOC, + 4 mois science phase + 2 mois Level2 production...)

## GRACE- 2

- Soumis au programme NASA Decadal Survey 2017-2027

## Projets: E-Motion<sup>(2)</sup>, MOBILE (ESA EE), GRICE (CNES)...



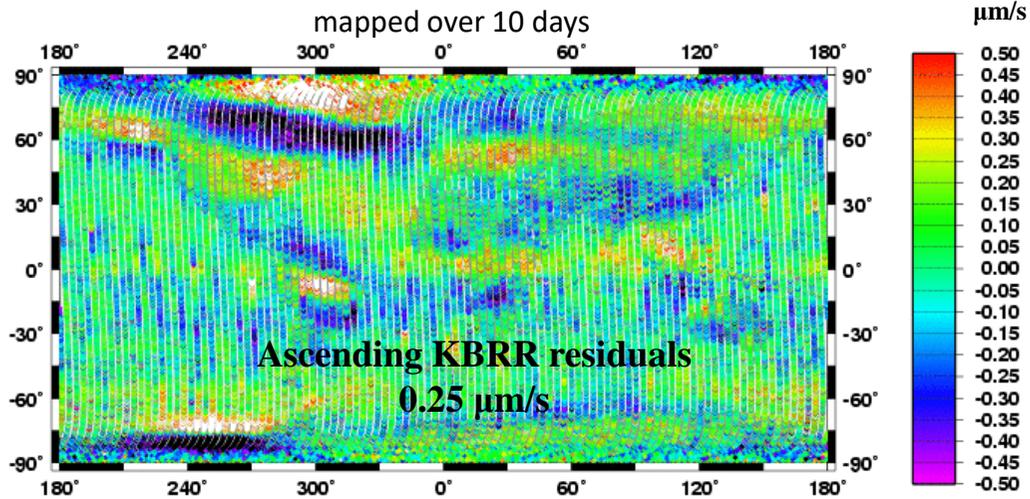
Monthly official products

L1B data	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2002												
2003												
2004												
2005												
2006												
2007												
2008												
2009												
2010												
2011												
2012												
2013												
2014												
2015												
2016												
2017												

■ No data due to accelerometer problems  
■ No data due to GRACE-B battery problems  
■ GRACE-A accelerometer data transferred to GRACE-B



# Le signal GRACE et ses transformations



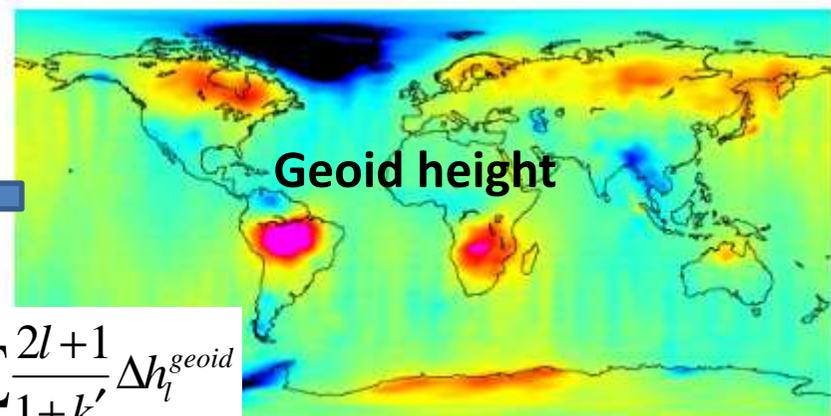
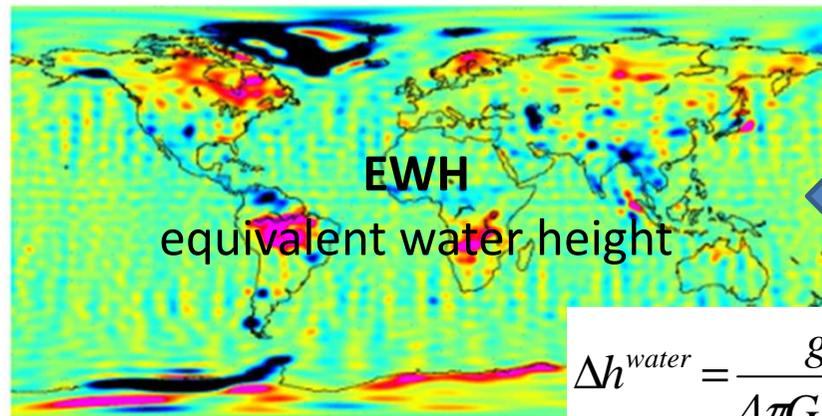
Distance inter-satellite dérivée

$$\rightarrow \Delta U_{AB} = \frac{1}{2} |\Delta v_{AB}|^2 - \dots$$

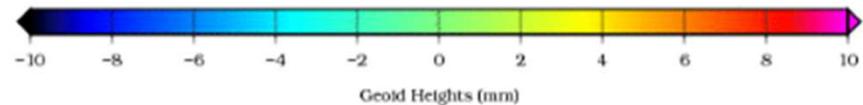
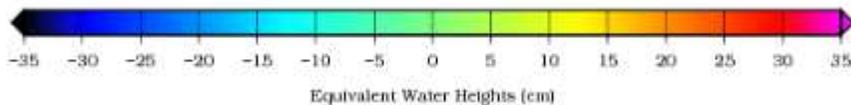
selon l'approche de l'intégrale de l'énergie

Projeté à la surface de la Terre

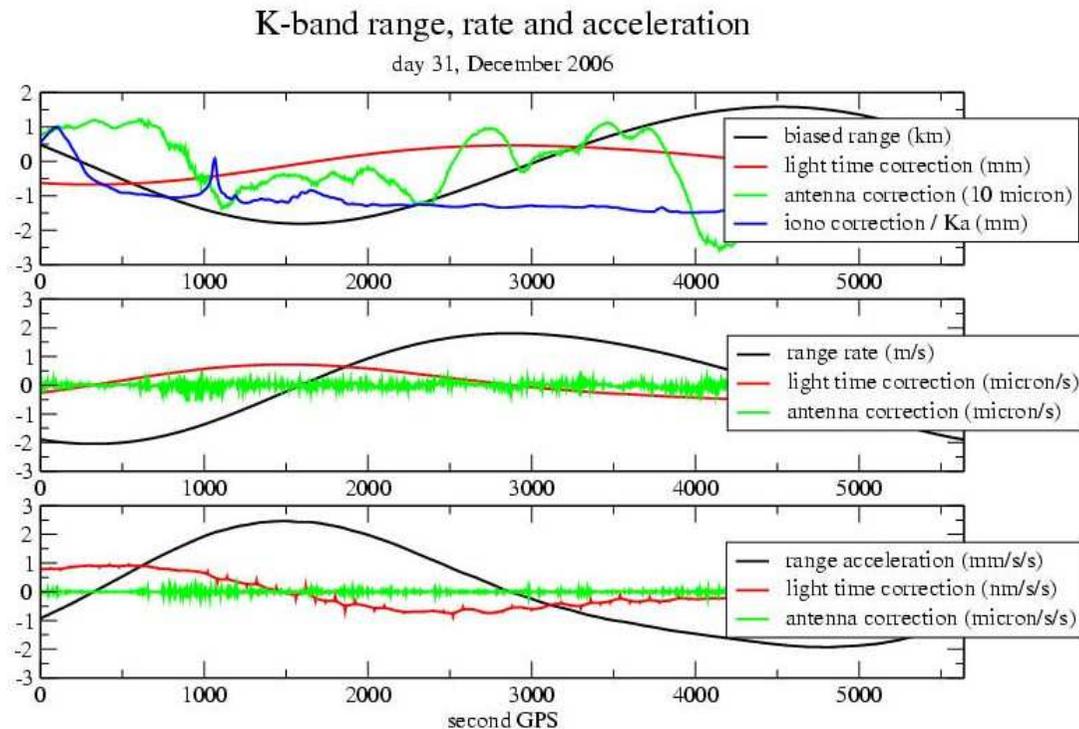
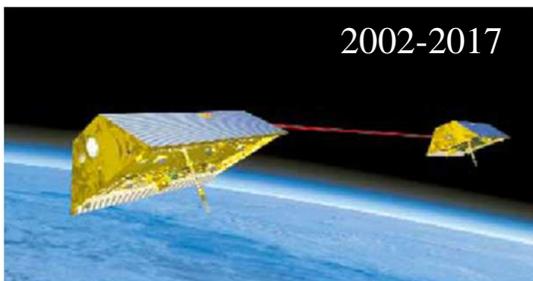
↓  $\times R$



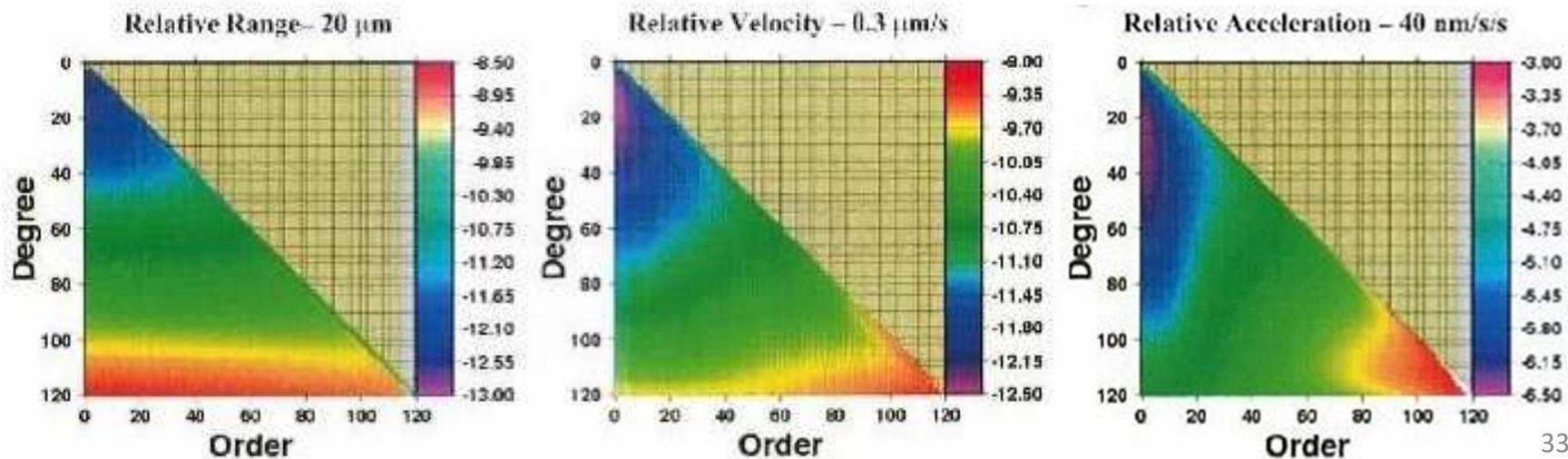
$$\Delta h^{water} = \frac{g}{4\pi G R \rho_w} \sum_{l=2}^{80} \frac{2l+1}{1+k'_l} \Delta h_l^{geoid}$$



# Sensibilité du type de mesure GRACE pour la détermination du champ de gravité



Etude analytique comparative des sensibilités des dérivées successives de la mesure de distance KBR



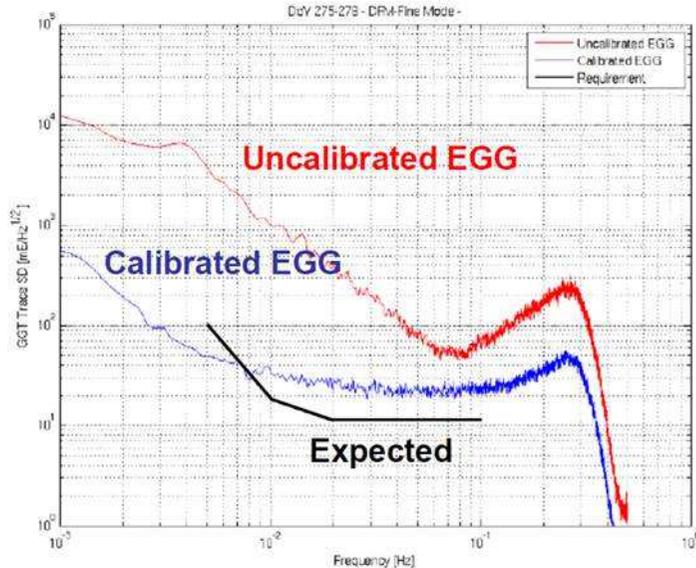
# Missions de type GOCE (Gradiométrie)

**GOCE** (17-3-2009 – 10-11-2013)

- Drag free at 260 km altitude
- 3 pairs of accelerometers sensitive to  $10^{-12} \text{ m s}^{-2} \text{ Hz}^{-1/2}$

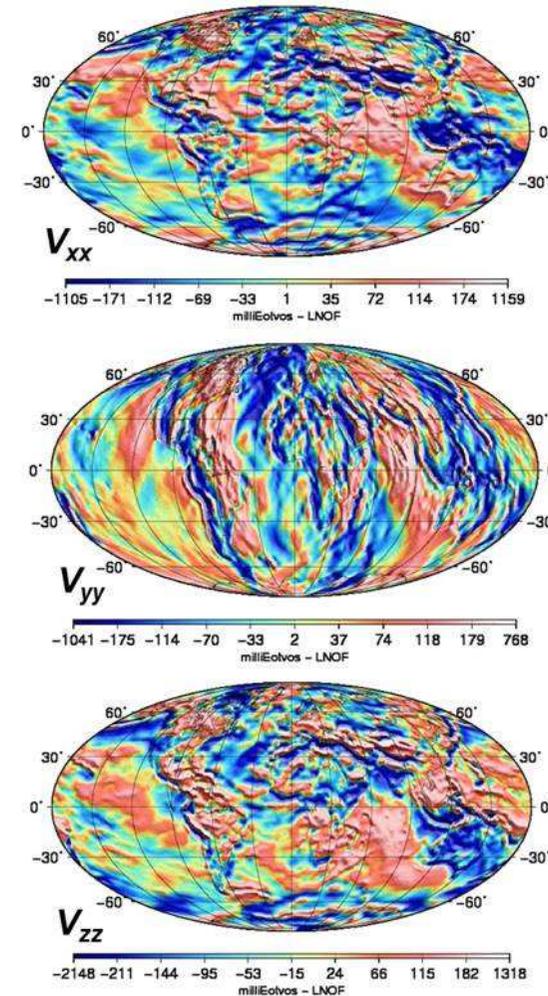


Trace of the Gravity Gradient



Measured :  $\sim 24 \text{ mE/Hz}^{1/2}$  (40-100 mHz)  
 Expected:  $11 \text{ mE/Hz}^{1/2}$

