Relativistic effects and IERS Conventions

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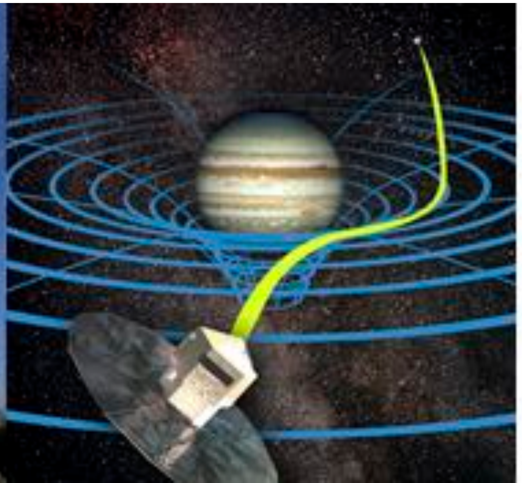
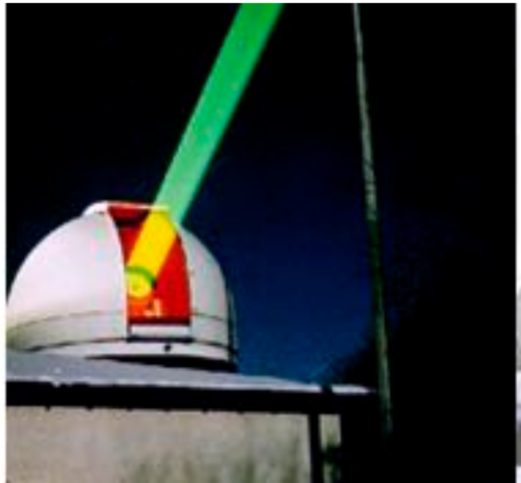
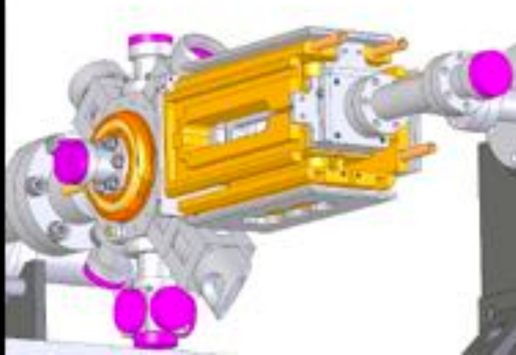


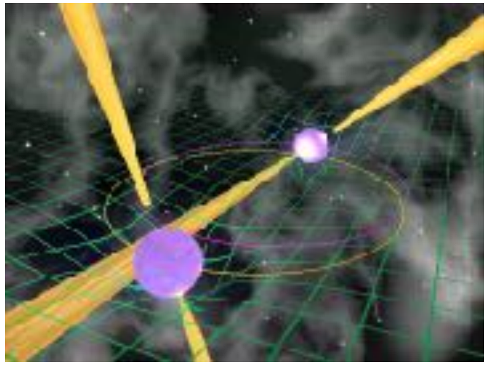
Where do we need gravity ?

Since Galileo Galilei, here…

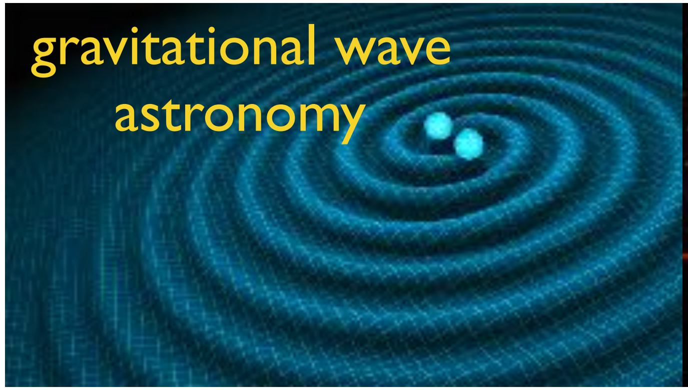
and with the beginning of Deep Space exploration in the 60th

weak field tests

1967 : discovery of pulsar.
=> first strong field tests



and tomorrow (even already today) :



General Outline

- Some basics concerning General Relativity

- Examples in Space/Ground Geodesy illustrating why we need Relativity

- Relativistic Reference Systems and Alternative to GR…

- How to describe observables in GR
  - light propagation
  - celestial mechanics

- What are the consequences on the IERS conventions ?

Basic principles of GR

1) Equivalence Principle:

- 3 facets: Universality of free fall, Local Position/Lorentz Invariance
- very well tested (10^{-13} with Eöt-wash experiments and Lunar Laser Ranging ; 10^{-4} with grav. redshift ; no variation of constants)[1]
- more accurate measurement needed: alternative (string) theories predict violation smaller[2] → MICROSCOPE accuracy 10^{-15}

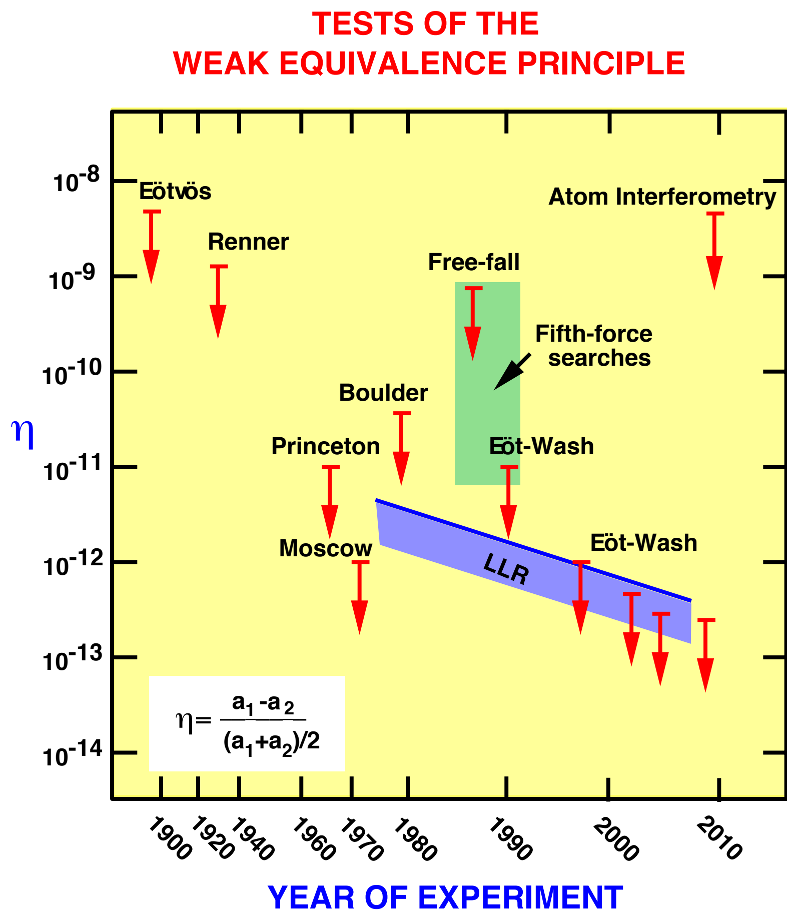
- Gravitation ⇔ space-time curvature (described by a metric $g_{\mu\nu}$)
- free-falling masses follow geodesics of this metric and ideal clocks measure proper time

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

[1] C. Will, LRR, 9, 2006

[2] T. Damour, CQG, 29-184001, 2012

Free Fall Experiments



400 CE Ioannes Philiponus: *"...let fall from the same height two weights of which one is many times as heavy as the other .... the difference in time is a very small one"*

1553 Giambattista Benedetti
proposed equality

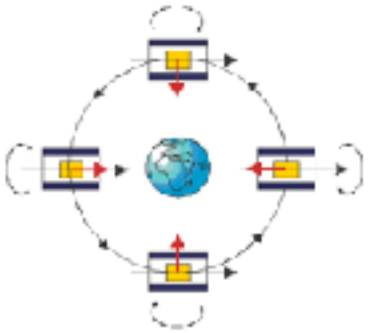
1586 Simon Stevin
experiments

1589-92 Galileo Galilei
Leaning Tower of Pisa?

1670-87 Newton
pendulum experiments

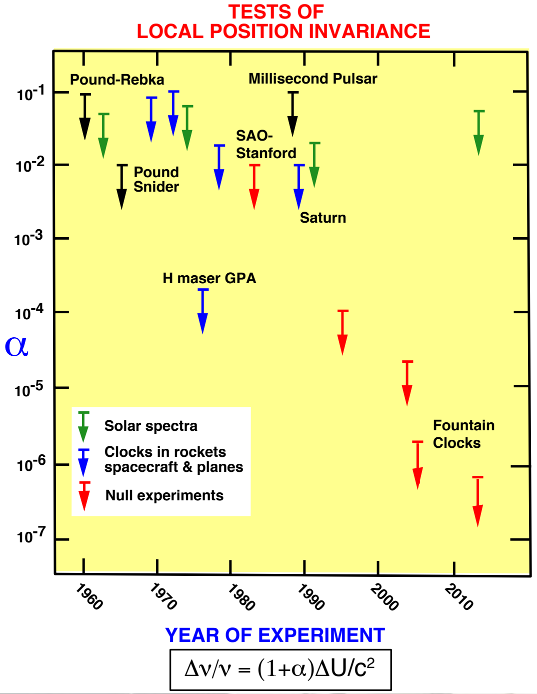
1889, 1908 Baron R. von Eötvös
torsion balance experiments (10^{-9})

1990s UW (Eöt-Wash) *10^{-13}*



CNES Microscope Mission : 10^{-15}

Local Position Invariance : redshift

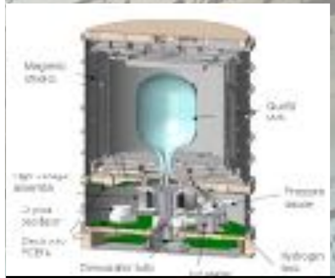


1959 : Pound & Rebka (10%)

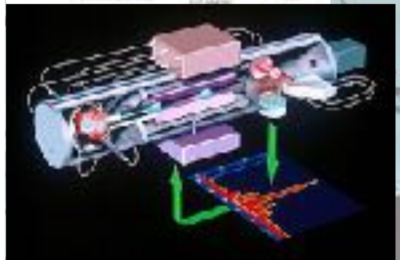
1980 : Gravity Probe A
Vessot (0.01%)

Launch : 1976 with Scout rocket
duration : 1h55mn
where : Wallops Island

Atomic Clock Ensemble in Space (ACES)