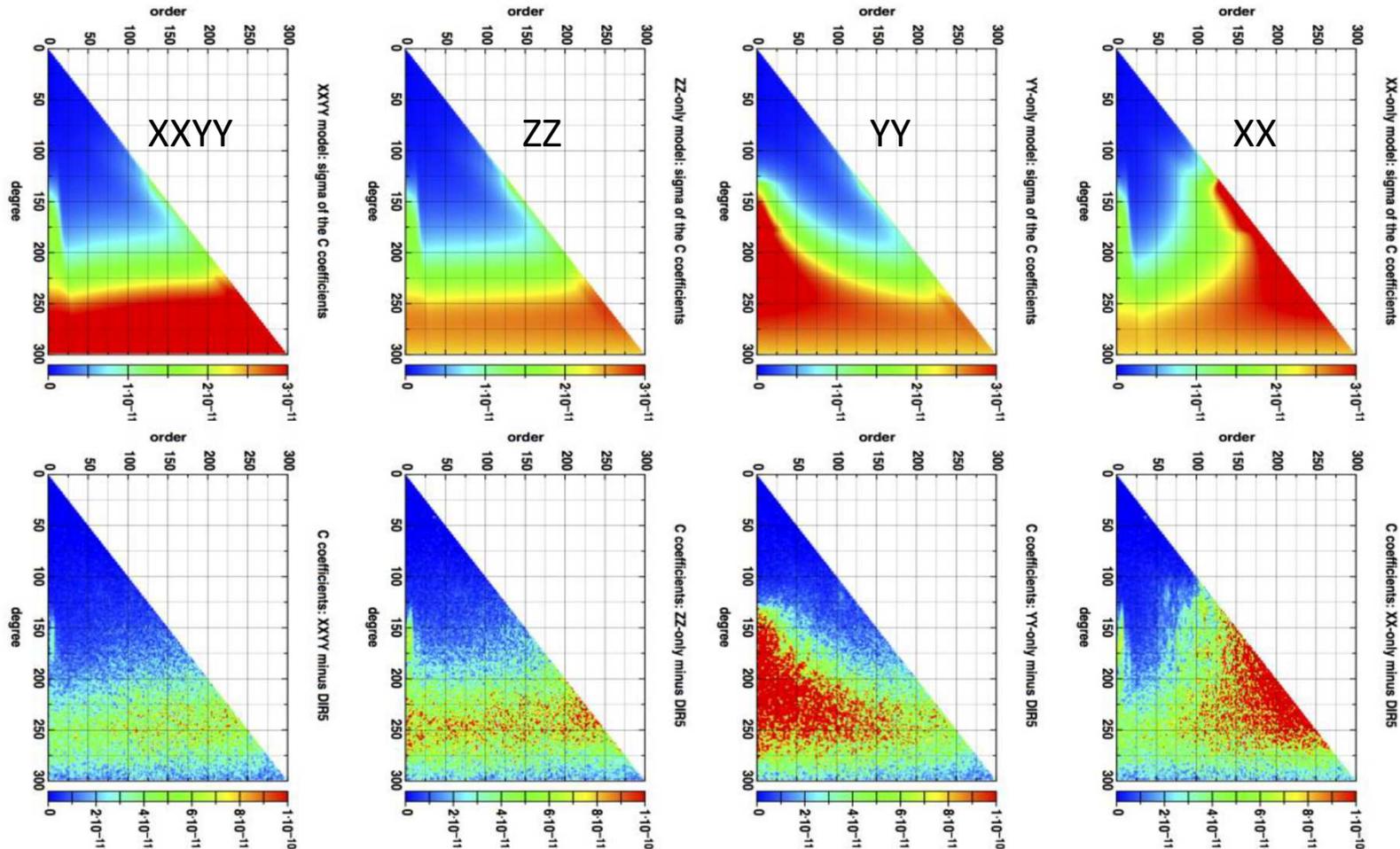
**No new gradiometric mission planned presently**



*Dérivées secondes du potentiel gravitationnel terrestre – effet de l'ellipsoïde soustrait en repère local North-West-Up (source IGN/IPGP)*

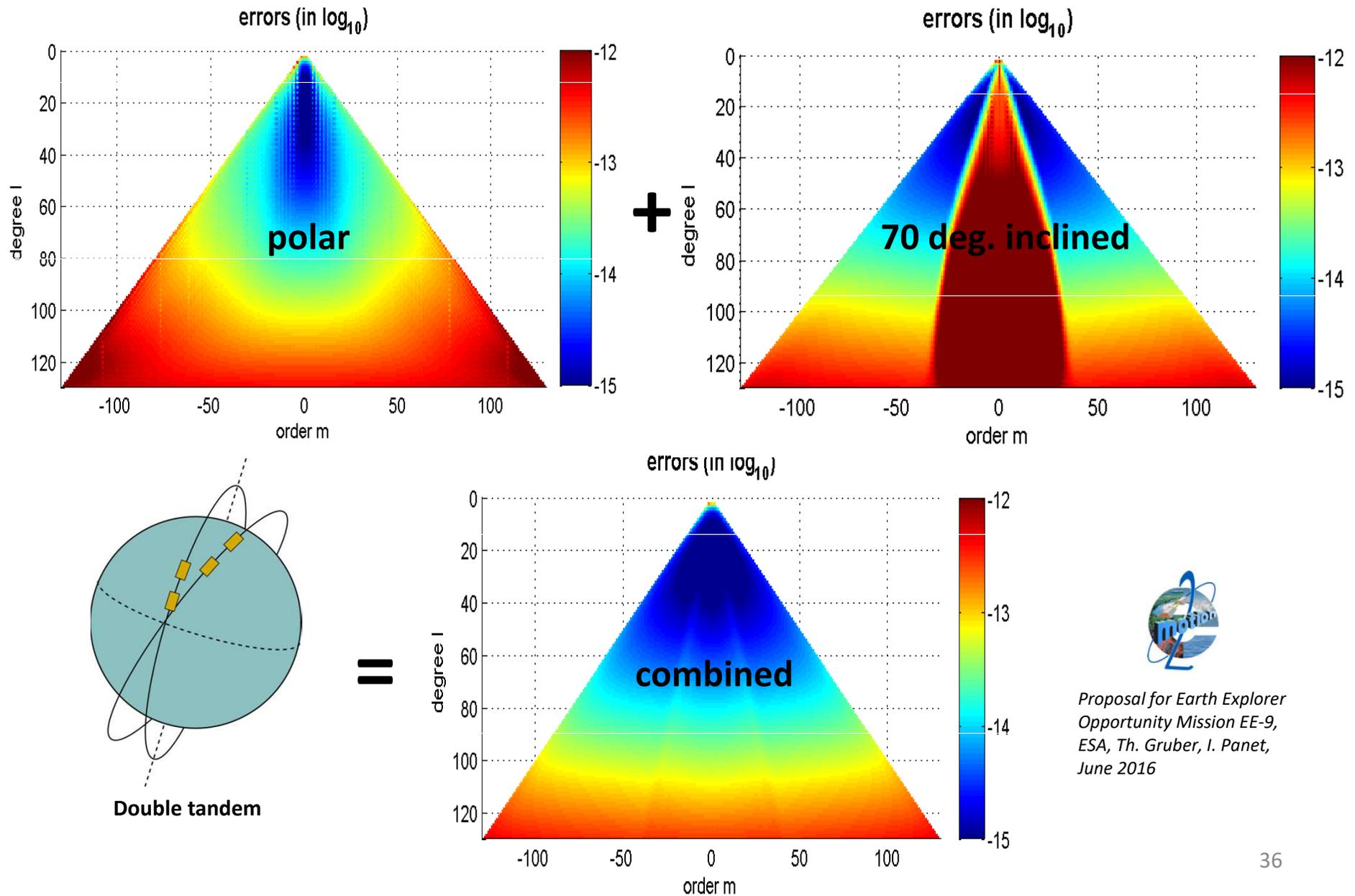
# Sensibilité des gradients sur GOCE

Standard deviation (sigma) per coefficient  $C_{lm}$  and  $S_{lm}$  of degree  $l$  and order  $m$  (upper frames) and coefficient differences with DIR-R5 (lower frames) for the XX-only, YY-only and ZZ-only and XXYY models (right to left).

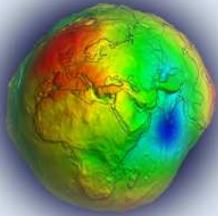


Source: Comparison of satellite-only gravity field models constructed with all and parts of the GOCE gravity gradient dataset, S. Bruinsma, Ch. Förste, S. Mulet, M.-H. Rio, O. Abrikosov, J.-Ch. Marty, Marine Geodesy (2016), DOI 10.1080/01490419.2016.1182090

# Etude prospective pour 2 couples de satellites (emotion<sup>2</sup>, 2016)



Proposal for Earth Explorer Opportunity Mission EE-9, ESA, Th. Gruber, I. Panet, June 2016



# ICGEM



## International Centre for Global Earth Models (ICGEM)

ICGEM Home

Gravity Field Models

Static Models

Temporal Models

Topographic Gravity Field Models

Calculation Service

3D Visualisation

Static Models

Temporal Models

Trend & Amplitude

Spherical Harmonics

Evaluation

Spectral domain

GNSS Leveling

FAQ

Theory

References

Latest Changes

Discussion Forum

GFZ Potsdam

Other Celestial Bodies  
(Moon, Venus, Mars)

Table of Models

3D Visualization

Calculation Service

Login

### Appointment of the new director:

The longtime director of the ICGEM service, Franz Barthelmes retired on December 31st 2017. We would like to acknowledge the invaluable contributions he provided to ICGEM service and GFZ family. As of January 1st 2018, E. Sinem Ince has been appointed as the new director of the ICGEM service.

ICGEM is one of five services coordinated by the International Gravity Field Service (IGFS) of the International Association of Geodesy (IAG). The other services are:

- BGI (Bureau Gravimetrique International), Toulouse, France
- ISG (International Service for the Geoid), Politecnico di Milano, Milano, Italy
- IGETS (International Geodynamics and Earth Tide Service), EOST, Strasbourg, France
- IDEMS (International Digital Elevation Model Service), ESRI, Redlands, CA, USA

### Services of ICGEM

- collecting and archiving of all existing global gravity field models
- web interface for getting access to global gravity field models
- web based visualization of the gravity field models their differences and their time variation
- web based service for calculating different functionals of the gravity field models
- web site for tutorials on spherical harmonics and the theory of the calculation service
- *new service since 2016*: providing a Digital Object Identifier (DOI) for the data set of the model (the coefficients)

### Some ICGEM related documents

- Definition of Functionals of the Geopotential and Their Calculation from Spherical Harmonic Models
- Article about Global Models
- Description of the ICGEM-format
- IGFS and ICGEM in Geodesists Handbook 2016
- ICGEM-Report 2003-2007
- ICGEM-Report 2007-2011
- ICGEM-Report 2011-2015
- ICGEM-Report 2015-2017
- The ICGEM-Poster 2018
- Presentation about global models and the ICGEM
- Presentation about the ICGEM (IAG-Assembly, 2013, Potsdam)
- Figure with the differences of some satellite-only models to a recent combination model

Please cite our service as:

Nr	Model	Year	Degree	Data	References	Download	Calculate	Show	DOI
168	Tongji-Grace02k	2018	180	S(GRACE)	Chen, Q. et al, 2018	gfc zip	Calculate	Show	✓
167	SGG-UGM-1	2018	2159	EGM2008, S(GOCE)	Liang, W. et al., 2018 & Xu, X. et al. (2017)	gfc zip	Calculate	Show	✓
166	GOSG01S	2018	220	S(GOCE)	Xu, X. et al., 2018	gfc zip	Calculate	Show	✓
165	IGGT_R1	2017	240	S(GOCE)	Lu, B. et al, 2017	gfc zip	Calculate	Show	✓
164	IFE_GOCE05s	2017	250	S(GOCE)	Wu, H. et al, 2017	gfc zip	Calculate	Show	✓
163	GO_CONS_GCF_2_SPW_R5	2017	330	S(GOCE)	Gatti, A. et al, 2016	gfc zip	Calculate	Show	✓
162	GAO2012	2012	360	A, G, S(GOCE), S(GRACE)	Demianov, G. et al, 2012	gfc zip	Calculate	Show	✓
161	XGM2016	2017	719	A, G, S(GOCO05s)	Pail, R. et al, 2017	gfc zip	Calculate	Show	✓
160	Tongji-Grace02s	2017	180	S(Grace)	Chen, Q. et al, 2016	gfc zip	Calculate	Show	✓
159	NULP-02s	2017	250	S(Goce)	A.N. Marchenko et al, 2016	gfc zip	Calculate	Show	✓
158	HUST-Grace2016s	2016	160	S(Grace)	Zhou, H. et al, 2016	gfc zip	Calculate	Show	✓
157	ITU_GRACE16	2016	180	S(Grace)	Akyilmaz, O. et al, 2016	gfc zip	Calculate	Show	✓
156	ITU_GGC16	2016	280	S(Goce), S(Grace)	Akyilmaz, O. et al, 2016	gfc zip	Calculate	Show	✓
155	EIGEN-6S4 (v2)	2016	300	S(Goce), S(Grace), S(Lageos)	Förste, C. and Bruinsma, S.L., 2016	gfc zip	Calculate	Show	✓
154	GOCO05c	2016	720	(see model), A, G, S	Fecher, T. et al, 2016	gfc zip	Calculate	Show	✓
153	GGM05C	2015	360	A, G, S(Goce), S(Grace)	Ries, J. et al, 2016	gfc zip	Calculate	Show	✓
152	GECO	2015	2190	EGM2008, S(Goce)	Gilardoni, M. et al, 2016	gfc zip	Calculate	Show	
151	GGM05G	2015	240	S(Goce), S(Grace)	Bettadpur, S. et al, 2015	gfc zip	Calculate	Show	
150	GOCO05s	2015	280	(see model), S	Mayer-Gürr, T. et al, 2015	gfc zip	Calculate	Show	
149	GO_CONS_GCF_2_SPW_R4	2014	280	S(Goce)	Gatti, A. et al, 2014	gfc zip	Calculate	Show	
148	EIGEN-6C4	2014	2190	A, G, S(Goce), S(Grace), S(Lageos)	Förste, Christoph et al, 2014	gfc zip	Calculate	Show	✓
147	ITSG-Grace2014s	2014	200	S(Grace)	Mayer-Gürr, T. et al, 2014	gfc zip	Calculate	Show	
146	ITSG-Grace2014k	2014	200	S(Grace)	Mayer-Gürr, T. et al, 2014	gfc zip	Calculate	Show	
145	GO_CONS_GCF_2_TIM_R5	2014	280	S(Goce)	Brockmann, J. M. et al, 2014	gfc zip	Calculate	Show	
144	GO_CONS_GCF_2_DIR_R5	2014	300	S(Goce), S(Grace), S(Lageos)	Bruinsma, S. L. et al, 2013	gfc zip	Calculate	Show	
143	JYY_GOCE04S	2014	230	S(Goce)	Yi, Weiyong et al, 2013	gfc zip	Calculate	Show	
142	GOGRA04S	2014	230	S(Goce), S(Grace)	Yi, Weiyong et al, 2013	gfc zip	Calculate	Show	
141	EIGEN-6S2	2014	260	S(Goce), S(Grace), S(Lageos)	Rudenko, Sergei et al, 2014	gfc zip	Calculate	Show	
140	GGM05S	2014	180	S(Grace)	Tapley, B.D. et al, 2013	gfc zip	Calculate	Show	
139	EIGEN-6C3stat	2014	1949	A, G, S(Goce), S(Grace), S(Lageos)	Förste, C. et al, 2012	gfc zip	Calculate	Show	
138	Tongji-GRACE01	2013	160	S(Grace)	Shen, Y. et al, 2013	gfc zip	Calculate	Show	
137	JYY_GOCE02S	2013	230	S(Goce)	Yi, Weiyong et al, 2013	gfc zip	Calculate	Show	
136	GOGRA02S	2013	230	S(Goce), S(Grace)	Yi, Weiyong et al, 2013	gfc zip	Calculate	Show	
135	ULux_CHAMP2013s	2013	120	S(Champ)	Weigelt, M. et al, 2013	gfc zip	Calculate	Show	
134	ITG-Goce02	2013	240	S(Goce)	Schall, Judith et al, 2014	gfc zip	Calculate	Show	
133	GO_CONS_GCF_2_TIM_R4	2013	250	S(Goce)	Pail, Roland et al, 2011	gfc zip	Calculate	Show	
132	GO_CONS_GCF_2_DIR_R4	2013	260	S(Goce), S(Grace), S(Lageos)	Bruinsma, S. L. et al, 2013	gfc zip	Calculate	Show	
131	EIGEN-6C2	2012	1949	A, G, S(Goce), S(Grace), S(Lageos)	Förste, C. et al, 2012	gfc zip	Calculate	Show	
130	DGM-1S	2012	250	S(Goce), S(Grace)	Farahani, H. Hashemi et al, 2013	gfc zip	Calculate	Show	
129	GOCO03s	2012	250	S(Goce), S(Grace)	Mayer-Gürr, T. et al, 2012	gfc zip	Calculate	Show	
128	GO_CONS_GCF_2_DIR_R3	2011	240	S(Goce), S(Grace), S(Lageos)	Bruinsma, S.L. et al, 2010	gfc zip	Calculate	Show	
127	GO_CONS_GCF_2_TIM_R3	2011	250	S(Goce)	Pail, R. et al, 2010	gfc zip	Calculate	Show	

# Plan

- *La modélisation harmonique*
- *Les modèles globaux de champ de gravité*
- ***GM et bas degrés***
- *Modélisations alternatives*
- *Les corrections de gravité*
- *Evolution des Conventions*

# Interprétation des bas degrés

- **Degré 0**

- Contraint le  $GM = 398600.4415 \pm .0008 \text{ km}^3 \text{ s}^{-2}$  (Ries et al., 1992)

- **Degré 1**

- Contraint le mouvement du géocentre → système de référence

**Fixed CoF**  
 ⇒ Effet sur l'orbite  
 ( $C_{10}, C_{11}, S_{11}$   
 coefficients)



**Fixed CoM**  
 ⇒ effet sur le réseau de stations  
 ( $T_x, T_y, T_z$  translations)



- **Degré 2**

- Contraint le tenseur d'inertie → rotation de la Terre (équations d'Euler-Liouville)

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} = \frac{1}{3} \text{Tr}(I) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + Ma_e^2 \begin{pmatrix} C_{2,0}/3 - 2C_{2,2} & -2S_{2,2} & -C_{2,1} \\ -2S_{2,2} & C_{2,0}/3 + 2C_{2,2} & -S_{2,1} \\ -C_{2,1} & -S_{2,1} & -2C_{2,0}/3 \end{pmatrix}$$

## Expression du coefficient de degré 0

Le degré 0 agit comme facteur d'échelle de la masse conventionnelle  $M$  de la Terre:

$$C_{0,0} = \frac{1}{M} \iiint_V dm = 1$$

$GM$  est défini par sa valeur en **Temps Coordonné Géocentrique** (TCG). Cette constante gravitationnelle géocentrique, incluant la masse de l'atmosphère, est en fait déduite de l'ajustement d'orbite des satellites Lageos-1 et -2 à partir des données de suivi laser :

$$GM = 3.986004418 \cdot 10^{14} \pm 8 \cdot 10^5 \text{ m}^3 \text{ s}^{-2} \text{ (IERS Conventions)}$$

Le TCG est une échelle de temps-coordonnée (le  $t$  des équations) liée au système de référence spatio-temporel géocentrique. Il diffère du **Temps Terrestre** (TT), temps-coordonné lié à la réalisation du **Temps Atomique International** (TAI, appelé temps propre mesurable) et rapporté au géoïde:

$$TT = TAI + 32.184 \text{ s} \text{ (pour assurer la continuité avec le TE)}$$

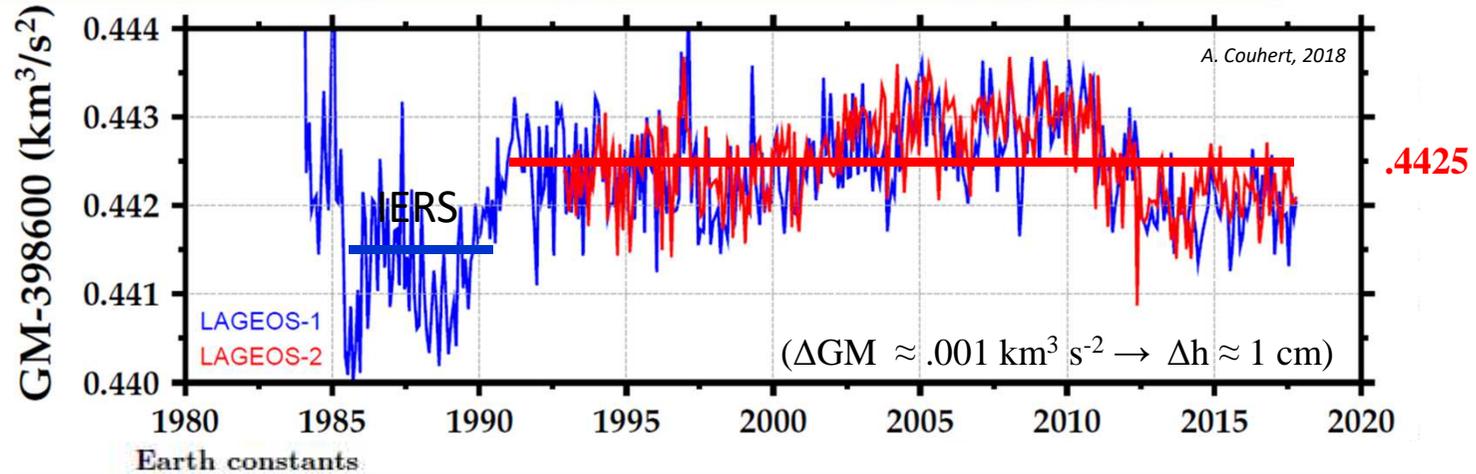
Dans cette échelle rapporté à un temps mesurable,  $GM$  s'obtient par transformation:

$$GM_{TT} = GM_{TCG} d(TT)/d(TCG) = GM_{TCG} (1 - L_G) = 3.986004415 \cdot 10^{14} \text{ m}^3 \text{ s}^{-2}$$

C'est la valeur adoptée dans les modèles EIGEN ainsi que dans EGM2008.

Table 1.1: IERS numerical standards.

Constant	Value	Uncertainty	Ref. Description
<b>Natural defining constants</b>			
$c$	$299792458 \text{ ms}^{-1}$	Defining	[1] Speed of light
<b>Auxiliary defining constants</b>			
$k$	$1.720209895 \times 10^{-2}$	Defining	[2] Gaussian gravitational constant
$L_G$	$6.969290134 \times 10^{-10}$	Defining	[3] $1 - d(\text{TT})/d(\text{TCG}) = W_0/c^2$



Earth constants	Value	Uncertainty	Ref. Description
$GM_{\oplus}^{\dagger}$	$3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$	$8 \times 10^5 \text{ m}^3 \text{ s}^{-2}$	[7] Geocentric gravitational constant
$a_E^{\dagger\dagger}$	$6378136.6 \text{ m}$	$0.1 \text{ m}$	[8] Equatorial radius of the Earth
$J_{2\oplus}^{\ddagger}$	$1.0826359 \times 10^{-3}$	$1 \times 10^{-10}$	[8] Dynamical form factor of the Earth
$1/f^{\ddagger}$	$298.25642$	$0.00001$	[8] Flattening factor of the Earth
$g_E^{\ddagger}$	$9.7803278 \text{ ms}^{-2}$	$1 \times 10^{-6} \text{ ms}^{-2}$	[8] Mean equatorial gravity
$W_0$	$62636856.0 \text{ m}^2 \text{ s}^{-2}$	$0.5 \text{ m}^2 \text{ s}^{-2}$	[8] Potential of the geoid
$R_0^{\dagger}$	$6363672.6 \text{ m}$	$0.1 \text{ m}$	[8] Geopotential scale factor ( $GM_{\oplus}/W_0$ )
$H$	$3273795 \times 10^{-9}$	$1 \times 10^{-9}$	[9] Dynamical flattening

$a_{\text{EGM2008}} = 6378\ 136.3 \text{ m}$   
 $a_{\text{EIGEN}} = 6378\ 136.46 \text{ m}$

# TCB-compatible value, computed from the TDB-compatible value in [5].  
 † The value for  $GM_{\oplus}$  is TCG-compatible. For  $a_E$ ,  $g_E$  and  $R_0$  the difference between TCG-compatible and TT-compatible is not relevant with respect to the uncertainty.  
 ‡ The values for  $a_E$ ,  $1/f$ ,  $J_{2\oplus}$  and  $g_E$  are “zero tide” values (see the discussion in Section 1.1 above). Values according to other conventions may be found in reference [8].  
 †† TDB-compatible value. An accepted definition for the TCB-compatible value of au is still under discussion.

## Expression des coefficients de degré 1

$$\text{Degré 1 : } C_{1,0} = \frac{1}{Ma_e} \iiint_V r' P_{1,0}(\sin \varphi') dm = \frac{1}{Ma_e} \iiint_V r' \sin \varphi' dm$$

$$= \frac{1}{Ma_e} \iiint_V z' dm = \frac{z_G}{a_e}$$

$$C_{1,1} = \frac{1}{Ma_e} \iiint_V r' P_{1,1}(\sin \varphi') \cos \lambda' dm = \frac{1}{Ma_e} \iiint_V r' \cos \varphi' \cos \lambda' dm$$

$$= \frac{1}{Ma_e} \iiint_V x' dm = \frac{x_G}{a_e}$$

$$S_{1,1} = \frac{1}{Ma_e} \iiint_V r' P_{1,1}(\sin \varphi') \sin \lambda' dm = \frac{1}{Ma_e} \iiint_V r' \cos \varphi' \sin \lambda' dm$$

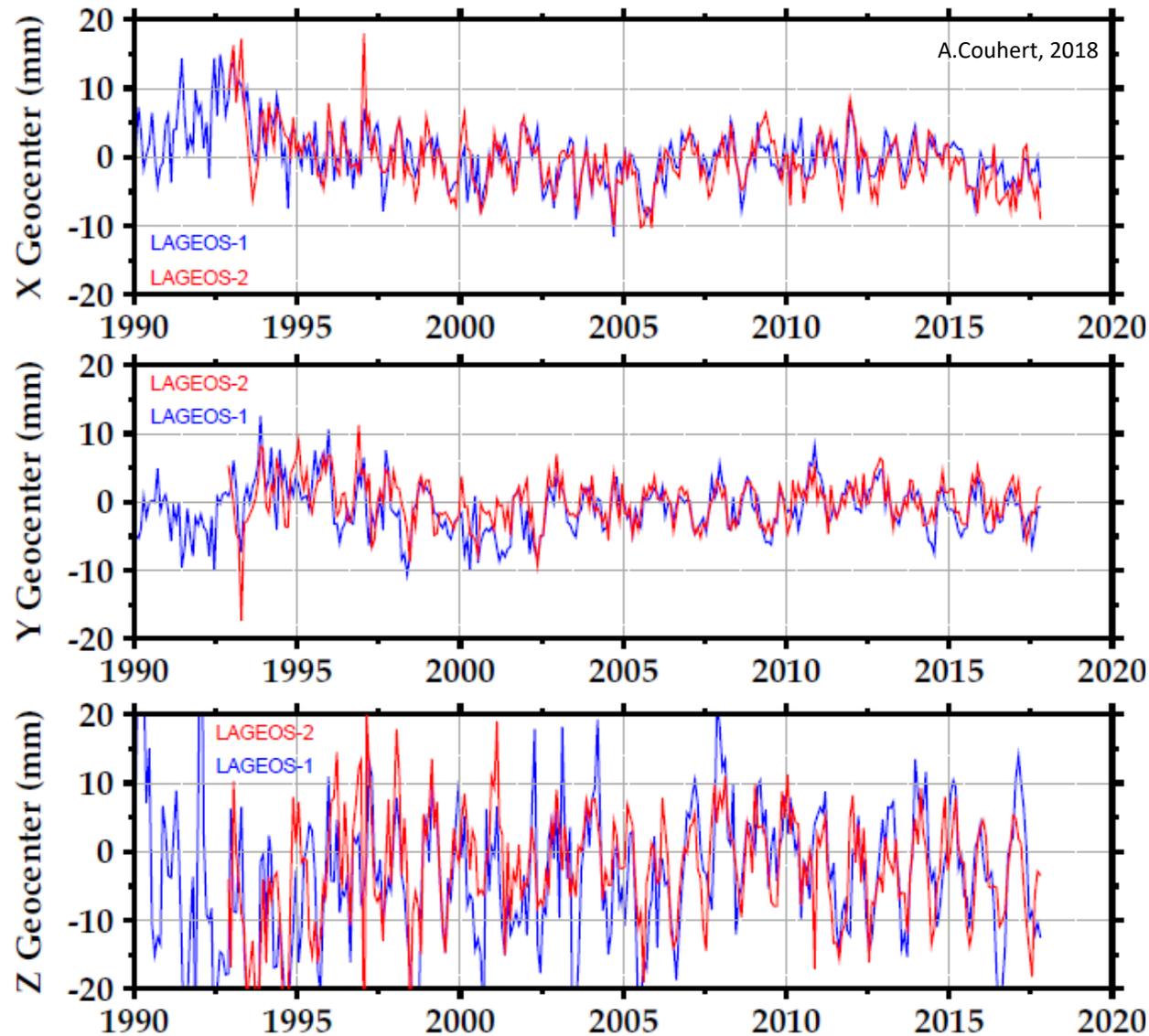
$$= \frac{1}{Ma_e} \iiint_V y' dm = \frac{y_G}{a_e}$$

Le degré 1 définit le centre des masses G de la Terre tel que :

$$G \begin{cases} x_G = a_e C_{1,1} \\ y_G = a_e S_{1,1} \\ z_G = a_e C_{1,0} \end{cases}$$

Le degré 1 varie principalement en fonction du transfert saisonnier des masses fluides superficielles. Les satellites orbitant autour du centre des masses, cette variation se répercute d'autant sur l'origine du système de référence terrestre (le centre de figure) dont l'ITRF est la réalisation.