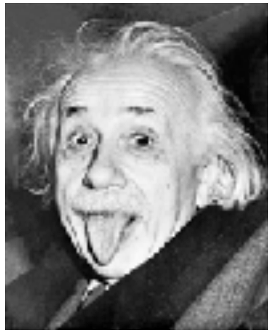


Goal :
improving by 35 GPA



To be launched soon

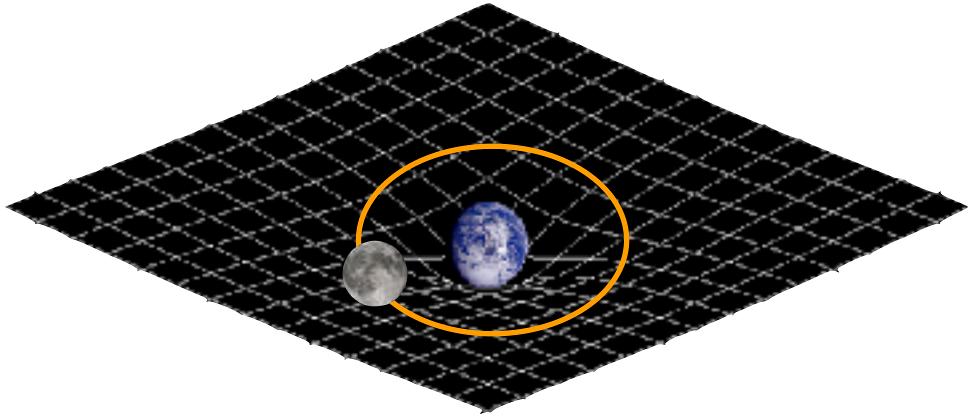
Basic principles of GR



II) Field equations (determination of the metric):

- Einstein Equations: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

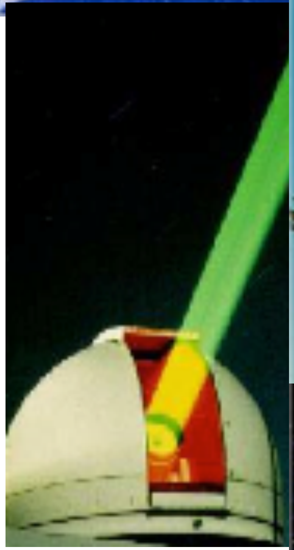
space-time curvature (metric) ⇔ matter-energy content

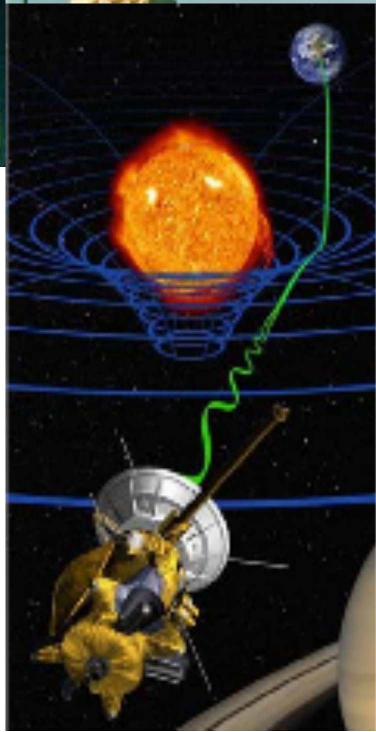


- important effects for space-mission:
  - dynamics ≠ from Newton (ex.: advance of the perihelion)
  - proper time (measured by ideal clocks) ≠ coordinate time
  - coordinate time delay for light propagation (Range/Doppler)
  - light deflection (VLBI, astrometry)

More and more precision !







Ground & space geodesy accuracy is increasing:

LLR & SLR → From cm to mm
GALILEO

Gravity Probe A to ACES/Pharao → factor 80 on Grav. Redshift

Ground & space astrometry:

Gaia, Gravity → from milli to micro-arcsecond

Navigation of interplanetary probes :

Cassini Experiment, use of Ka Band
MORE Experiment on BepiColombo → factor 10 on Doppler
JUNO Experiment 2016, JUICE towards 2030


Need to describe light propagation/dynamics more precisely in relativistic framework : go to 2PN theory !

Timespan and accuracy are increasing :

- One can *catch* more relativistic effects
- Better sensibility to test Relativity

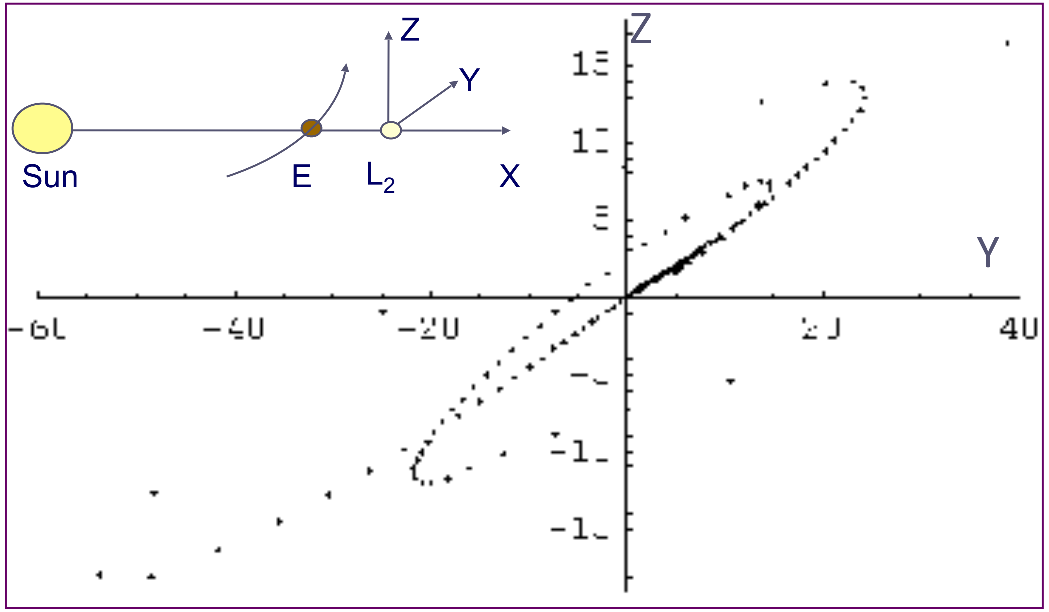
A first but illustrative example : Gaia

Light deflection (first order)

body	Mass (μas)	J_2 (μas)
Sun	1.75″	-
Mercury	83	-
Venus	493	-
Earth	574	-
Mars	116	-
Jupiter	16270	240
Saturn	5780	90
Uranus	2080	-
Neptun	2533	-

2d order, Sun : 10 μas

Celestial Mechanics : Gaia orbit => 1-2 mm/s = aberration of 1 μas



Relativistic effects in km over 200 days…

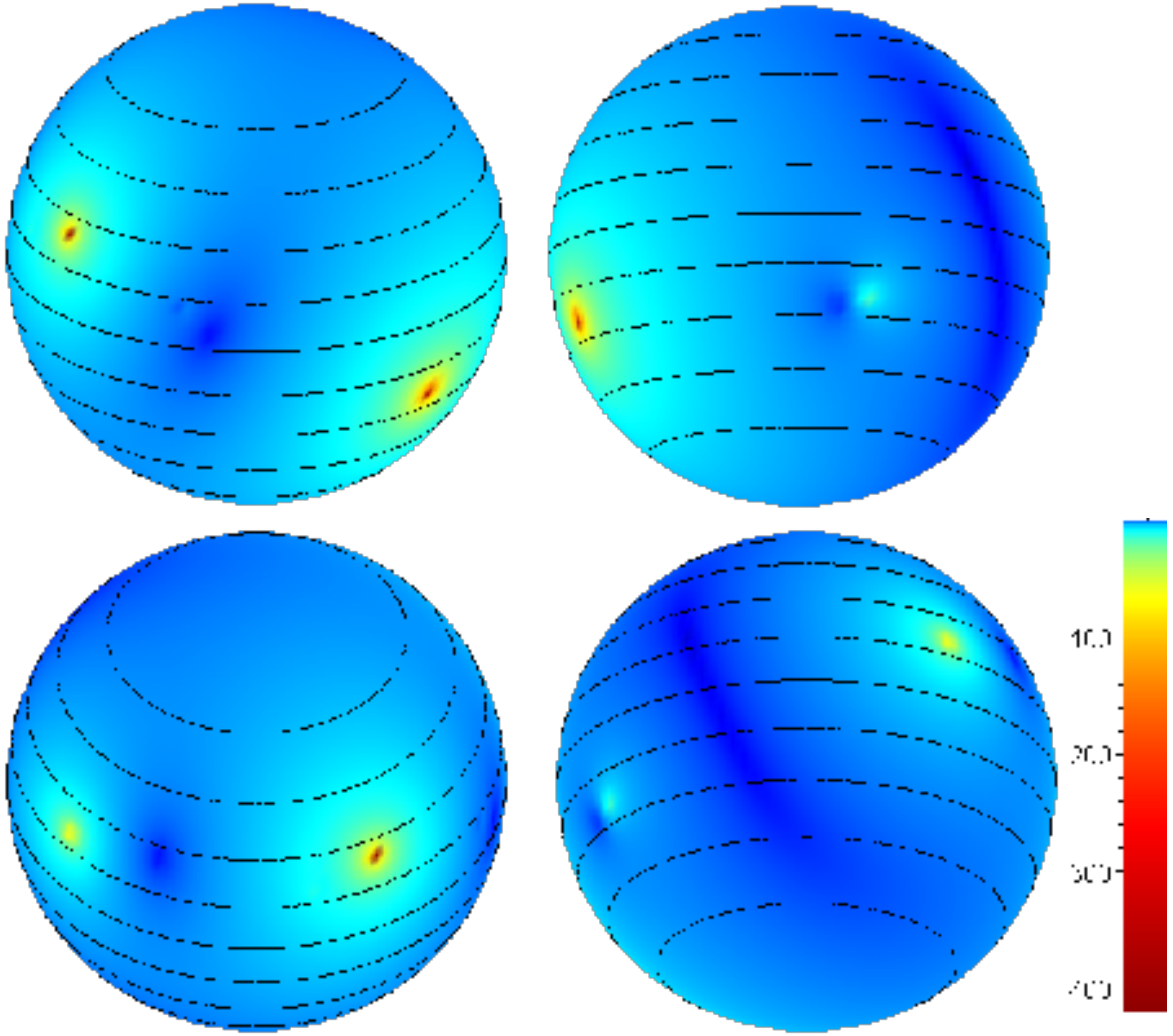
Time Metrology: need to synchronize onboard clock with the ground at an accuracy of 1 μs over the mission.

But we have periodic differences of several μs between the real data analysis time scale and the Gaia proper time….

- It is not any more possible to speak about corrections….
- All modeling must be natively relativistic

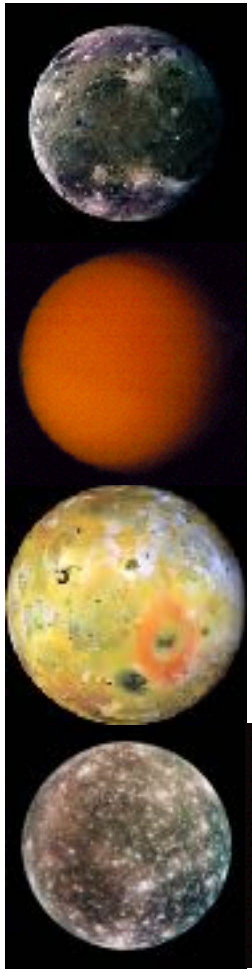
Light deflection : how much ?