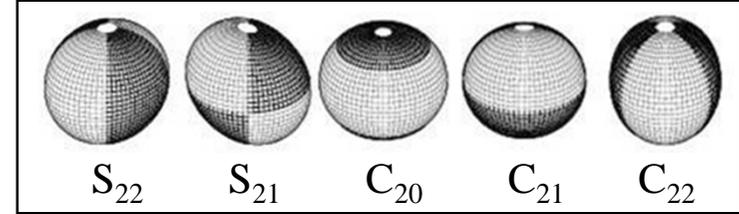
# Séries temporelles du degré 1, converties en géocentre, obtenues à partir des données LAGEOS-1 and -2



## Expression des coefficients de degré 2

$$C_{2,0} = \frac{1}{Ma_e^2} \iiint_V r'^2 P_{2,0}(\sin \varphi') dm = \frac{1}{Ma_e^2} \iiint_V r'^2 \frac{(3 \sin^2 \varphi' - 1)}{2} dm$$

$$= \frac{1}{Ma_e^2} \iiint_V \left[ z'^2 - \left( \frac{x'^2 + y'^2}{2} \right) \right] dm = \frac{(I_{xx} + I_{yy})/2 - I_{zz}}{Ma_e^2}$$



$$C_{2,1} = \frac{1}{Ma_e^2} \iiint_V \frac{1}{3} r'^2 P_{2,1}(\sin \varphi') \cos \lambda' dm = \frac{1}{Ma_e^2} \iiint_V r'^2 \sin \varphi' \cos \varphi' \cos \lambda' dm = \frac{1}{Ma_e^2} \iiint_V x' z' dm = \frac{-I_{xz}}{Ma_e^2}$$

$$S_{2,1} = \frac{1}{Ma_e^2} \iiint_V \frac{1}{3} r'^2 P_{2,1}(\sin \varphi') \sin \lambda' dm = \frac{1}{Ma_e^2} \iiint_V r'^2 \sin \varphi' \cos \varphi' \sin \lambda' dm = \frac{1}{Ma_e^2} \iiint_V y' z' dm = \frac{-I_{yz}}{Ma_e^2}$$

$$C_{2,2} = \frac{1}{Ma_e^2} \iiint_V \frac{1}{12} r'^2 P_{2,2}(\sin \varphi') \cos 2\lambda' dm = \frac{1}{Ma_e^2} \iiint_V \frac{1}{4} r'^2 \cos^2 \varphi' \cos 2\lambda' dm = \frac{1}{4Ma_e^2} \iiint_V (x'^2 - y'^2) dm = \frac{I_{yy} - I_{xx}}{4Ma_e^2}$$

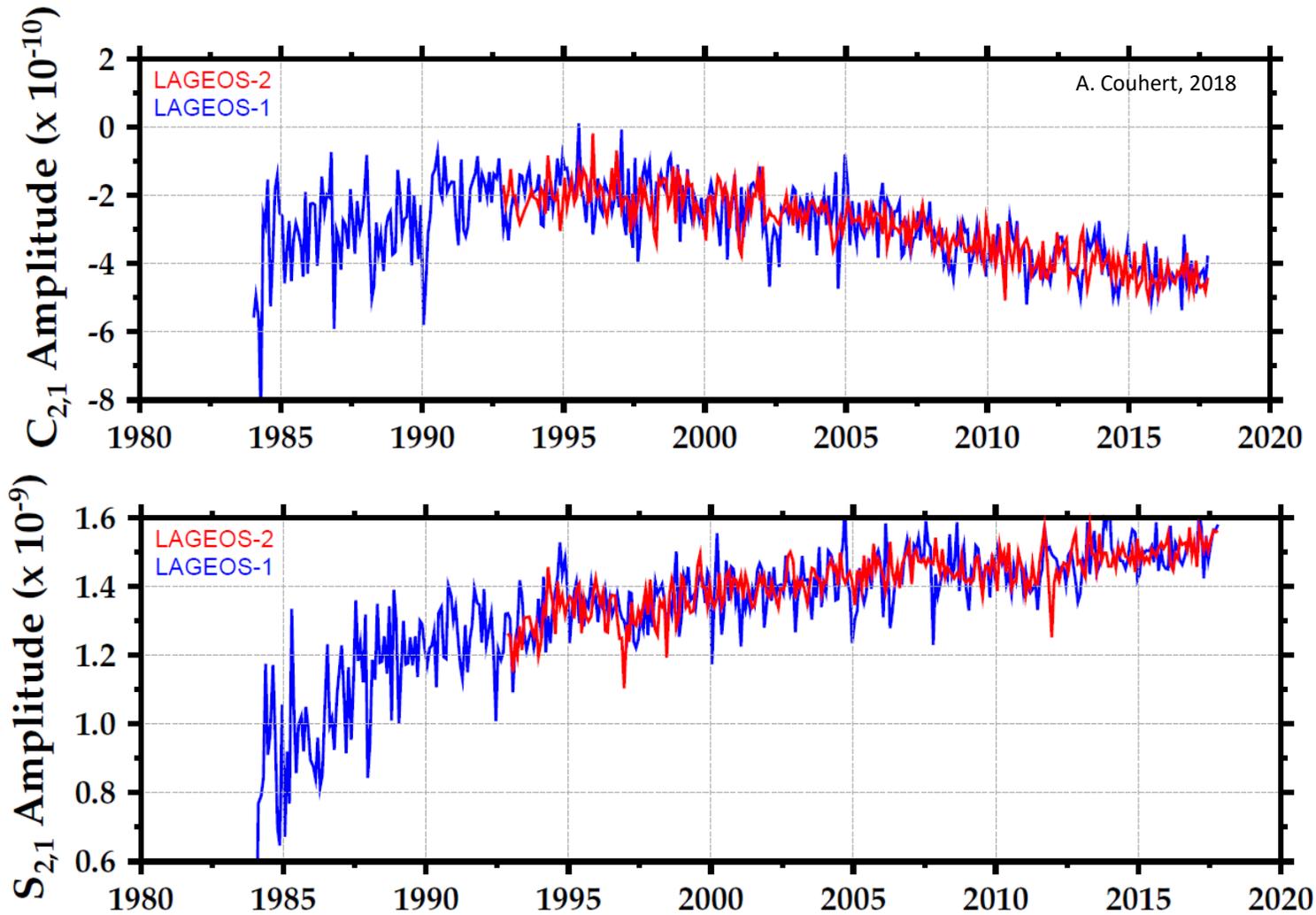
$$S_{2,2} = \frac{1}{Ma_e^2} \iiint_V \frac{1}{12} r'^2 P_{2,2}(\sin \varphi') \sin 2\lambda' dm = \frac{1}{Ma_e^2} \iiint_V \frac{1}{4} r'^2 \cos^2 \varphi' \sin 2\lambda' dm = \frac{1}{2Ma_e^2} \iiint_V x' y' dm = \frac{-I_{xy}}{2Ma_e^2}$$

Le degré 2 conditionne le tenseur d'inertie  $I$  de la Terre :

$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} = \frac{1}{3} \text{Tr}(I) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + Ma_e^2 \begin{pmatrix} C_{2,0}/3 - 2C_{2,2} & -2S_{2,2} & -C_{2,1} \\ -2S_{2,2} & C_{2,0}/3 + 2C_{2,2} & -S_{2,1} \\ -C_{2,1} & -S_{2,1} & -2C_{2,0}/3 \end{pmatrix}$$

La trace de  $I$ :  $\text{Tr}(I) = I_{xx} + I_{yy} + I_{zz}$ , est invariante dans l'hypothèse de la conservation de la masse.

# Séries temporelles des termes $C_{2,1}/S_{2,1}$ à partir des données LAGEOS-1 and -2



# Le degré 2: rotation de la Terre et moment angulaire

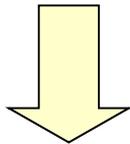
**Observations:**  
pole motion and LOD  
from *GPS, SLR, VLBI*



$$p + \frac{i}{\sigma_0} \dot{p} = \frac{k_0}{k_0 - k_2} (\chi_1 + i\chi_2)$$

*Euler-Liouville equations*

$$\frac{\Delta LOD}{LOD} = \chi_3$$



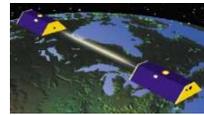
*Level of comparison*

**Geodetic excitation function**

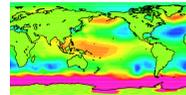
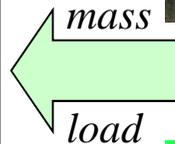
$$\chi = \chi^{motion} + (1 + k'_2) \chi^{mass}$$

**Geophysical excitation function**

*GRACE + LAGEOS-1/-2 → Hydrology*

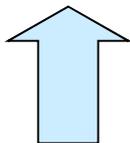


$$(\chi_1 + i\chi_2)^{mass} = -\sqrt{\frac{5}{3}} \frac{MR^2}{C - A} (\bar{C}_{21} + i\bar{S}_{21})$$



$$\chi_3^{mass} = -\frac{2\sqrt{5}}{3} \frac{MR^2}{C_m} \bar{C}_{20}$$

*Atmospheric or ocean bottom pressure*



**Models:**  
meteorology,  
oceanography...



*Winds*

$$(\chi_1 + i\chi_2)^{motion} = \frac{(h_1 + ih_2)}{\Omega(C - A)}$$

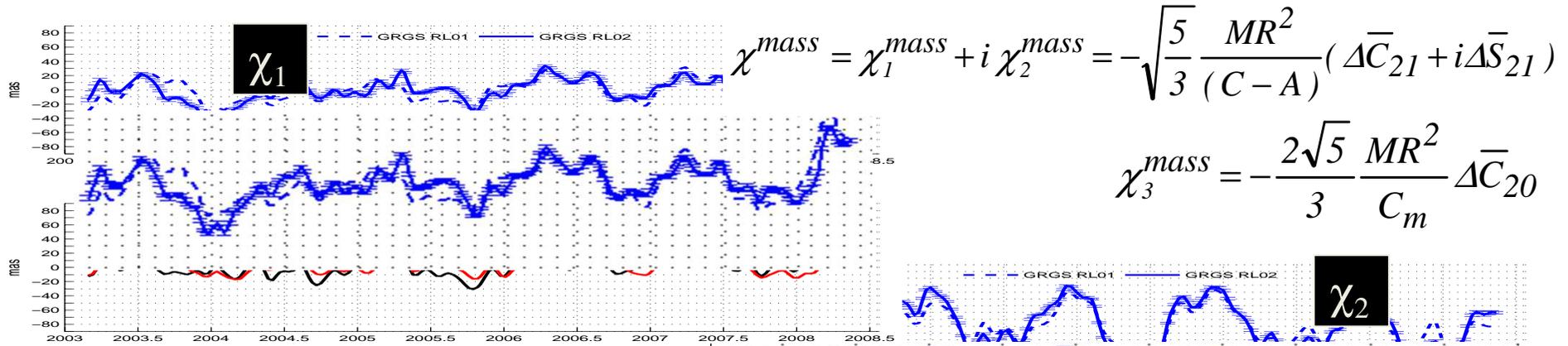


*Oceanic currents*

$$\chi_3^{motion} = \frac{h_3}{C_m \Omega}$$

*h: angular momentum of fluid layers*

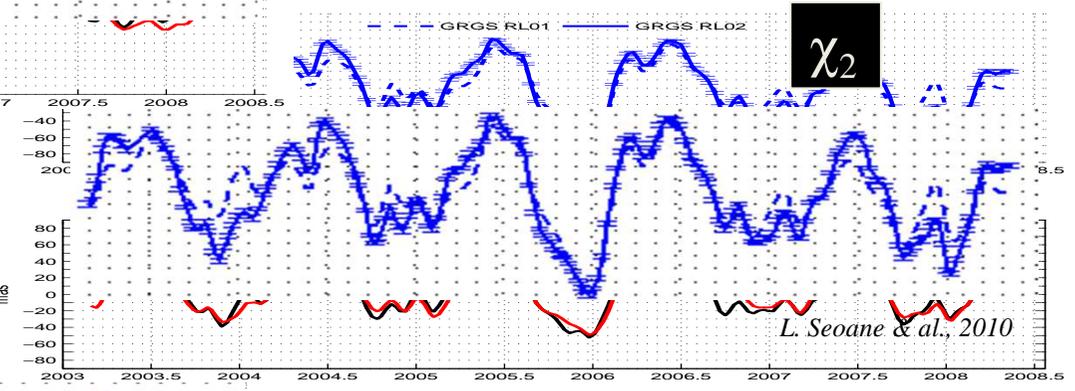
# Mass excitation from C21/S21



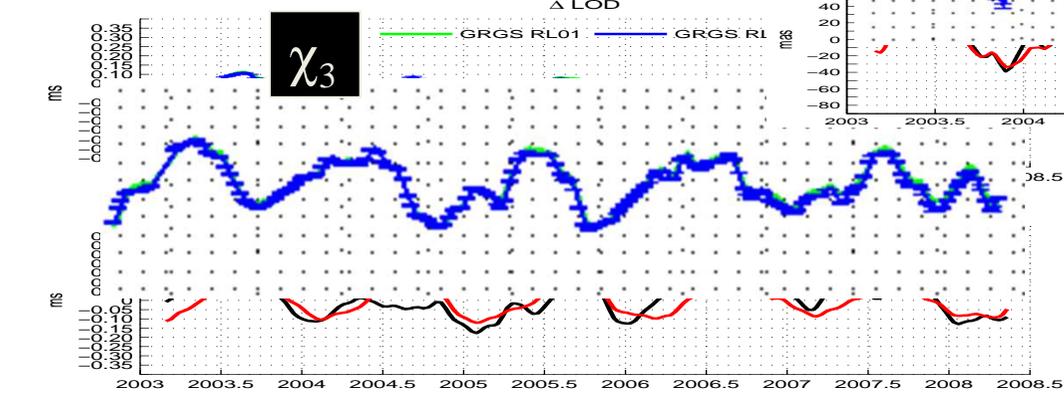
$$\chi^{mass} = \chi_1^{mass} + i \chi_2^{mass} = -\sqrt{\frac{5}{3}} \frac{MR^2}{(C-A)} (\Delta \bar{C}_{21} + i \Delta \bar{S}_{21})$$

$$\chi_3^{mass} = -\frac{2\sqrt{5}}{3} \frac{MR^2}{C_m} \Delta \bar{C}_{20}$$

LOD rms from GPS/C04:  $\sim 0.008$  ms  $\leftrightarrow$   $\sim 4$  mm



Pole rms from GPS/C04:  $\sim 0.03$  mas  $\leftrightarrow$   $\sim 1$  mm



G/LOD-WC: geodetic excitation – motion excitation

$$\chi^{geodetic} - \chi^{motion} \leftrightarrow (1 + k'_2) \chi^{mass}$$

PAOH: mass excitation from atmosphere/ocean/hydrology models (NCEP + ECCO + GLDAS)

EIGEN-GRGS-RL02: mass excitation from GRACE/Lageos + models (ECMWF + MOG2D)

# Plan

- *La modélisation harmonique*
- *Les modèles globaux de champ de gravité*
- *GM et bas degrés*
- ***Modélisations alternatives***
- *Les corrections de gravité*
- *Evolution des Conventions*

# Harmoniques ellipsoïdales

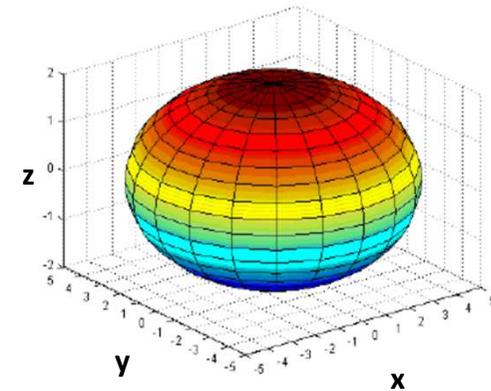
Soit l'équation de l'ellipsoïde:  $\frac{x^2}{a^2} + \frac{y^2}{a^2 - h^2} + \frac{z^2}{a^2 - k^2} = 1$  avec  $h^2 = a^2 - b^2$ ,  $k^2 = a^2 - c^2$ ,  $0 < h < k$

Solutions de l'équation de Lamé (solution de l'équation de Laplace en coordonnées ellipsoïdales):

- cas oblong ( $\rho = a$ ,  $a = b$ ,  $h = 0$ ,  $k^2 = a^2 - c^2$ )

$$\begin{cases} x = \sqrt{\rho^2 - k^2} \cos \varphi \cos \lambda \\ y = \sqrt{\rho^2 - k^2} \cos \varphi \sin \lambda \\ z = \rho \sin \varphi \end{cases}$$

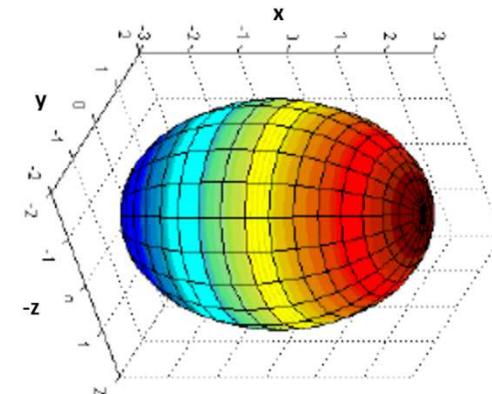
$$U = GM \sum_{l=0}^{\infty} \sum_{m=0}^n \frac{Q_{lm}(i(\rho^2 - k^2))}{Q_{lm}(ia^2)} P_{lm}(\sin \varphi) (A_{lm} \cos m\lambda + B_{lm} \sin m\lambda)$$



- cas prolong ( $\rho = b$ ,  $b = c$ ,  $h^2 = a^2 - b^2$ ,  $k = h$ )

$$\begin{cases} x = \rho \cos \varphi \cos \lambda \\ y = \sqrt{\rho^2 - h^2} \cos \varphi \sin \lambda \\ z = \sqrt{\rho^2 - h^2} \sin \varphi \end{cases}$$

$$U = GM \sum_{l=0}^{\infty} \sum_{m=0}^n \frac{Q_{lm}(\rho)}{Q_{lm}(a)} P_{lm}(\sin \varphi) (A_{lm} \cos m\lambda + B_{lm} \sin m\lambda)$$



$Q_{nm}$  : fonctions associées de Legendre de seconde espèce

# Modélisation polyhédrique

Par transformation de l'intégrale de volume en intégrale de surface (densité  $\rho$  constante):

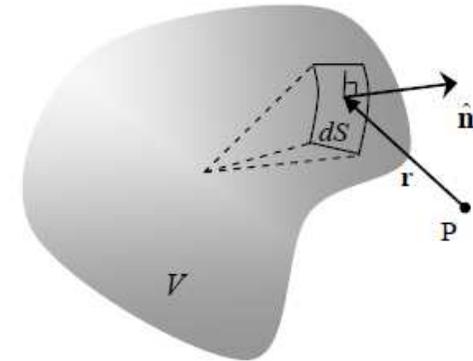
$$V = \iiint_{\text{Volume}} \frac{G}{r} dm = G \iiint_{\text{Volume}} \frac{\rho}{r} dV = \frac{G\rho}{2} \iint_{\text{Surface}} \vec{n} \cdot \vec{r} dS$$

Polyèdre à faces triangulaires planes (densité constante):

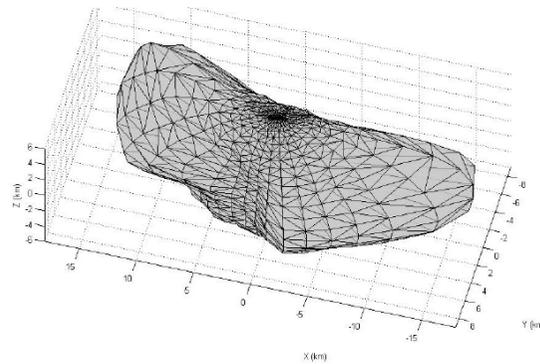
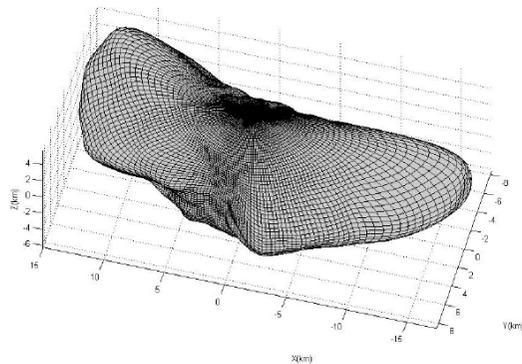
$$V = \frac{G\rho}{2} \iint_{\text{Surface}} \vec{n} \cdot \vec{r} dS = \frac{G\rho}{2} \sum_{i=1}^n \iint_{\text{Triangle}} \vec{n} \cdot \vec{r} dS$$

Polyèdre à prismes droits (densité  $\rho_i$  variable):

$$V = G \iiint_{\text{Volume}} \frac{\rho}{r} dV = \frac{G}{2} \sum_{i=1}^n \rho_i \iint_{\text{Prisme}} \vec{n} \cdot \vec{r} dS$$



La surface différentielle  $dS$  en relation avec le potentiel au point P



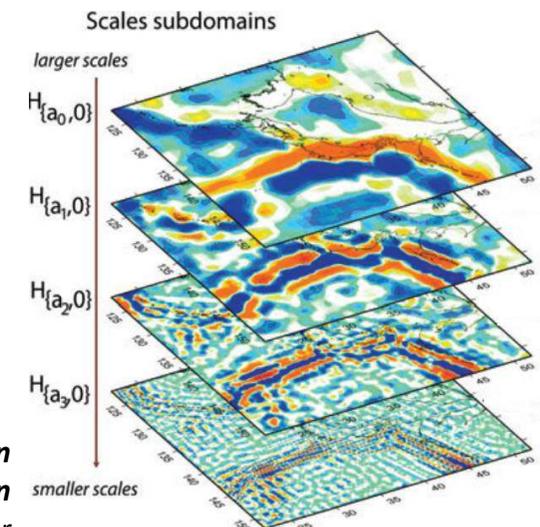
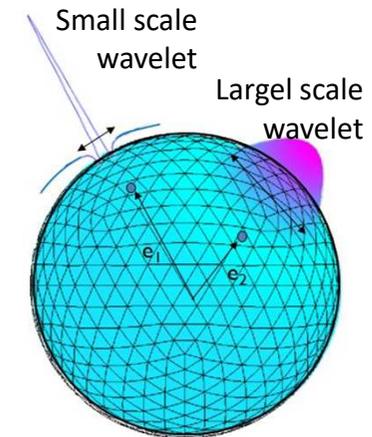
Xuanyu Hu, Thesis, 2012, OSU

Balmino G., *Gravitational potential harmonics from the shape of an homogeneous body*,  
*Celestial Mechanics and Dynamical Astronomy*, vol.60, pp.331-364, 1994

# Ondelettes de Poisson

*Des fonctions localisées spatialement et spectralement*  
(Holschneider et al, 2003)

- Intégrer au sein d'un modèle unique des observations distribuées localement, avec différentes résolutions
- Augmenter localement la résolution d'un modèle global ou régional (zoom)
- Une optimisation locale du rapport signal/bruit
- Estimation itérative du modèle par sous-domaines
- Modélisation multi-échelles 4D



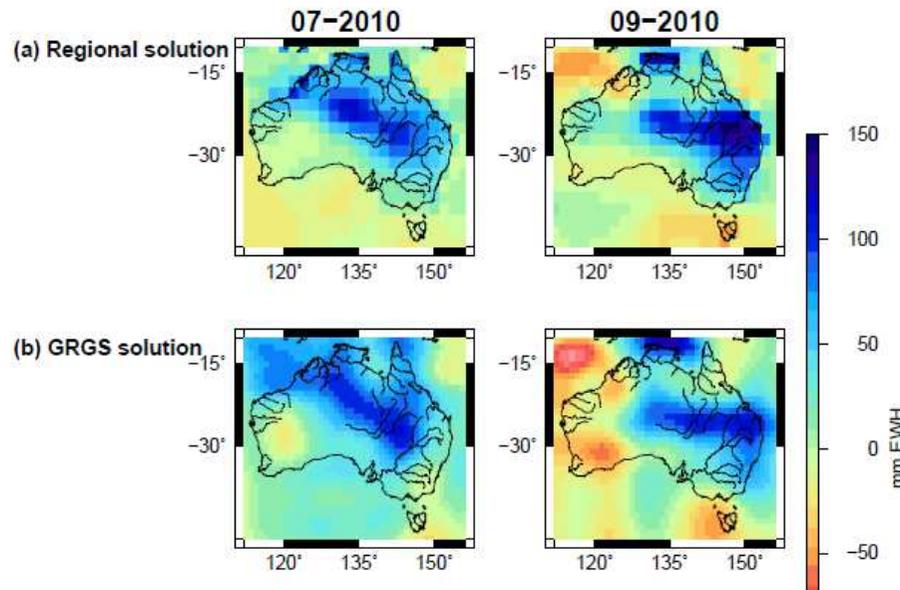
**Wavelet modelling of the gravity field by domain decomposition  
methods: an example over Japan**

Isabelle Panet, Yuki Kuroishi and Matthias Holschneider  
*Geophys. J. Int.* (2011) **184**, 203–219

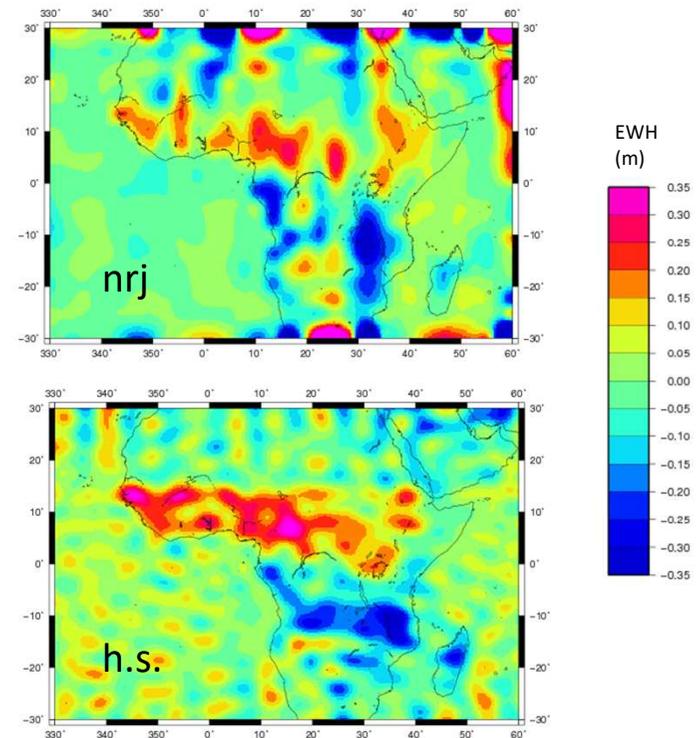
# Approche par l'intégrale de l'énergie

Cas appliqué aux mesures GRACE de vitesse inter-satellite

$$\underbrace{\Delta \left( \frac{1}{2} \vec{v}^2 \right)}_{E_{kin}} + \underbrace{\Delta (-U)}_{E_{pot}} = \underbrace{\int_{\Delta t} \vec{f} \cdot \vec{v} dt}_{W_{nc \text{ forces}}} + \underbrace{\int_{\Delta t} \vec{g} \cdot (\vec{\Omega} \wedge \overline{OM}) dt}_{W_{nc \text{ grav}} = \int -\frac{\partial U}{\partial t} dt}$$