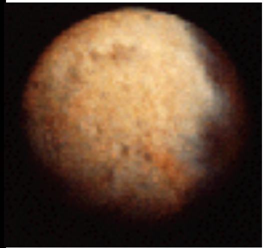
- Monopole light deflection: distribution over the sky on 25.01.2006 at 16:45
  equatorial coordinates



body	(μas)	>1μas
Sun	$1.75''$	180°
Mercury	83	$9'$
Venus	493	4.5°
Earth	574	125°
Moon	26	5°
Mars	116	$25'$
Jupiter	16270	90°
Saturn	5780	17°
Uranus	2080	$71'$
Neptune	2533	$51'$

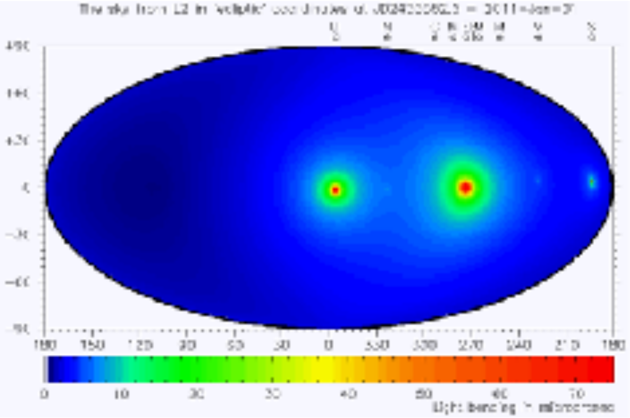
Light deflection : how much ?





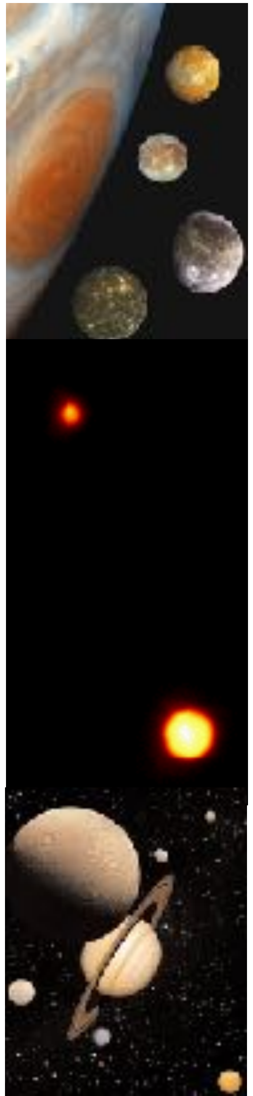
body	(muas)	>1muas
Sun	1.75 10^6	180°
Mercury	83	9°
Venus	493	4.5°
Earth	574	125°
Moon	26	5°
Mars	116	25°
Jupiter	16270	90°
Saturn	5780	17°
Uranus	2080	71°
Neptune	2533	51°

Order of magnitude for monopole light deflection.



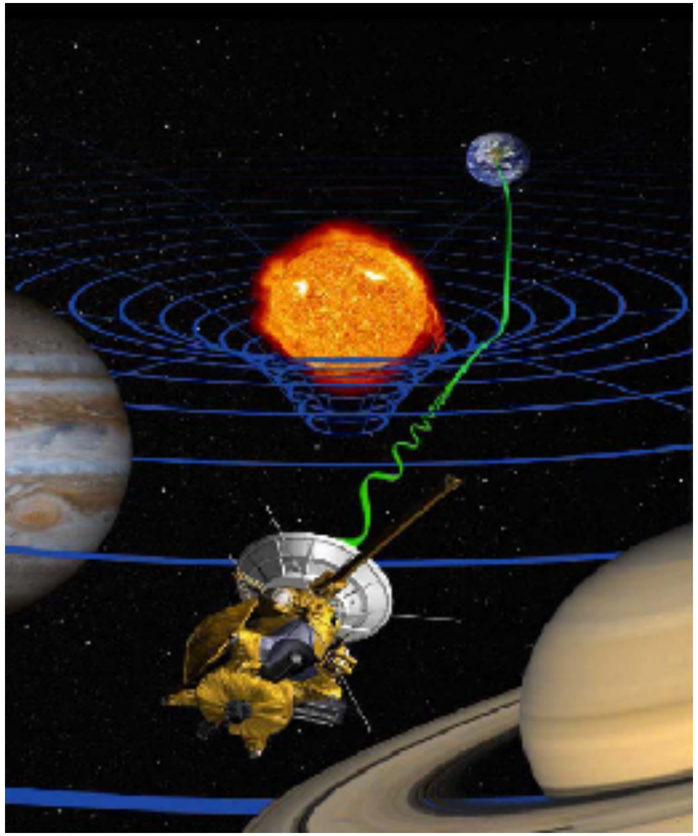
Minor bodies :

Ganymede	35
Titan	32
Io	30
Callisto	28
Pluto	7
Charon	4
Titania	3
Ceres	1



A second but historic example : Space navigation

$$\Delta t \approx \frac{2GM(1+\gamma)}{c^3}\left[\ln\left(\frac{4r_A r_B}{r_0^{\,2}}\right)+1\right]$$



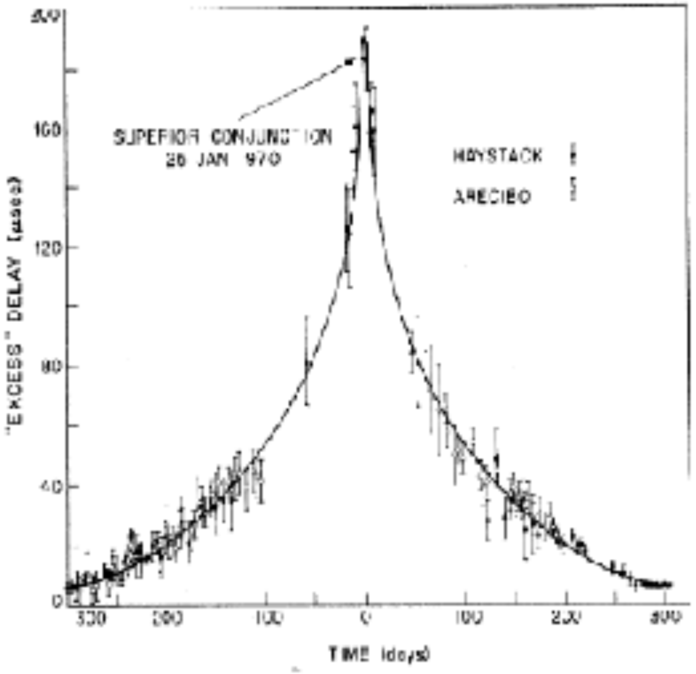
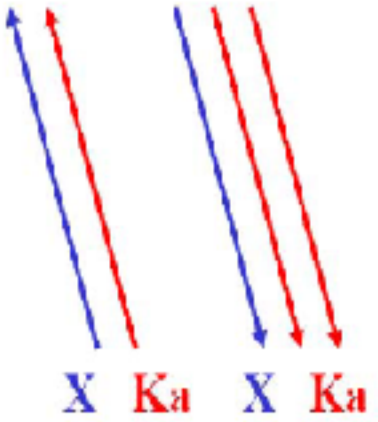
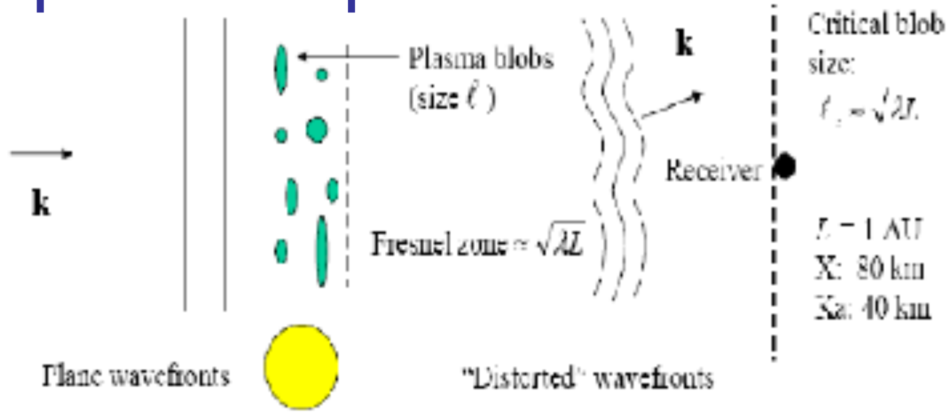


FIG. 1. Typical sample of post-fit residuals for Earth-Venus time-delay measurements, displayed relative to the "excess" delays predicted by general relativity. Corrections were made for known topographic trends on Venus. The bars represent the original estimates of the measurement standard errors. Note the dramatic increase in accuracy that was obtained with the radar-system improvements incorporated at Haystack just prior to the inferior conjunction of November 1970.

Shapiro, I.I. et al, Phys.Rev.Lett, 26, 1132 (1971)