Regional GRACE-based estimates of water mass variations over Australia : validation and interpretation  
L. Seoane, G. Ramillien, F. Frappart, M. Leblanc  
Hydrol. Earth Syst. Sci., 2013



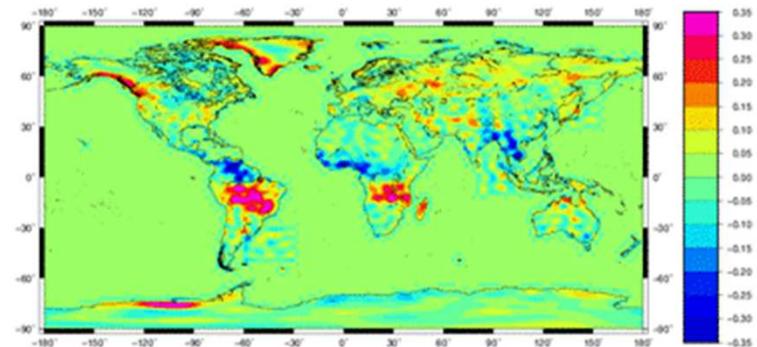
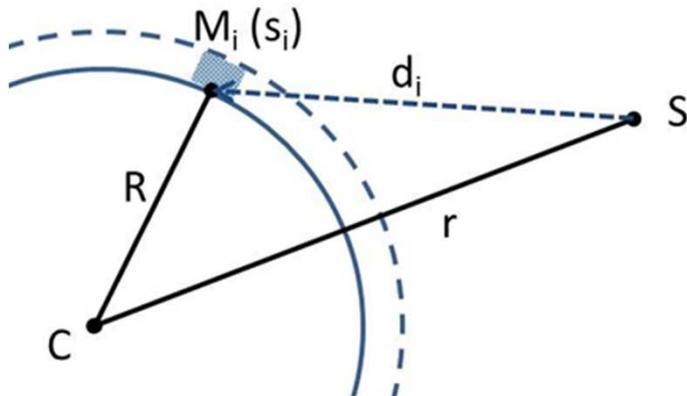
Comparaison d'une solution mensuelle harmonique sphérique vs. Intégrale de l'énergie sur l'Afrique en octobre 2010  
S. Bourgogne, 2017

# Masses surfaciques (mascons)

L'accélération gravitationnelle de masses surfaciques est calculée directement par l'équation de Newton dans le repère tournant lié au corps central:

$$\ddot{\vec{r}} = -G\rho \sum_i \frac{s_i h_i}{d_i^3} \vec{d}_i$$

- où
- $G$  : constante gravitationnelle
  - $\rho$  : densité (ex.: 1000 kg m<sup>-2</sup> pour l'eau)
  - $s_i$  : surface du  $i^e$  élément
  - $h_i$  : hauteur due  $i^e$  élément
  - $\vec{d}_i$  : vecteur du  $i^e$  élément au satellite S exprimé dans le repère tournant lié au corps central
  - $d_i$  : distance au  $i^e$  élément au satellite S



Solution mascons

# Approche directe

Intégration numérique (du second ordre) de l'équation fondamentales de la dynamique en repère inertiel:

$$\bar{r} = \iint \ddot{r} dt \quad \text{avec} \quad \ddot{r} = \sum_n (\bar{r}, \dot{\bar{r}}, p_i)$$

À partir d'un jeu de conditions initiales (orbite et paramètres des modèles dynamiques et géométriques):

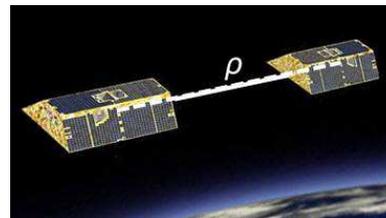
$$\bar{r}_0, \dot{\bar{r}}_0, p_i = (\bar{C}_{lm}, \bar{S}_{lm}, p_{dyn})$$

Formation de la quantité calculée ( $Q_{calc}$ ):

$$\rho = \| \bar{r}_B - \bar{r}_A \|$$

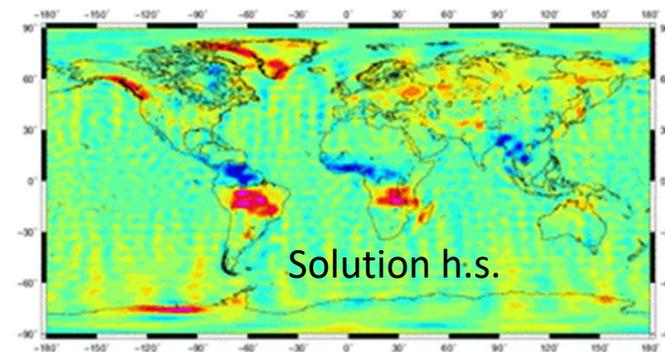
$$\dot{\rho} = \frac{(\bar{v}_B - \bar{v}_A) \cdot (\bar{r}_B - \bar{r}_A)}{\rho}$$

$$\ddot{\rho} = \frac{(\bar{a}_B - \bar{a}_A) \cdot (\bar{r}_B - \bar{r}_A) + \| \bar{v}_B - \bar{v}_A \|^2 - \dot{\rho}^2}{\rho}$$



Formation des résidus de mesures et ajustement des paramètres par inversion de l'équation de Taylor au premier ordre:

$$\Delta Q = Q_{obs} - Q_{calc} = \frac{\partial Q}{\partial \bar{r}_0} \Delta \bar{r}_0 + \frac{\partial Q}{\partial \dot{\bar{r}}_0} \Delta \dot{\bar{r}}_0 + \sum_i \frac{\partial Q}{\partial p_i} \Delta p_i$$

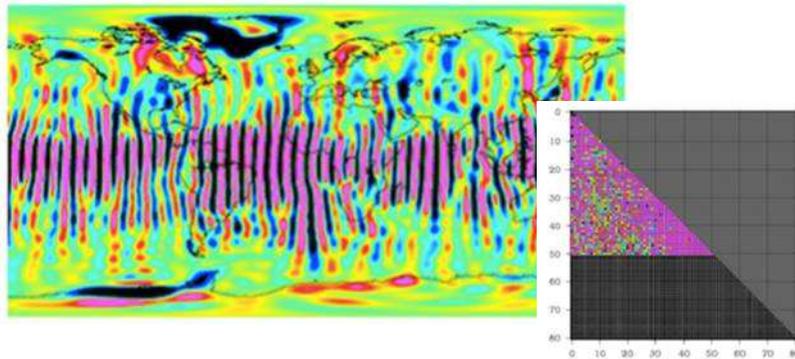


Solutions mensuelles GRGS, Mars 2007

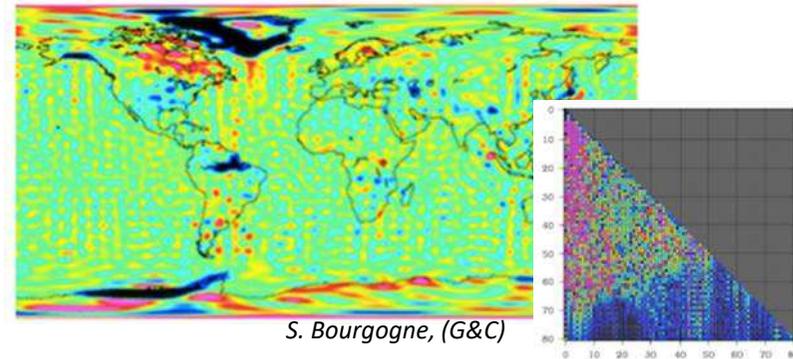
# Le besoin de filtrage

- **Filtrage par valeurs propres**

Inversion de 2500 coefficients (champ 50\*50)

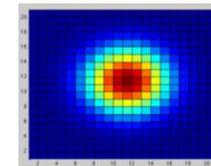


Inversion de 2500 valeurs propres (champ 80\*80)

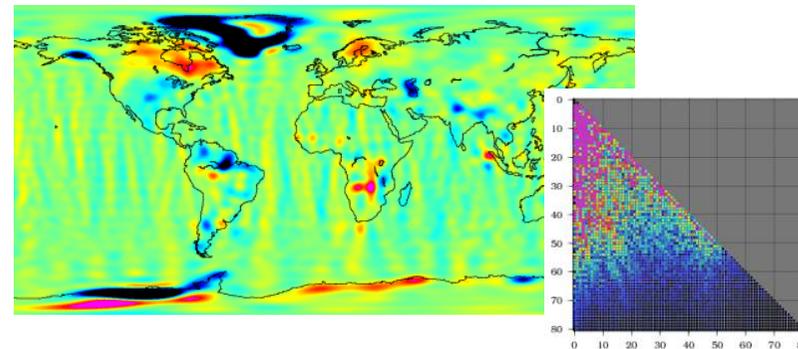


S. Bourgogne, (G&C)

- **Filtre gaussien** (produit de convolution du signal par un filtre 2D, en général de rayon 400 km pour les modèles GRACE)



- **Filtres DDK** (filtre anisotrope - convolution par matrice de covariance synthétique à partir de la cartographie des observations inter-satellites)



filtrage DDK5  
J. Kusche (Univ. Bonn)

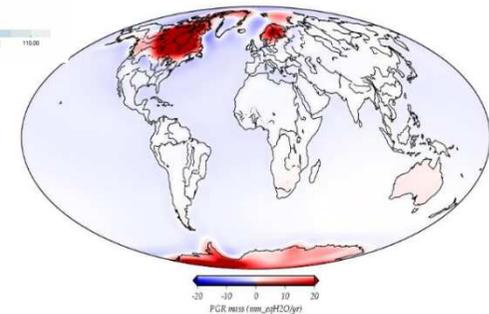
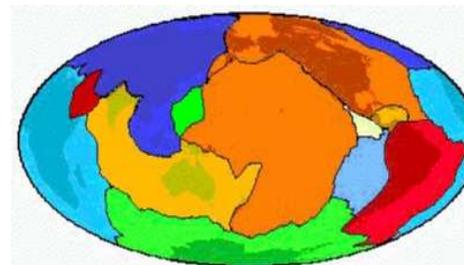
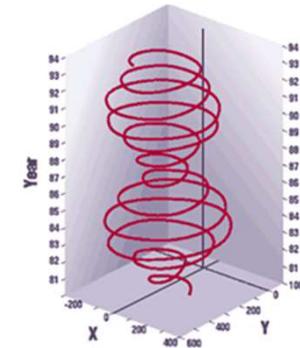
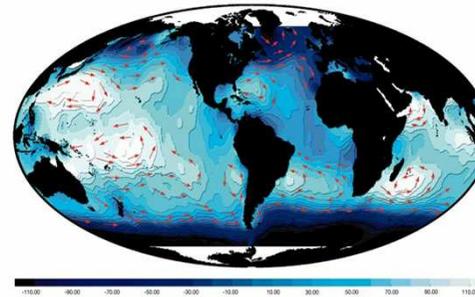
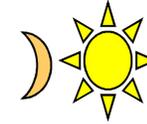
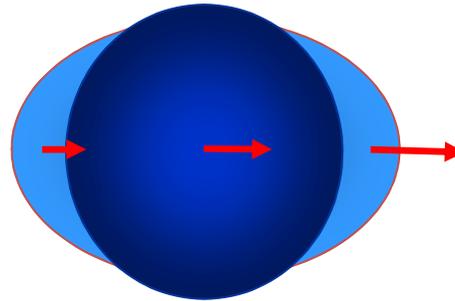
Problème du filtrage a posteriori: « mange du signal »

# Plan

- *La modélisation harmonique*
- *EGM2008 et les modèles plus récents*
- *GM et bas degrés*
- *Modélisations alternatives*
- ***Les corrections de gravité***
- *Evolution des Conventions*

# Les variations du champ de gravité

- les marées terrestres
- les marées océaniques
- les marées polaires terrestre et océanique
- les courants océaniques
- la pression atmosphérique
- l'hydrologie
- le rebond post-glaciaire
- la tectonique
- les tremblements de Terre



## 6.2.2 Treatment of the permanent tide

In the case of a “zero tide” geopotential model, the model of tidal effects to be added should not once again contain a time independent part. One must not then use the expression (6.6) as it stands for modeling  $\Delta\bar{C}_{20}$ ; its permanent part must first be restored. This is Step 3 of the computation, which provides  $\Delta\bar{C}_{20}^{zt}$ , to be used with a “zero tide” geopotential model.

$$\Delta\bar{C}_{20}^{zt} = \Delta\bar{C}_{20} - \Delta\bar{C}_{20}^{perm} \quad (6.13)$$

where  $\Delta\bar{C}_{20}$  is given by Equation (6.6) and where  $\Delta\bar{C}_{20}^{perm}$  is the time-independent part:

$$\Delta\bar{C}_{20}^{perm} = A_0 H_0 k_{20} = (4.4228 \times 10^{-8})(-0.31460)k_{20}. \quad (6.14)$$

In the case of EGM2008, the difference between the zero-tide and tide-free value of  $C_{20}$  is  $-4.1736 \times 10^{-9}$ . Assuming the same values for  $A_0$ ,  $H_0$  and  $k_{20}$ , the tide-free value of  $C_{20}$  corresponding to Table 6.2 would be  $-0.48416531 \times 10^{-3}$ .

The use of “zero tide” values and the subsequent removal of the effect of the permanent tide from the tide model is presented for consistency with the 18th IAG General Assembly Resolution 16.