Cassini probe experiment

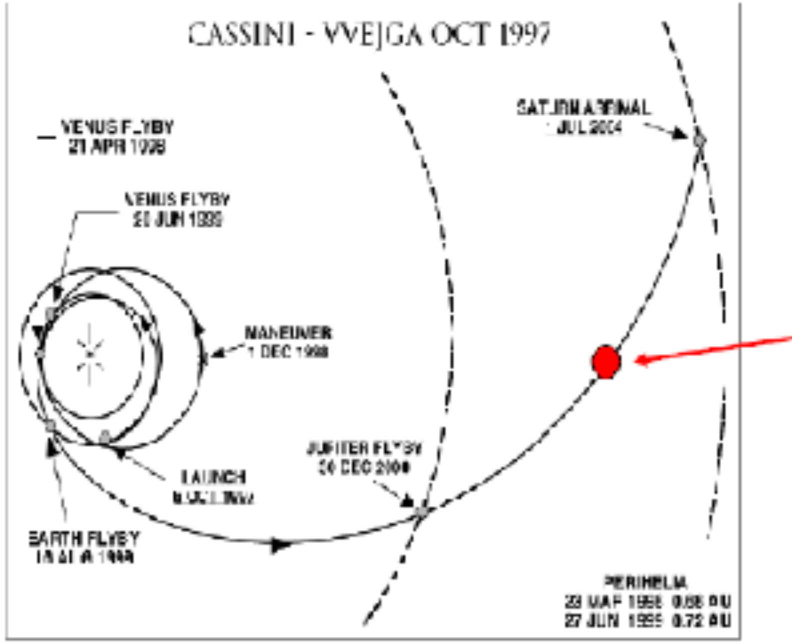




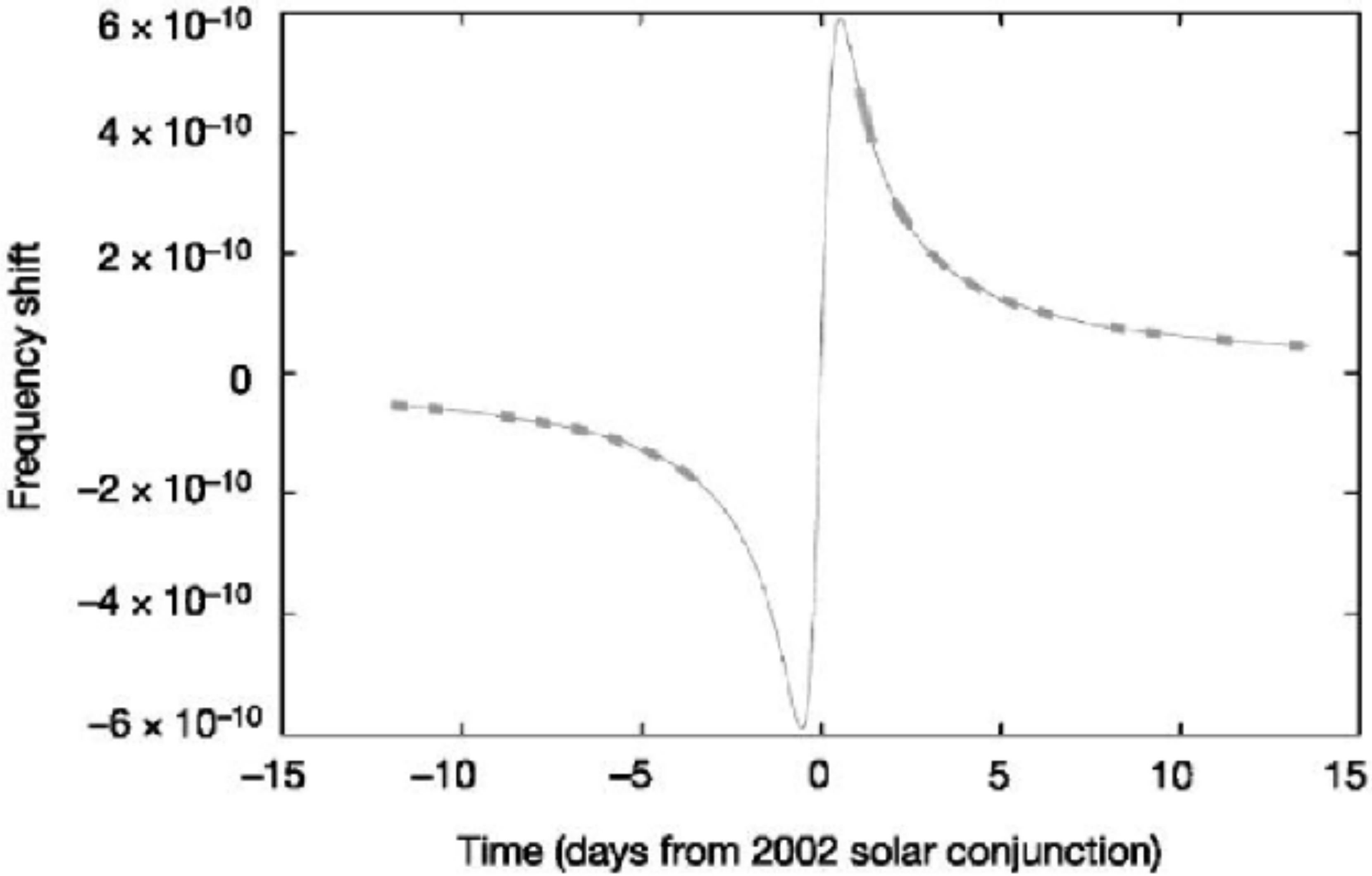
DSS 25 - Goldstone

Doppler effect

$$\frac{\Delta\nu}{\nu} \approx 4(1+\gamma)\frac{M_{Sol}}{b}\frac{db}{dt}$$



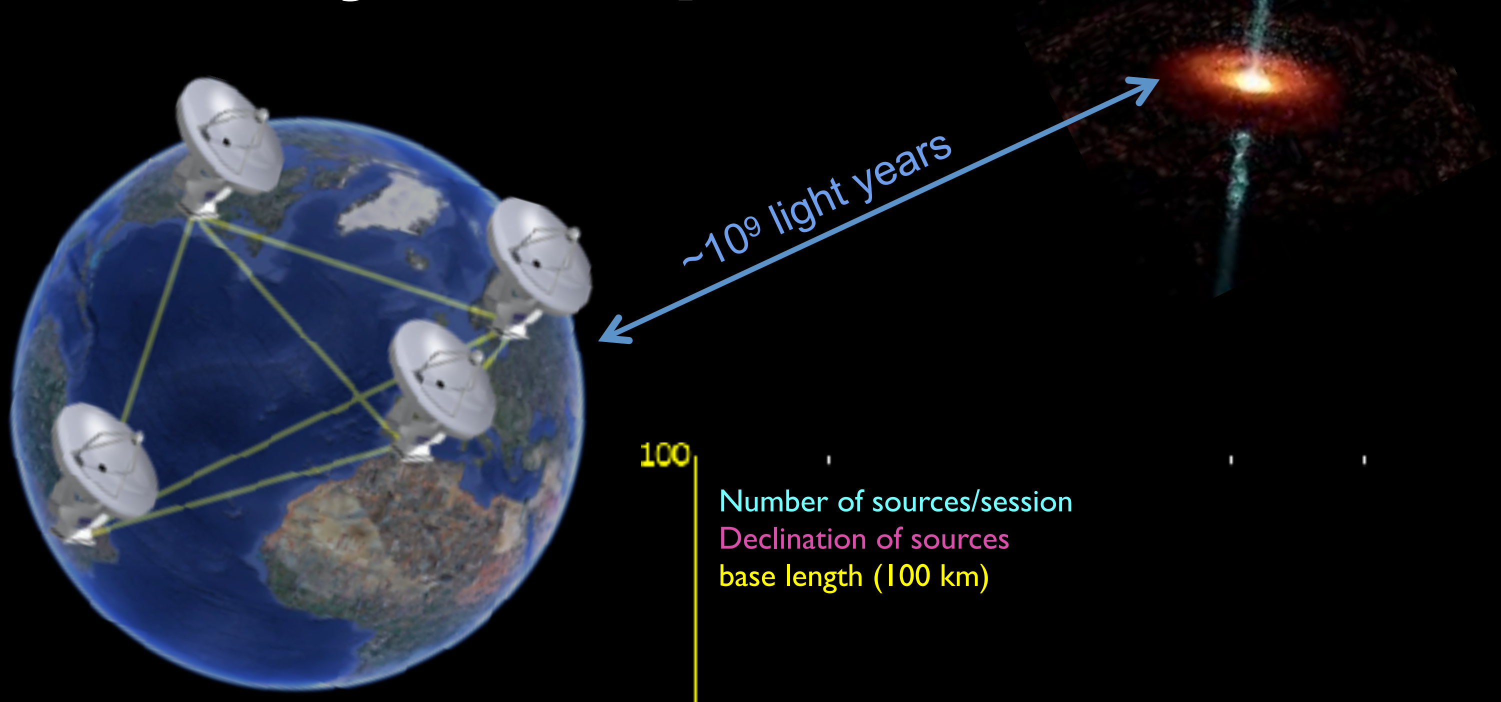
$$\delta\left(\frac{f_r - f_e}{f_0}\right) \approx \frac{d}{dt}\left[2(1+\gamma)\frac{GM}{c^3}\ln\left(\frac{4r_S r_G}{b^2}\right)\right] \approx -4(1+\gamma)\frac{GM}{c^3 b}\frac{db}{dt}$$

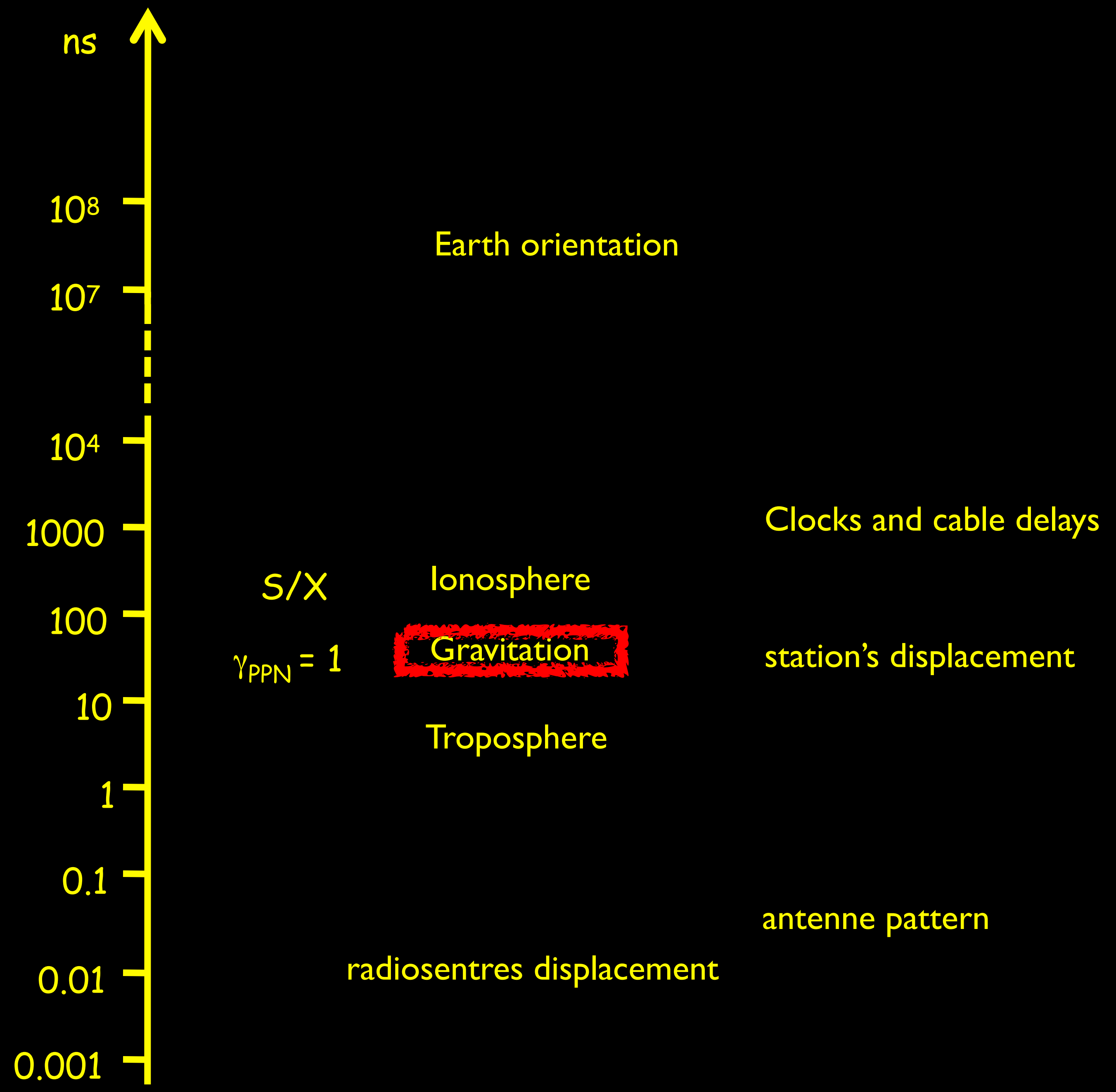


$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

Bertotti, B. et al, Nature, 425, 374 (2003)

A third but funny example : VLBI

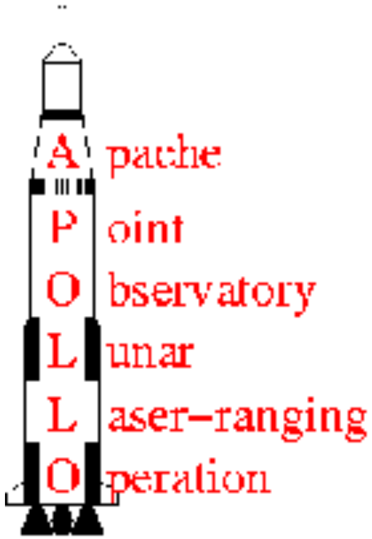



ns
10^8
Earth orientation
10^7
10^4
1000
Clocks and cable delays
S/X
Ionosphere
100
$\gamma_{PPN} = 1$
Gravitation
station's displacement
10
Troposphere
1
0.1
antenne pattern
0.01
radiosentres displacement
0.001

Lunar Laser Ranging and Nordtvedt effect

$$\frac{m_g}{m_i} = 1 - \eta_N \frac{E_g}{m_i c^2} = 1 + \frac{6}{5}\eta_N \frac{G m_i c^2}{R}$$

with $\eta_N - 4\beta - \gamma - 3$



$\frac{E_g}{m_i c^2}$ $< 10^{-27}$ for lab experiment

$= 3.6 \quad 10^{-6}$ Sun

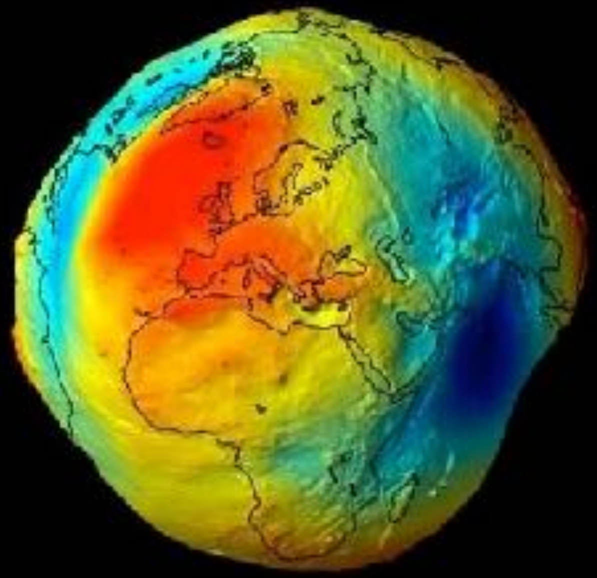
10^{-8} Jupiter

$4.6 \quad 10^{-10}$ Earth

$2 \quad 10^{-11}$ Moon

$$\eta_N = (4.4 \pm 4.5) 10^{-4}$$

Post Scriptum : light propagation between Earth and Moon…
Shapiro delay = 8 meters



Space geodesy and relativity

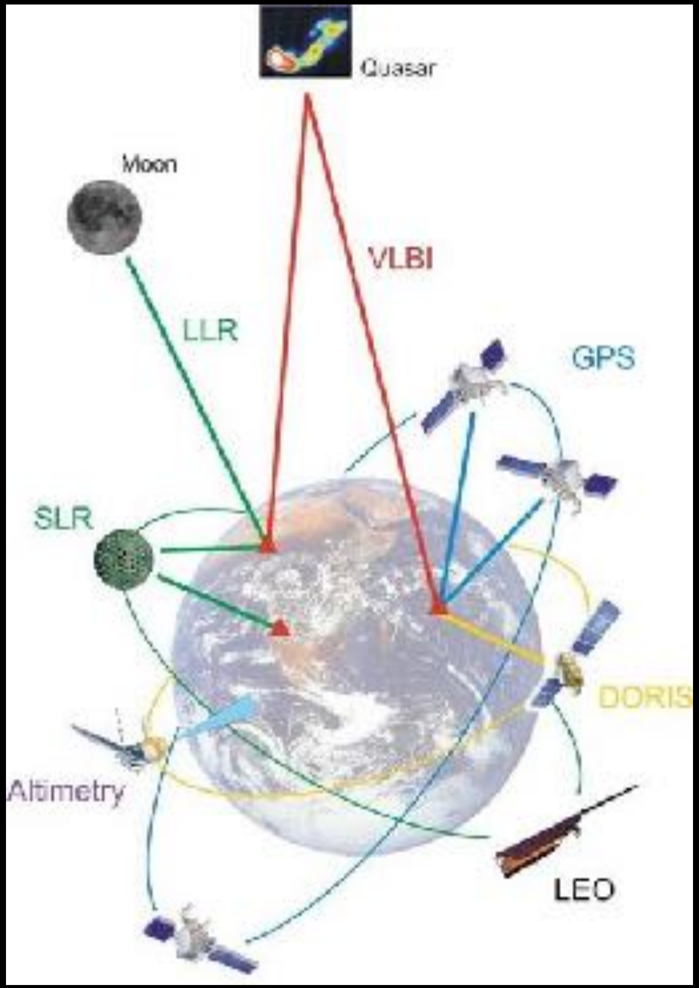
Several techniques used

Understand internal dynamics of the Earth

Determine gravity field

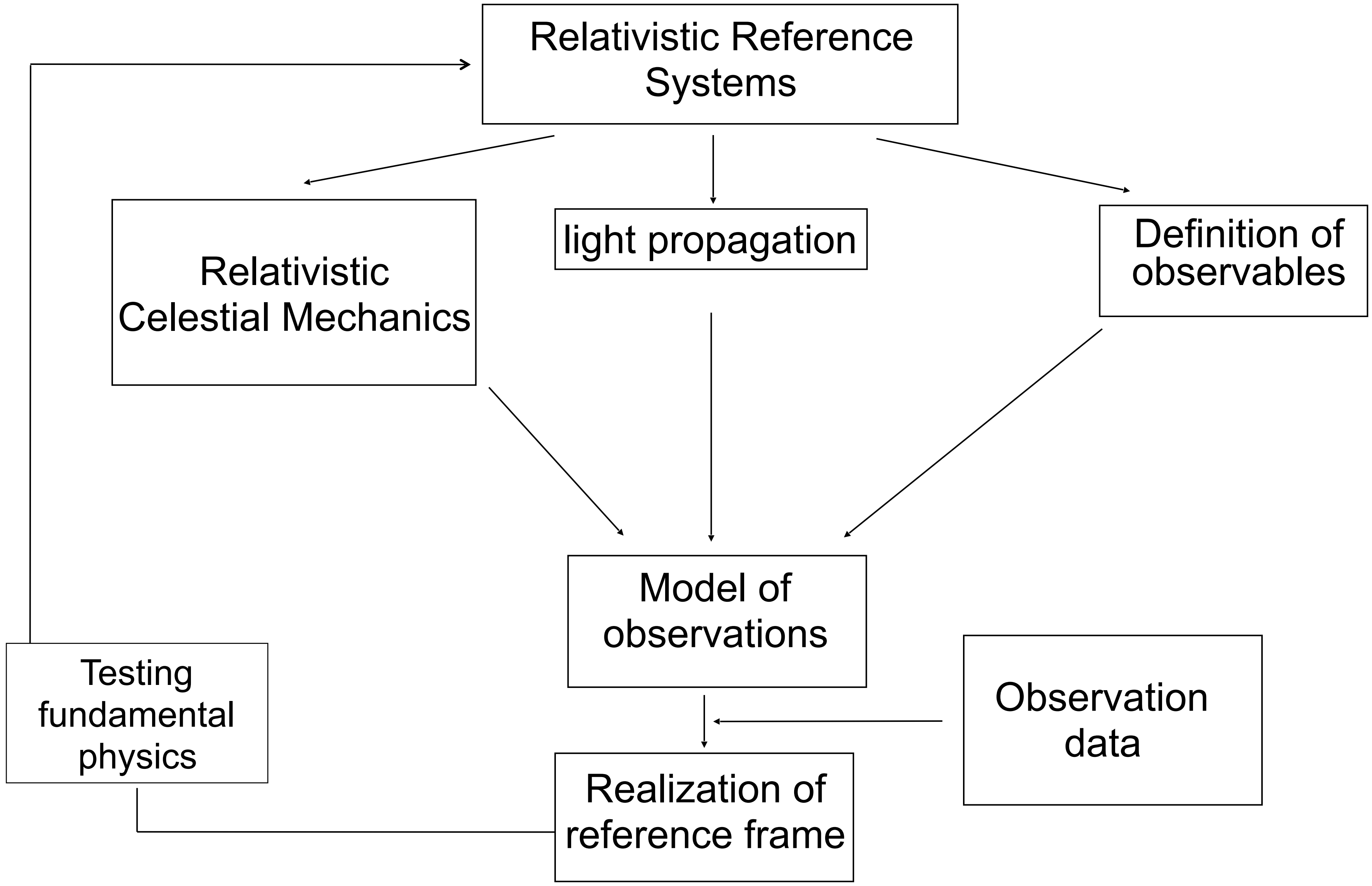
Good positioning

Lense-Thirring Effect detected (1% level)by Ciufolini & Pavlis on LAGEOS, Nature 2004



PS : Schwarzchild radius of the Earth = 9mm

How works Fundamental Relativistic Astronomy



[Klioner 2003, CLPL 2008]

IAU Reference Systems and relativity

THE IAU 2000 RESOLUTIONS FOR ASTROMETRY, CELESTIAL MECHANICS, AND METROLOGY IN THE RELATIVISTIC FRAMEWORK: EXPLANATORY SUPPLEMENT

M. Soffel,[1] S. A. Klioner,[1] G. Petit,[2] P. Wolf,[2] S. M. Kopeikin,[3] P. Bretagnon,[4] V. A. Brumberg,[5] N. Capitaine,[6] T. Damour,[7] T. Fukushima,[8] B. Guinot,[6] T.-Y. Huang,[9] L. Lindegren,[10] C. Ma,[11] K. Nordtvedt,[12] J. C. Ries,[13] P. K. Seidelmann,[14] D. Vokrouhlický,[15] C. M. Will,[16] and C. Xu[17]