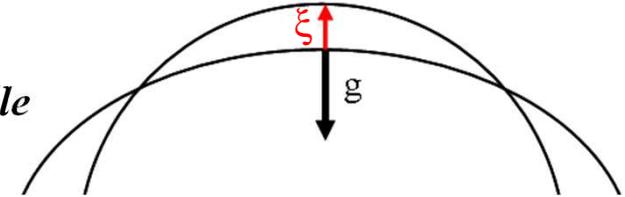
# Le potentiel de marée terrestre : théorie de Love (1909)

**Equipotentielle de surface :**  $U(r, \varphi, \lambda) = C$

Le potentiel de marée introduit une **déformation de l'équipotentielle**

telle que :  $U(r+\xi, \varphi, \lambda) + U_p(r, \varphi, \lambda) = C$

soit :  $U(r, \varphi, \lambda) + \frac{\partial U}{\partial r} \xi + U_p(r, \varphi, \lambda) = C$  , d'où :  $\xi = -\frac{U_p}{\frac{\partial U}{\partial r}} = \frac{U_p}{g}$



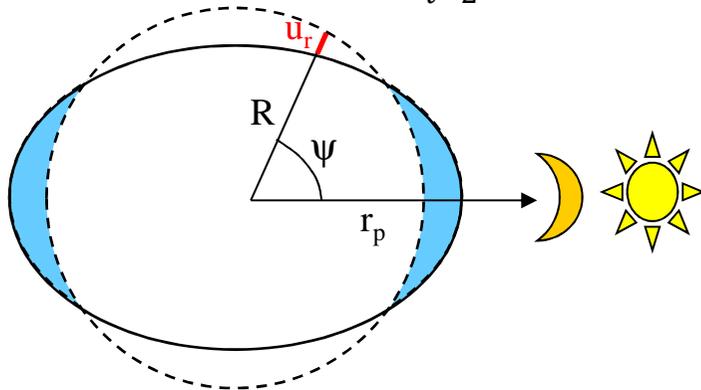
$U$  : potentiel terrestre

$U_p$  : potentiel excitateur

Dans une hypothèse de Terre élastique, le déplacement de la croûte terrestre doit être proportionnel à l'excitation.

**Excitation :**  $U_p = \frac{Gm_p}{R} \sum_{l=2}^3 \left(\frac{R}{r_p}\right)^{l+1} P_{l0}(\cos\Psi)$

**Déplacement :**  $u_r = \sum_{l=2}^3 h_l \frac{U_{pl}}{g}$  ,  $h_l$  : nombre de Love (sans dimension) de déformation verticale



L'incrément de potentiel externe engendré par la déformation élastique de la Terre est proportionnel au potentiel d'excitation. Il s'exprime au degré 2 à la surface de la Terre:

$$\Delta U_{20} = k_2 U_p(R)$$

$k_2$  : nombre de Love (sans dimension) de potentiel

## Le potentiel de marée permanente

Par trigonométrie sphérique entre  $P(r_p, \delta_p, \alpha_p)$  et  $M(r, \varphi, \lambda)$ , on établit la relation :

$$\cos \psi = \sin \varphi \sin \delta_p + \cos \varphi \cos \delta_p \cos(\lambda - \alpha_p)$$

et  $P_{20}(\cos \psi)$  s'explique :

$$\begin{aligned} P_{20}(\cos \psi) &= P_{20}(\sin \varphi) P_{20}(\sin \delta_p) \\ &+ \frac{1}{3} P_{21}(\sin \varphi) P_{21}(\sin \delta_p) \cos(\lambda - \alpha_p) \\ &+ \frac{1}{12} P_{22}(\sin \varphi) P_{22}(\sin \delta_p) \cos(2(\lambda - \alpha_p)) \end{aligned}$$

soit :

$$\begin{aligned} P_{20}(\cos \psi) &= \frac{1}{4} (3 \sin^2 \varphi - 1) (3 \sin^2 \delta_p - 1) \\ &+ \frac{3}{4} \sin 2\varphi \sin 2\delta_p \cos(\lambda - \alpha_p) \\ &+ \frac{3}{4} \cos^2 \varphi \cos^2 \delta_p \cos(2(\lambda - \alpha_p)) \end{aligned}$$

Dans le cas de la lune ou du soleil, la moyenne de  $P_{20}(\sin \delta_p)$  s'exprime :

$$\left[ P_{20}(\sin \delta_p) \right]_{mean} \approx \frac{3}{4} \sin^2 \varepsilon_0 - \frac{1}{2} = \frac{1}{2} \left( P_{20}(\sin \varepsilon_0) - \frac{1}{2} \right), \quad \varepsilon_0 = 23^\circ 26' 21.4'', \text{ inclinaison de l'écliptique}$$

$$\text{D'où: } [\Delta U_2]_{perm.} = \sum_{p=L,S} \frac{Gm_p}{R} k_2 \left( \frac{R}{r_p} \right)^3 P_{20}(\cos \psi) \approx R^2 \left( \frac{Gm_L}{r_L^3} + \frac{Gm_S}{r_S^3} \right) k_2 P_{20}(\sin \varphi) \left( \frac{3}{4} \sin^2 \varepsilon_0 - \frac{1}{2} \right)$$

$$\Rightarrow [\Delta C_{20}]_{perm.} \approx \frac{R^3}{GM} \left( \frac{Gm_L}{r_L^3} + \frac{Gm_S}{r_S^3} \right) k_2 \left( \frac{3}{4} \sin^2 \varepsilon_0 - \frac{1}{2} \right) = \frac{-.4180 \cdot 10^{-8}}{\sqrt{5}}$$

# IERS Conventions

## Modèle "Tide Free"

néglige tout impact de la lune et du soleil, comme si ces corps n'existaient pas.

Le potentiel de marée terrestre doit contenir tous les termes, permanents ( $U_{p0}$ ) et périodiques.

## Modèle "Zero Tide"

Le potentiel terrestre tient compte de la déformation géométrique de marée permanente ( $-h_2U_{p0}$ ).

## Modèle "Mean Tide"

Le potentiel terrestre tient compte de la déformation + du potentiel induit de marée permanente ( $-h_2U_{p0}+k_2U_{p0}$ ).

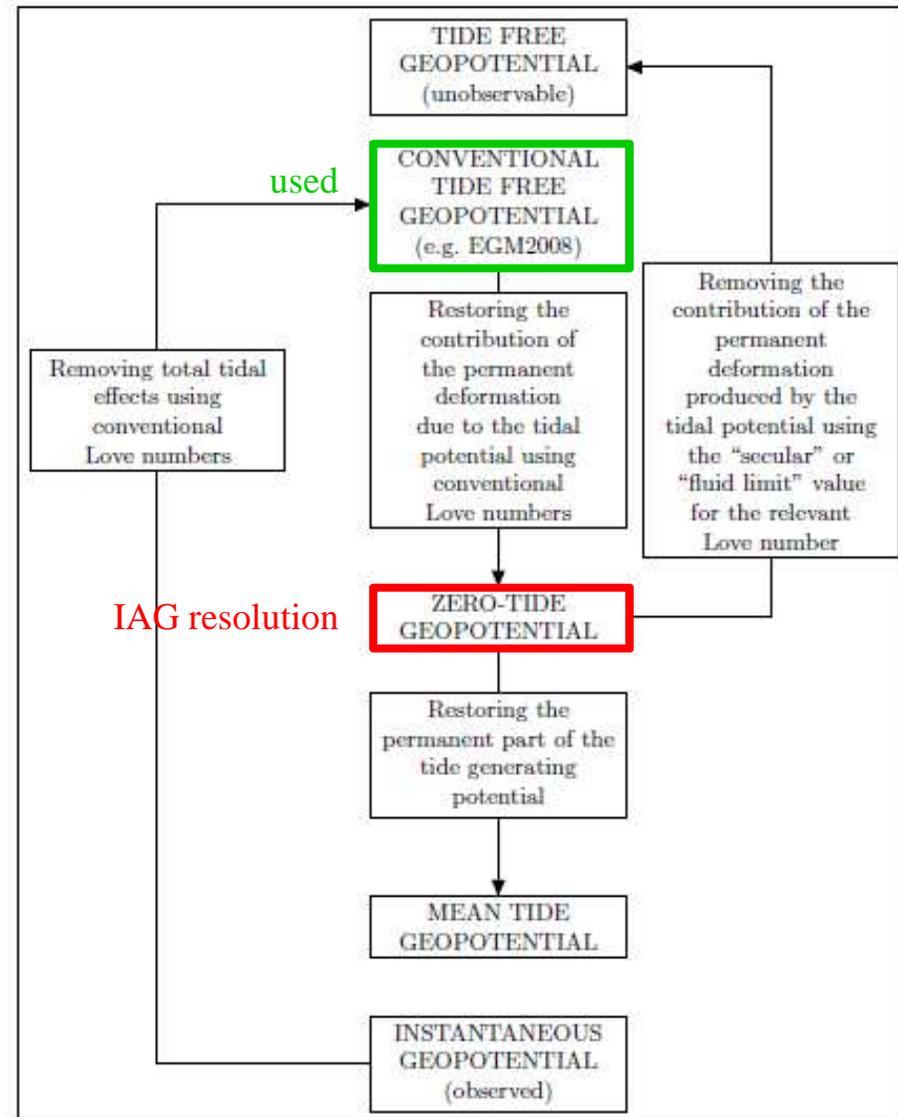


Figure 1.2: Treatment of observations to account for tidal effects in the geopotential (see Chapter 6).

Les modèles globaux (tels EIGEN) sont classiquement donnés en système "Tide Free", quoique la résolution 16 de l'IAG (à la 18<sup>e</sup> Assemblée Générale, 1984) recommande le système "Zero Tide".

# La déformation de marée permanente sur le géoïde

**In mean tide system:**

$$a_e = 6\,378\,136.70 \text{ m}$$

$$1/f = 298.25227$$

**In zero tide system:**

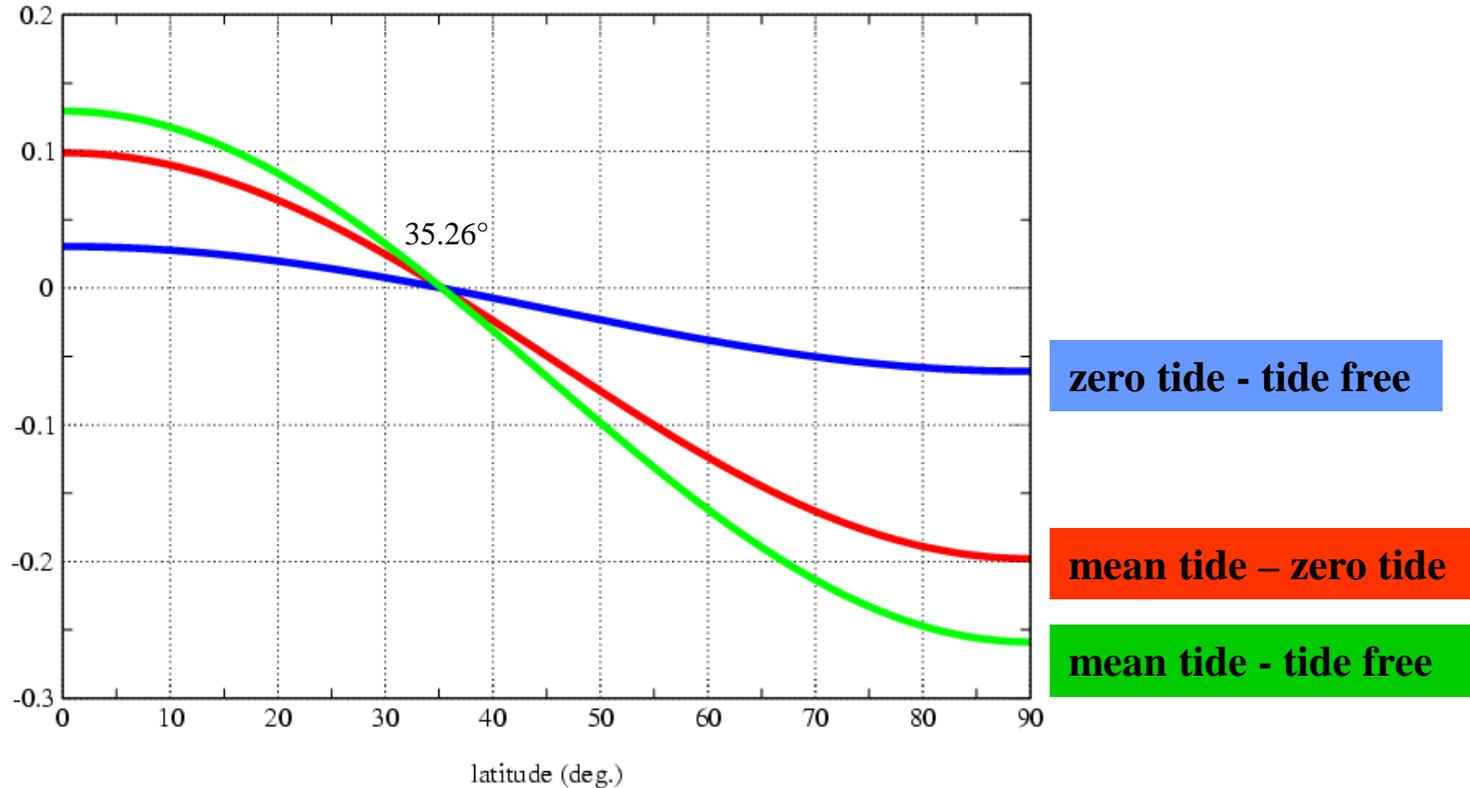
$$a_e = 6\,378\,136.60 \text{ m}$$

$$1/f = 298.25642$$

**In tide free system:**

$$a_e = 6\,378\,136.57 \text{ m}$$

$$1/f = 298.25767$$



$$R[\Delta C_{20}]_{perm} = -.198 \text{ m} \rightarrow \begin{cases} N_{mean} - N_{zero} = -.198 (1.5 \sin^2 \varphi - .5) \\ N_{zero} - N_{free} = -.198 k_2 (1.5 \sin^2 \varphi - .5) \\ \hline N_{mean} - N_{free} = -.198 (1+k_2) (1.5 \sin^2 \varphi - .5) \end{cases}$$

## Consideration about the Earth's figure axis

Values for the  $C_{21}$  and  $S_{21}$  coefficients are included in the conventional geopotential model. The  $C_{21}$  and  $S_{21}$  coefficients describe the position of the Earth's figure axis. When averaged over many years, the figure axis should closely coincide with the observed position of the rotation pole averaged over the same time period. Any differences between the averaged positions of the mean figure and the mean rotation pole would be due to long-period fluid motions in the atmosphere, oceans, or Earth's fluid core (Wahr, 1987; 1990). At present, there is no independent evidence that such motions are important. The conventional values for  $C_{21}(t)$  and  $S_{21}(t)$  are intended to give a mean figure axis that corresponds to the mean pole position consistent with the terrestrial reference frame defined in Chapter 4.