Received 2002 August 9; accepted 2003 July 2

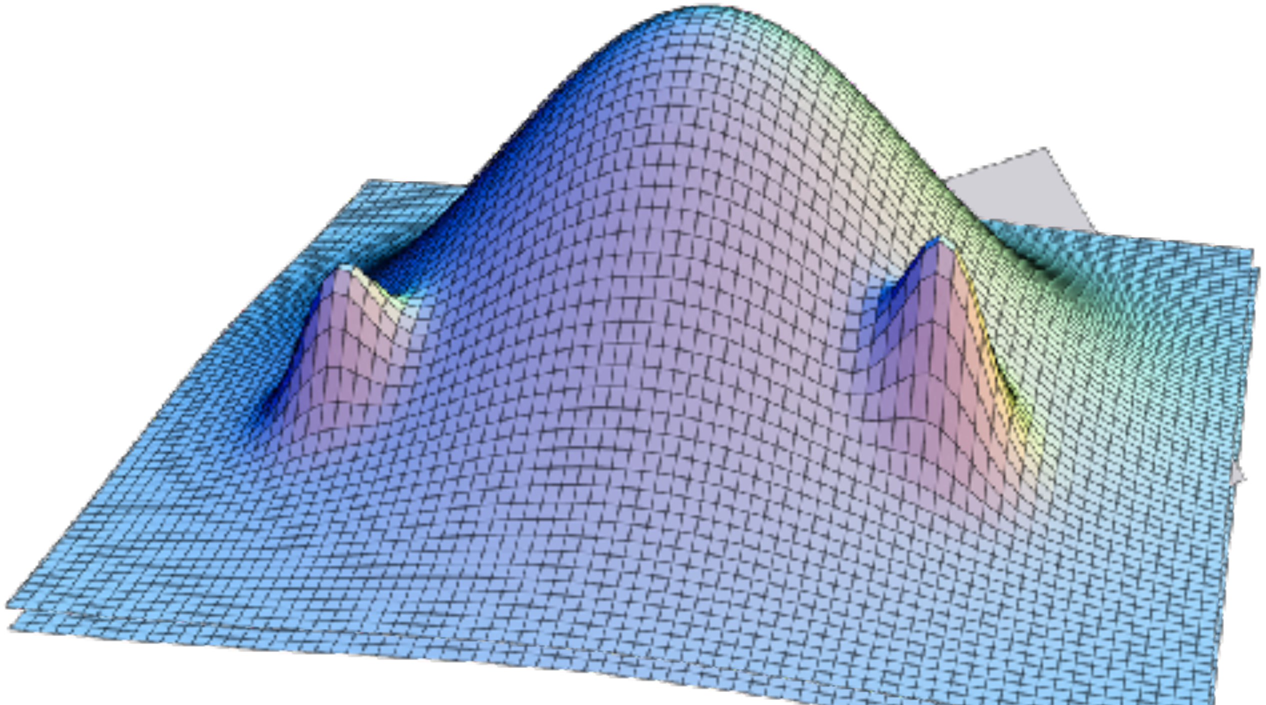
- First attempt : IAU 1976
- IAU 2000:
  - Fully relativistic (General Relativity, not PPN)
  - BCRS: time scale TCB
  - GCRS: time scale TCG
  - Time transformation between TCG & TCB
- IAU 2006: redefinition of time scale TDB

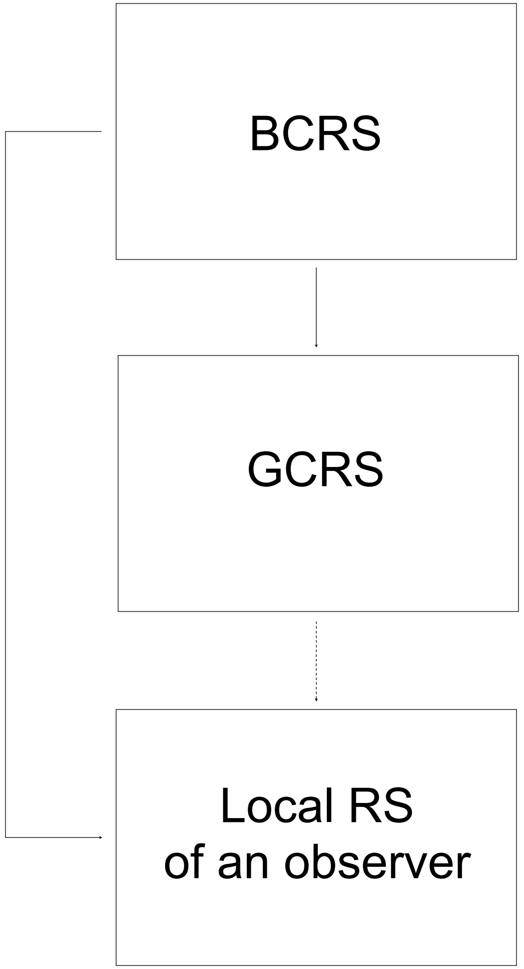
Reference systems theory

- In relativistic astronomy the
  - BCRS (Barycentric Celestial Reference System)
  - GCRS (Geocentric Celestial Reference System)
  - Local reference system of an observer

  play an important role.

- All these reference systems are defined by
  the form of the corresponding metric tensor.





Bini, 2003
Klioner, 2004

Barycentric Celestial Reference System

The BCRS is a particular reference system in the curved space-time of the Solar system

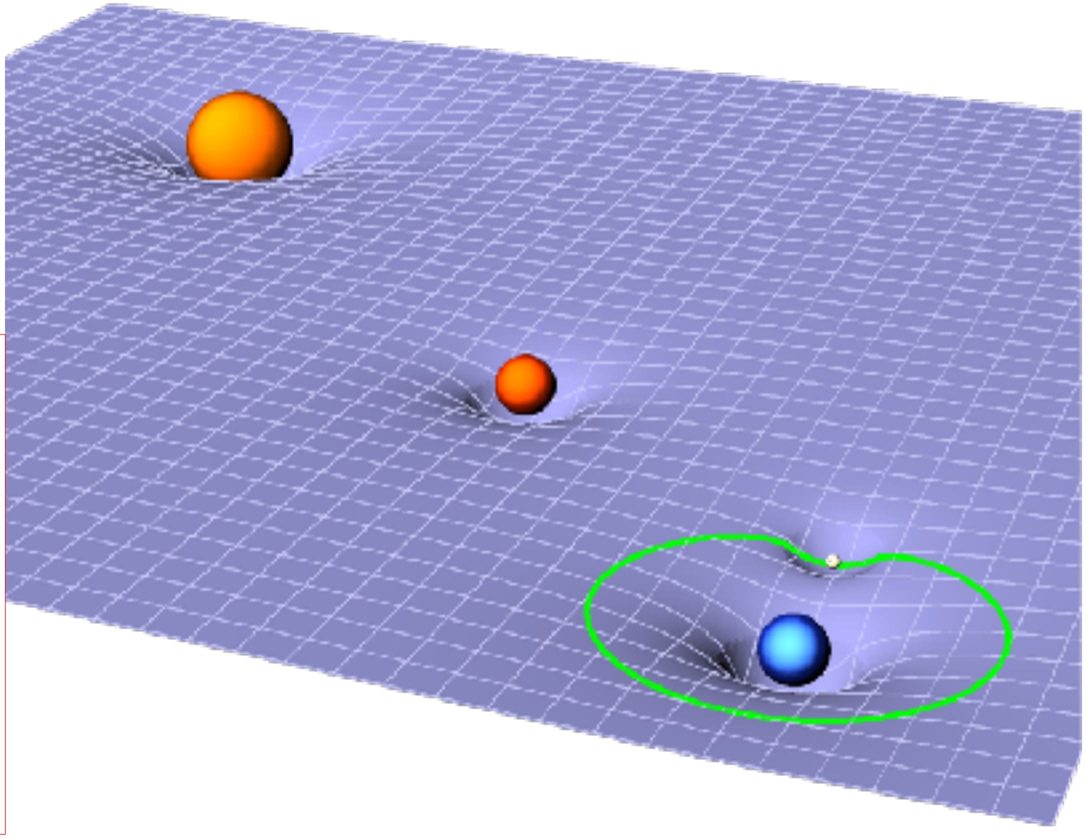
- One can use any
- but one should fix one :

ICRF by VLBI

$$g_{00} = -1 + \frac{2}{c^2} w(t, \boldsymbol{x}) - \frac{2}{c^4} w^2(t, \boldsymbol{x}),$$

$$g_{0i} = -\frac{4}{c^3} w^i(t, \boldsymbol{x}),$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2}{c^2} w(t, \boldsymbol{x}) \right).$$



Used to describe motion of celestial body and description of light propagation

Ephemeride

Astrometry

Tests of the gravitational dynamics

- How to test the form of the metric/the Einstein field equations ? Two frameworks widely used so far:

I) Parametrized Post-Newtonian Formalism[1]

- powerful phenomenology making an interface between theoretical development and experiments
- metric parametrized by 10 dimensionless coefficients
- γ and β whose values are1 in GR

$$ds^2 = (1 + 2\phi_N + 2\beta\phi_N^2 + \ldots)dt^2 - (1 - 2\gamma\phi_N + \ldots)d\vec{x}^2$$

II) Fifth force formalism[2]

- modification of Newton potential of the form of a Yukawa potential

$$\phi(r) = \frac{GM}{c^2 r}\left(1 + \alpha e^{-r/\lambda}\right)$$

[1] C. Will, LRR, 9, 2006
"Theory and Experiment in Grav. Physics", C. Will, 1993

[2] E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
"The Search for Non-Newtonian gravity", E. Fischbach, C. Talmadge, 1998

PPN parameters and their significance

Parameter	What it measures, relative to general relativity	Value in GR	Value in scalar tensor theory	Value in semi-conservative theories
γ	How much space curvature produced by unit mass?	1	$(1+\omega)/(2+\omega)$	γ
β	How "nonlinear" is gravity?	1	$1 + \Lambda$	β
ξ	Preferred-location effects?	0	0	ξ
α_1	Preferred-frame effects?	0	0	α_1
α_2		0	0	α_2
α_3		0	0	0
ζ_1	Is momentum conserved?	0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

Light propagation is crucial in the modeling of Sol. Sys. observations

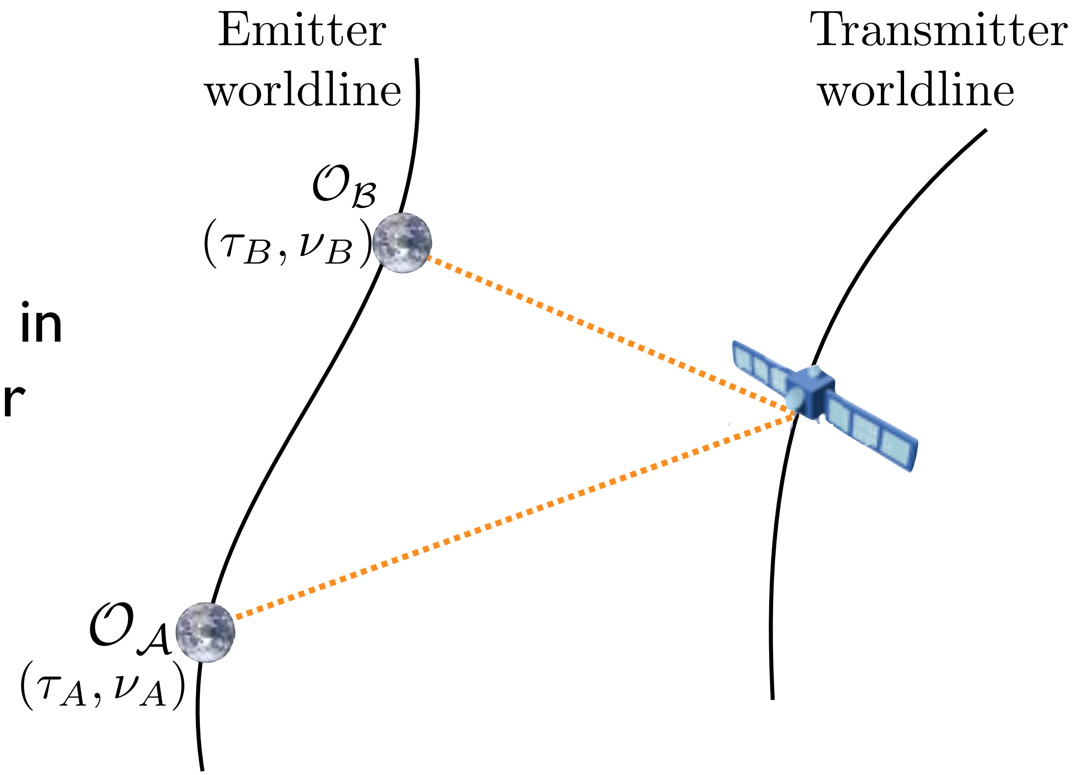
1) Range observable

- Difference in proper time

$$\text{Range} = c(\tau_B - \tau_A)$$

- Depends on the difference in coord. time (amongst other parameters)

$$t_B - t_A$$



Light propagation is crucial in the modeling of Sol. Sys. observations

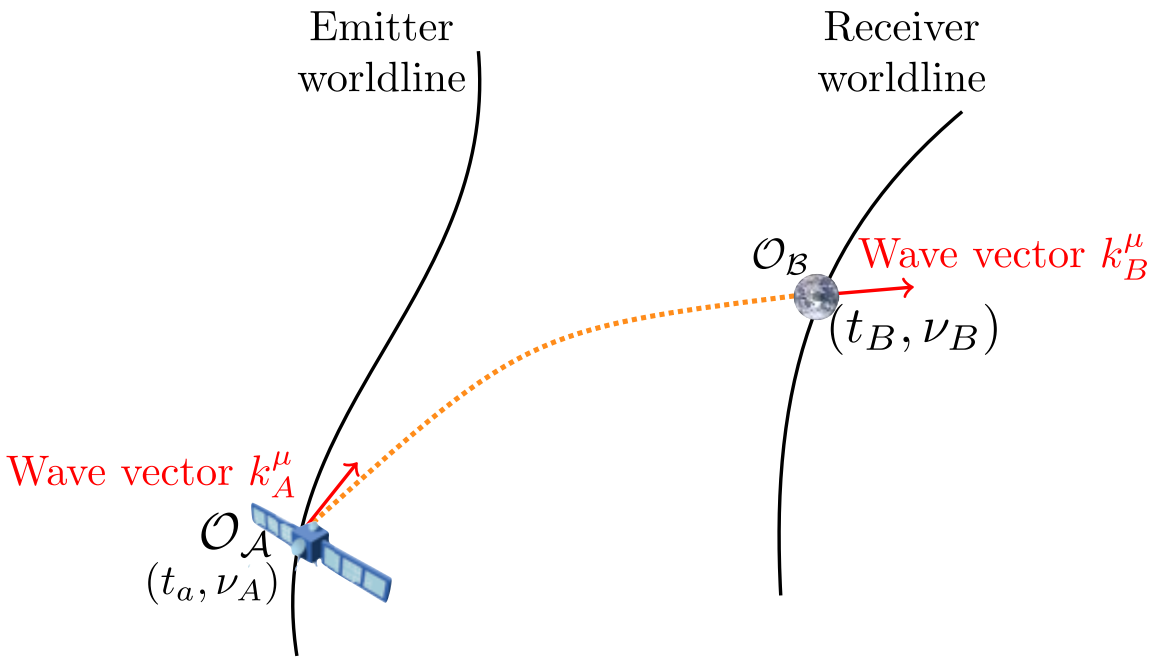
2) Doppler observable

- Ratio of proper frequency $$D = \frac{\nu_B}{\nu_A} = \left(\frac{d\tau}{dt}\right)_A \left(\frac{d\tau}{dt}\right)_B^{-1} \frac{k_0^B}{k_0^A} \frac{1+\beta^i_B \hat{k}_i^B}{1+\beta^i_A \hat{k}_i^A}$$

with $\beta^i = v^i/c$ and

$$\hat{k}_i = \frac{k_i}{k_0}$$

- Wave vector at emission and reception needed



Light propagation is crucial in the modeling of Sol. Sys. observations

3) Astrometric observables

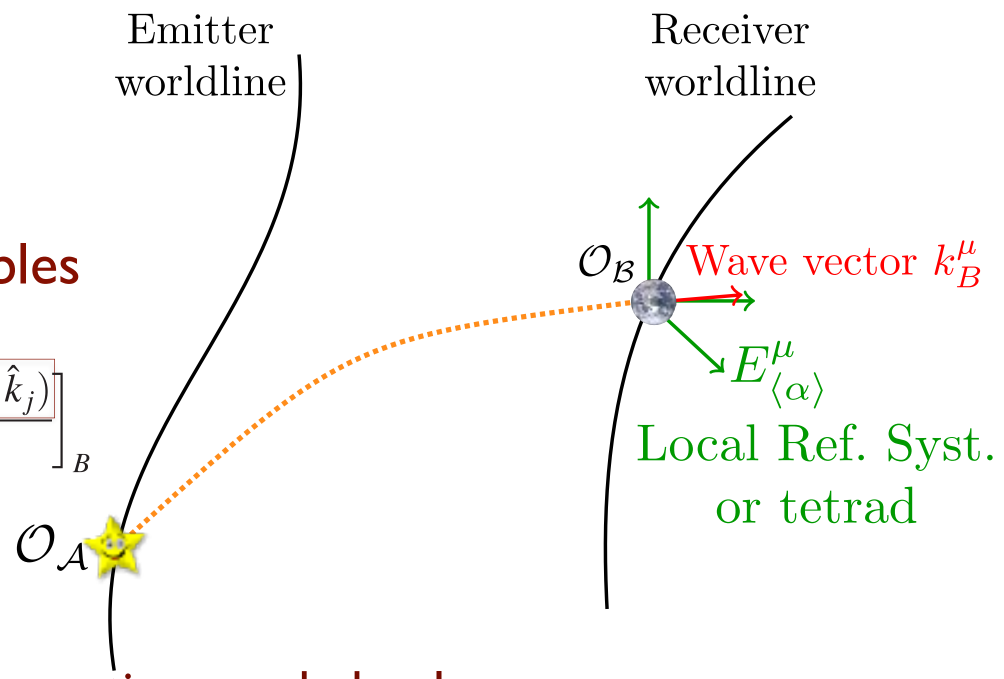
Direction of observation of the light ray in a local reference system (or tetrad)

$$n^{\langle i\rangle} = -\frac{E^0_{\langle i\rangle} + E^j_{\langle i\rangle}\hat{k}^B_j}{E^0_{\langle 0\rangle} + E^j_{\langle 0\rangle}\hat{k}^B_j}$$

4) Differential astrometric observables

$$\sin^2\frac{\phi}{2} = -\frac{1}{4}\left[\frac{(g_{00}+2g_{0k}\beta^k+g_{kl}\beta^k\beta^l)g^{ij}(\hat{k}'_i-\hat{k}_i)(\hat{k}'_j-\hat{k}_j)}{(1+\beta^m\hat{k}_m)(1+\beta^l\hat{k}_l{}')}\right]_B$$

Angle between 2 incoming light rays



Wave vector at reception needed only

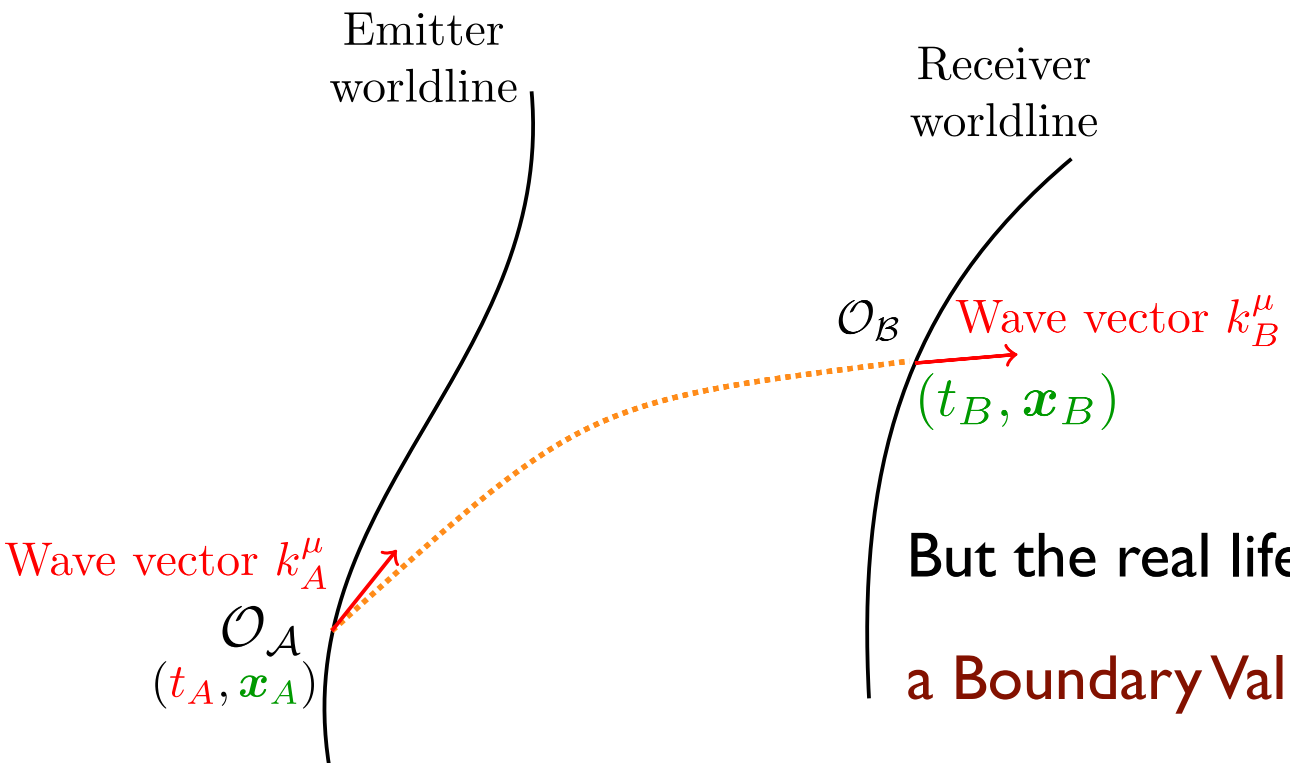
How to determine the light propagation ?