This choice for  $C_{21}$  and  $S_{21}$  is realized as follows. First, to use the gravitational potential coefficients to solve for a satellite orbit, it is necessary to rotate from the Earth-fixed frame, where the coefficients are pertinent, to an inertial frame, where the satellite motion is computed. This transformation between frames should include polar motion. We assume the polar motion parameters used are relative to the IERS Reference Pole. Then the values

$$\begin{aligned}C_{21}(t) &= \sqrt{3}\bar{x}_p(t)C_{20} - \bar{x}_p(t)C_{22} + \bar{y}_p(t)S_{22} \\ \bar{S}_{21}(t) &= -\sqrt{3}\bar{y}_p(t)\bar{C}_{20} - \bar{y}_p(t)\bar{C}_{22} - \bar{x}_p(t)\bar{S}_{22}\end{aligned}\tag{6.5}$$

where  $\bar{x}_p(t)$  and  $\bar{y}_p(t)$  (in radians) represent the IERS conventional mean pole (see Section 7.1.4), provide a mean figure axis which coincides with the mean pole consistent with the TRF defined in Chapter 4. Any recent value of  $\bar{C}_{20}$ ,  $\bar{C}_{22}$  and  $\bar{S}_{22}$  is adequate for  $10^{-14}$  accuracy in Equation (6.5), e.g. the values of the present conventional model ( $-0.48416948 \times 10^{-3}$ ,  $2.4393836 \times 10^{-6}$  and  $-1.4002737 \times 10^{-6}$  respectively) can be used.

The models for the low degree terms are generally consistent with the past long-term trends, but these are not strictly linear in reality. There may be decadal variations that are not captured by the models. In addition, they may not be consistent with more recent surface mass trends due to increased ice sheet melting and other results of global climate change.

<sup>4</sup><http://www.igg.uni-bonn.de/apmg/fileadmin/itg-grace03.html>

# Le pôle moyen IERS

**IERS conventions 2010:** polynôme cubique sur la période 1976-2010 ( $t_0 = 2000$ ):

$$\bar{x}_p = 55.974 \text{ mas} + (t - t_0) 1.8243 \text{ mas/yr} + (t - t_0)^2 0.18413 \text{ mas/yr}^2 + (t - t_0)^3 0.007024 \text{ mas/yr}^3$$

$$\bar{y}_p = 346.346 \text{ mas} + (t - t_0) 1.7896 \text{ mas/yr} - (t - t_0)^2 0.10729 \text{ mas/yr}^2 - (t - t_0)^3 0.000908 \text{ mas/yr}^3$$

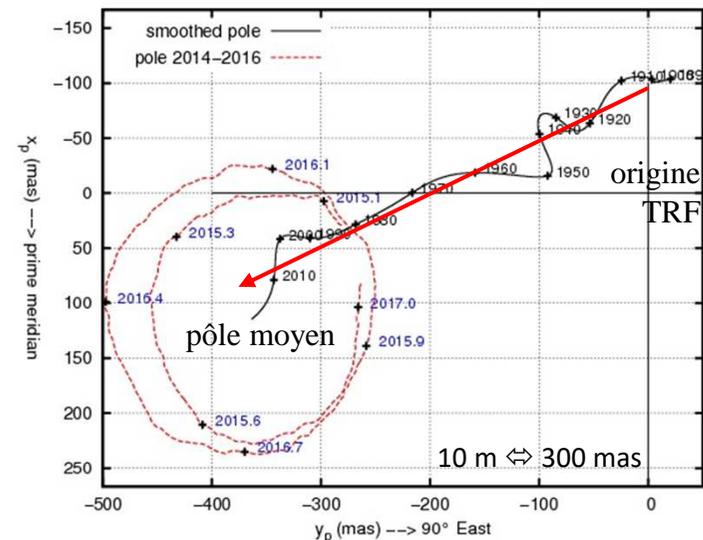
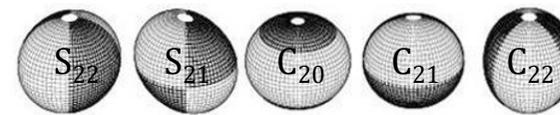
Complété par une extrapolation linéaire après 2010 ( $t_0 = 2000$ ):

$$\bar{x}_p = 23.513 \text{ mas} + (t - t_0) 7.6141 \text{ mas/yr}$$

$$\bar{y}_p = 358.891 \text{ mas} - (t - t_0) 0.6287 \text{ mas/yr}$$

Les modèles récents de champ de gravité contiennent des termes  $C_{21}/S_{21}$ . Toutefois, lorsque ces termes sont manquants, il est possible de les introduire géométriquement en assimilant l'axe du pôle moyen (PM) à l'axe d'inertie en Z (où ces coefficients sont nuls par définition) par rotation du potentiel de degré 2. Il apparait alors des termes  $C_{21}/S_{21}$  dans le TRF tels que:

$$\begin{aligned} \bar{C}_{21} &= \sqrt{3} \bar{x}_p \bar{C}_{20} - \bar{x}_p \bar{C}_{22} + \bar{y}_p \bar{S}_{22} \\ \bar{S}_{21} &= -\sqrt{3} \bar{y}_p \bar{C}_{20} - \bar{y}_p \bar{C}_{22} - \bar{x}_p \bar{S}_{22} \end{aligned}$$



## Remarques:

- les rotations de TRF à J2000 (référentiel du calcul d'orbite) sont globales et conformes aux Conventions (SOFA);
- les modèles variables GRACE contiennent des termes  $C_{21}/S_{21}$  ajustés, ce qui rend cette correction caduque;
- les coordonnées du pôle moyen sont utilisées pour calculer la déformation visco-élastique de marée polaire.

# La correction de système de référence terrestre

**Explication au premier ordre:**  $U_2 = \frac{GM}{r} \left(\frac{a_e}{r}\right)^2 \bar{C}_{20} \bar{P}_{20}(\sin\varphi)$  avec  $\bar{P}_{20}(\sin\varphi) = \sqrt{5} \frac{3\sin^2\varphi - 1}{2}$

Soit en appliquant la transformation du pôle TRF au pôle moyen:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{TRF} = \begin{pmatrix} 1 & 0 & -\bar{x}_p \\ 0 & 1 & \bar{y}_p \\ \bar{x}_p & -\bar{y}_p & 1 \end{pmatrix} \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}_{PM}$$

il vient au premier ordre:

$$\sin^2\varphi = \sin^2\varphi_0 + 2\sin\varphi_0\cos\varphi_0(\bar{x}_p\cos\lambda_0 - \bar{y}_p\sin\lambda_0)$$

et  $\bar{P}_{20}$  se développe en:

$$\bar{P}_{20}(\sin\varphi) = \bar{P}_{20}(\sin\varphi_0) + \sqrt{3}\bar{P}_{21}(\sin\varphi_0)(\bar{x}_p\cos\lambda_0 - \bar{y}_p\sin\lambda_0)$$

D'où au 1<sup>er</sup> ordre, l'apparition de termes  $\bar{C}_{21}/\bar{S}_{21}$  dans le TRF:

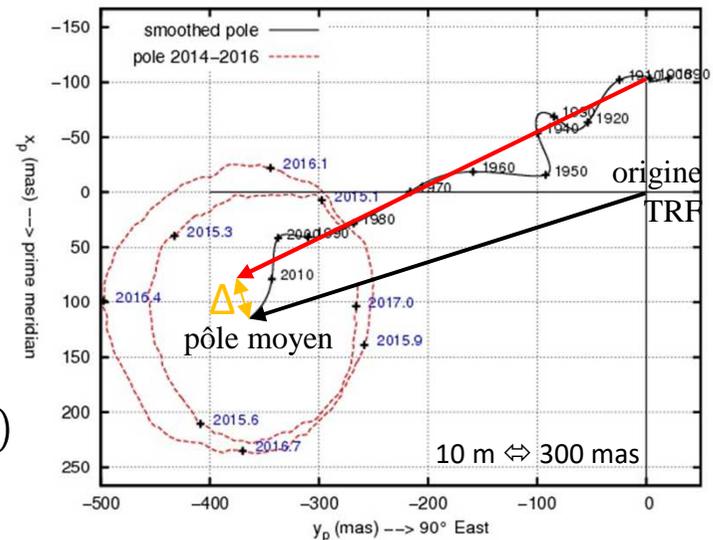
$$\bar{C}_{21} = \sqrt{3} \bar{x}_p \bar{C}_{20} ; \quad \bar{S}_{21} = -\sqrt{3} \bar{y}_p \bar{C}_{20}$$

## Recommandation:

- Appliquer la correction pour les modèles dépourvus de termes (annuels)  $C_{21}/S_{21}$

## Question (relative à la marée polaire):

- Quelle définition du pôle moyen se rapproche le plus du pôle d'inertie?
- Comment introduire une réponse élastique long terme sur les écarts de pôle moyen?



Valeurs du pôle moyen 2010.5 et des coefficients  $C_{21}/S_{21}$  ajustés (en vert) ou transformés (en rouge)

	$\bar{x}_p$ (")	$\bar{y}_p$ (")	$\bar{C}_{21}(10^{-9})$	$\bar{S}_{21}(10^{-9})$
IERS (2010.5)	.082	.345	→ -0.337	1.400
EIGEN-GRGS RL03 annuel	.084	.350	← -0.346	1.418
EGM2008	.050	.341	← -0.206	1.384

# Plan

- *La modélisation harmonique*
- *EGM2008 et les modèles plus récents*
- *GM et bas degrés*
- *Modélisations alternatives*
- *Les corrections de gravité*
- *Evolution des Conventions*

## Evolution des recommandations

- La valeur conventionnelle du GM est-elle à revoir?
- Faut-il introduire des coefficients de degré 1?
- Clarifier la recommandation « tide free » versus « zero tide »
- Quelles variations du pôle moyen ( $C_{21}/S_{21}$ ) prendre en compte?
- Quel(s) modèle(s) de champ de gravité statique/variable utiliser?
- Assurer la cohérence des conventions entre les « piliers » de la géodésie !

# COST-G

## International **C**ombination **S**ervice for **T**ime-variable **G**ravity Field Solutions (IGFS/IAG Service proposal)

### *Products and Goals*

COST-G provides:

- Combined gravity field solutions in SH coefficients (Level-2 products) derived from a weighted combination of individual normal equations (NEQs) generated by the different ACs,
- Spatial grids (Level-3 products) of the Combined Solutions for hydrological, oceanic and polar ice sheets applications.

COST-G performs a quality control of the combined models as well as of the individual AC contributions before combination.

### *Permanent components*

COST-G accomplishes its objectives through the following permanent components:

- Analysis Centers: AIUB, CNES/GRGS, CSR, GFZ, IFG, JPL, LUH, TUG...
- Analysis Center Coordinator: AIUB
- Validation Center: CNES/GRGS

# Cohérence entre les domaines de la géodésie spatiale

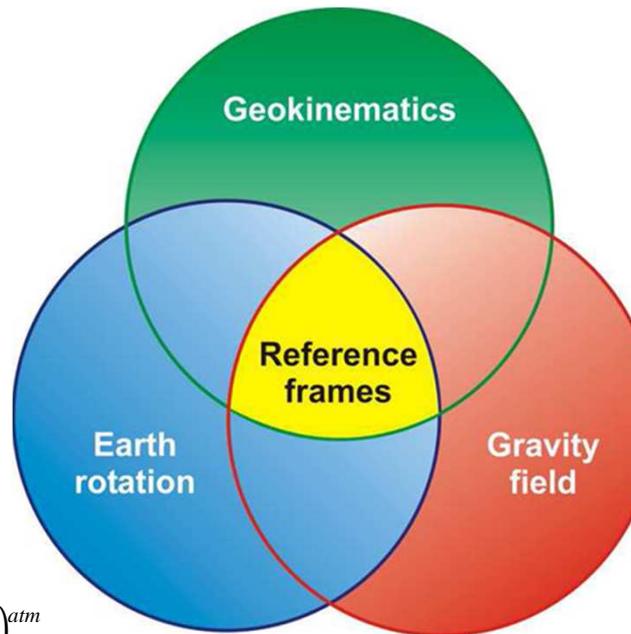
Déformations (Love)  $u_r = \sum_{\lambda=2}^3 \frac{h_\lambda}{g} \Delta U_\lambda, u_\varphi = \sum_{\lambda=2}^3 \frac{\lambda_\lambda}{g} \frac{\partial \Delta U_\lambda}{\partial \varphi}, u_\lambda = \sum_{\lambda=2}^3 \frac{\lambda_\lambda}{g \cos \varphi} \frac{\partial \Delta U_\lambda}{\partial \lambda}$

$$h(\varphi, \lambda) = \sum_{l=1}^{\infty} h'_l \frac{U_l}{g} = \frac{4\pi GR}{g} \sum_{l=1}^{\infty} \frac{h'_l}{2l+1} q_l(\varphi, \lambda)$$

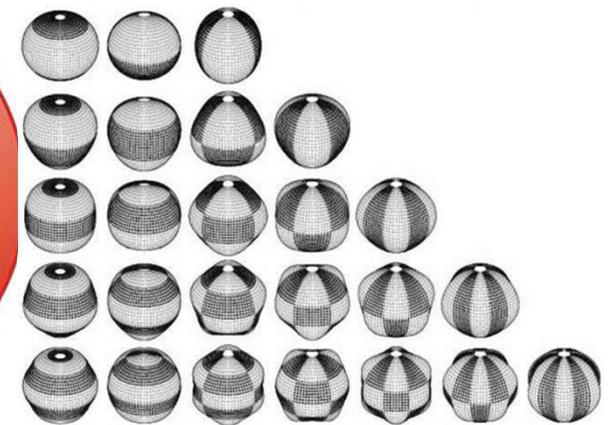
Rotation (Euler-Liouville)

$$\frac{\Delta LOD}{LOD_{ref}} = \frac{h_3}{C_m \Omega} - (1+k'_2) \frac{2}{3C_m} MR^2 \sqrt{5} \Delta \bar{C}_{20}^{atm}$$

$$p + \frac{i}{\sigma_0} \dot{p} = \frac{k_0}{k_0 - k_2} \frac{h}{\Omega(C_m - A_m)} - \sqrt{\frac{5}{3}} \frac{k_0}{k_0 - k_2} \frac{MR^2}{C_m - A_m} (\bar{C}_{21} + i\bar{S}_{21})^{atm}$$



Potentiel (Legendre)



$$U = \frac{GM}{a_e} \sum_{n=2}^{\infty} \left( \frac{a_e}{r} \right)^{n+1} \sum_{m=0}^n \bar{P}_{n,m}(\sin \varphi) (\bar{C}_{n,m}^* \cos m\lambda + \bar{S}_{n,m}^* \sin m\lambda)$$