- At the geometric optics approximation: photons follow null geodesics

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0 \qquad k^\mu k_\mu = 0$$

with $k^\mu = \frac{dx^\mu}{d\lambda}$ the tangent vector

an Initial Value Problem



But the real life :

a Boundary Value Problem

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

  see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007

- Exact analytical solution for some metrics: Schwarzschild and Kerr (solution with Jacobian/ Weierstrass elliptic functions)

see for example: de Jans, Mem. de l'Ac. Roy. de Bel., 1922
B. Carter, Com. in Math. Phys. 10, 280, 1968
A. Cadez, U. Kostic, PRD 72, 104024, 2005
A. Cadez, et al, New Astr. 3, 647, 1998

- Analytical solutions for weak gravitational field:
  - 1 pM Schwarzschild metric

    see E. Shapiro, PRL 13, 26, 789, 1964

  - moving monopoles at 1pM order

    see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
    S. Klioner, A & A, 404, 783, 2003

  - static extended bodies with multipolar expansion at 1pM

    see S. Kopeikin, J. of Math. Phys., 38, 2587
    S. Zschocke, PRD 92, 063015, 2015

  - 2 pM Schwarzschild metric

    see G. Richter, R. Matzner, PRD 28, 3007, 1983
    S. Klioner, S. Zschocke, CQG 27, 075015, 2010

- Use of the eikonal equation:
  - perturbative solution for spherically symmetric space-time

    see for example N. Ashby, B. Bertotti, CQG 27, 145013, 2010

… and the Time Transfer Functions

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- The Time Transfer Functions - TTF - are defined by

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) \qquad t_B - t_A = \mathcal{T}_e(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B)$$

- The TTF is solution of an eikonal equation well adapted to a perturbative expansion

- The derivatives of the TTF are of crucial interest since

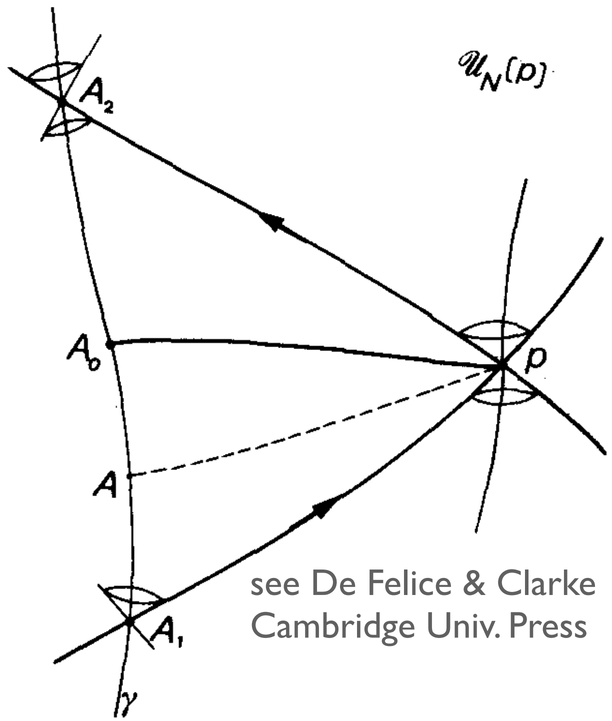
$$\hat{k}_i^A = c\frac{\partial \mathcal{T}_r}{\partial x_A^i} \qquad \hat{k}_i^B = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1} \qquad \frac{k_0^B}{k_0^A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}$$

Range, Doppler, astrometric observables can be written in terms of the TTF and its derivatives

Synge's World Function as TTF progenitor

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

Suppose the existence of two event-points x_A and x_B on a manifold. We assume that they are located in a convex neighbourhood in such a way that they are connected by a unique geodesic.



see De Felice & Clarke
Cambridge Univ. Press

One can define a Synge's World Function between x_A and x_B (Ruse 1931, Synge 1931, 1964)

$$\Omega\left(x_A, x_B\right) = \frac{\epsilon_{AB}}{2}\int_0^1 g_{\mu\nu}(x^\alpha(\lambda))\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}d\lambda\,,$$

where λ is an affine parameter, $\epsilon_{AB} = -1, 0, 1$

Very difficult to determine it… Schwarzschild (Buchdhal 1979).
But an iterative Post-Minkowskian expansion has been found

Synge's World Function as TTF progenitor

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

World Function property I: Hamilton-Jacobi equations

$$\frac{1}{2}\,g^{\alpha\beta}(x_A)\frac{\partial\Omega}{\partial x_A^\alpha}(x_A,x_B)\frac{\partial\Omega}{\partial x_A^\beta}(x_A,x_B)=\Omega(x_A,x_B)\,,$$

$$\frac{1}{2}\,g^{\alpha\beta}(x_B)\frac{\partial\Omega}{\partial x_B^\alpha}(x_A,x_B)\frac{\partial\Omega}{\partial x_B^\beta}(x_A,x_B)=\Omega(x_A,x_B)\,.$$

World Function property II: Tangent vectors at x_A and x_B

$$\left(g_{\mu\nu}\frac{dx^\nu}{d\lambda}\right)_A=-\frac{\partial\Omega}{\partial x_A^\mu}(x_A,x_B)\,,\qquad \left(g_{\mu\nu}\frac{dx^\nu}{d\lambda}\right)_B=\frac{\partial\Omega}{\partial x_B^\mu}(x_A,x_B)\,.$$

World Function property III: particular case of light rays

$$\epsilon_{AB}=0 \qquad \Leftrightarrow \qquad \Omega\left(x_A,x_B\right)=0$$

Synge's World Function as TTF progenitor

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

Let us introduce the emission TTF as follows

$$\Omega\left(x_A^0, \boldsymbol{x}_A, x_A^0 + c\mathcal{T}_e(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B), \boldsymbol{x}_B\right) \equiv 0$$

If we differentiate with respect to x_A^0, x_A^i and x_B^i

$$c\,\frac{\partial\Omega}{\partial x_B^0}(x_A, x_B)\,\frac{\partial\mathcal{T}_e}{\partial x_B^i}(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B) + \frac{\partial\Omega}{\partial x_B^i}(x_A, x_B) = 0\,,$$

$$\frac{\partial\Omega}{\partial x_A^i}(x_A, x_B) + c\,\frac{\partial\Omega}{\partial x_B^0}(x_A, x_B)\,\frac{\partial\mathcal{T}_e}{\partial x_A^i}(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B) = 0\,,$$

$$\frac{\partial\Omega}{\partial x_A^0}(x_A, x_B) + \frac{\partial\Omega}{\partial x_B^0}(x_A, x_B)\left[1 + \frac{\partial\mathcal{T}_e}{\partial t_A}(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B)\right] = 0\,,$$

Same reasoning on reception TTF

$$\Omega\left(x_B^0 - c\mathcal{T}_r(t_B, \boldsymbol{x}_A, \boldsymbol{x}_B), \boldsymbol{x}_A, x_B^0, \boldsymbol{x}_B\right) \equiv 0\,.$$

Fundamental properties of TTF's

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

It leads to the fundamental theorem for TTF

$$\left(\frac{k_i}{k_0}\right)_B = -c\,\frac{\partial \mathcal{T}_e}{\partial x_B^i} = -c\,\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1-\frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1}, \quad \left(\frac{k_i}{k_0}\right)_A = c\,\frac{\partial \mathcal{T}_e}{\partial x_A^i}\left[1+\frac{\partial \mathcal{T}_e}{\partial t_A}\right]^{-1} = c\,\frac{\partial \mathcal{T}_r}{\partial x_A^i},$$

$$\frac{(k_0)_B}{(k_0)_A} = \left[1+\frac{\partial \mathcal{T}_e}{\partial t_A}\right]^{-1} = 1-\frac{\partial \mathcal{T}_r}{\partial t_B}.$$

But how to calculate a TTF ?

I. First calculate the world function, then apply $\Omega\left(x_A, x_B\right)$ is equal to 0 and use a Lagrange inversion (2004)

II. Realize that $\left[g^{\mu\nu}k_\mu k_\nu\right]_{x_A/x_B} \equiv 0$, so (2008)

$$\Rightarrow g^{00}(x_B^0 - c\mathcal{T}_r, \boldsymbol{x}_A) + 2c\,g^{0i}(x_B^0 - c\mathcal{T}_r, \boldsymbol{x}_A)\frac{\partial \mathcal{T}_r}{\partial x_A^i} + c^2\,g^{ij}(x_B^0 - c\mathcal{T}_r, \boldsymbol{x}_A)\frac{\partial \mathcal{T}_r}{\partial x_A^i}\frac{\partial \mathcal{T}_r}{\partial x_A^j} = 0$$

$$g^{00}(x_A^0 + c\mathcal{T}_e, \boldsymbol{x}_B) - 2c\,g^{0i}(x_A^0 + c\mathcal{T}_e, \boldsymbol{x}_B)\frac{\partial \mathcal{T}_e}{\partial x_B^i} + c^2\,g^{ij}(x_A^0 + c\mathcal{T}_e, \boldsymbol{x}_B)\frac{\partial \mathcal{T}_e}{\partial x_B^i}\frac{\partial \mathcal{T}_e}{\partial x_B^j} = 0.$$

TTF is a dedicated World Function to light ray.
General Post-Minkowskian expansions are possible

Post-Minkowskian expansion of the TTF

see P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- A pM expansion of the TTF: $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}_r^{(n)}$

- Computation with an iterative procedure involving integrations over a straight line between the emitter and the spatial position of the receiver !

- Example at 1 pM: $\mathcal{T}_r^{(1)} = \frac{R_{AB}}{2c} \int_0^1 \left[g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} + N_{AB}^i N_{AB}^j g_{(1)}^{ij} \right]_{z^\alpha(\lambda)} d\lambda$

  with $z^\alpha(\lambda)$ the straight Mink. null path between em. and rec.

- Main advantages:

  - analytical computations relatively easy

  - very well adapted to numerical evaluation

Analytical results in Schwarzschild space-time

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014
P. Teyssandier, 2014, arXiv: 1407.4361

- A "simplified" iterative method has been developed for static spherically symmetric geometry

$$ds^2 = \left(1 - 2\frac{m}{r} + 2\beta\frac{m^2}{r^2} - \frac{3}{2}\beta_3\frac{m^3}{r^3} + \dots\right)dt^2 - \left(1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \dots\right)d\boldsymbol{x}^2$$

- In GR: $\gamma = \beta = \epsilon = \beta_3 = \gamma_3 = 1$

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \sum_{n>1}\mathcal{T}^{(n)}$

and the corresponding derivatives have been computed up to the 3rd pM order

Analytical results in Schwarzschild space-time

- A pM expansion of the TTF: $\mathcal{T} = \frac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

$$\mathcal{T}^{(1)} = \frac{(1+\gamma)m}{c} \ln \frac{r_A + r_B + |\boldsymbol{x}_B - \boldsymbol{x}_A|}{r_A + r_B - |\boldsymbol{x}_B - \boldsymbol{x}_A|}$$

see E. Shapiro, PRL 13, 26, 789, 1964
What is recommended by IERS !

$$\mathcal{T}^{(2)} = \frac{m^2}{r_A r_B} \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c} \left[\kappa \frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} - \frac{(1+\gamma)^2}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B} \right]$$

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
S. Klioner, S. Zschocke, CQG 27, 075015, 2010

$$\mathcal{T}^{(3)} = \frac{m^3}{r_A r_B} \left(\frac{1}{r_A} + \frac{1}{r_B} \right) \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c(1 + \boldsymbol{n}_A.\boldsymbol{n}_B)} \left[\kappa_3 - (1+\gamma)\kappa \frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} + \frac{(1+\gamma)^3}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B} \right]$$

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014

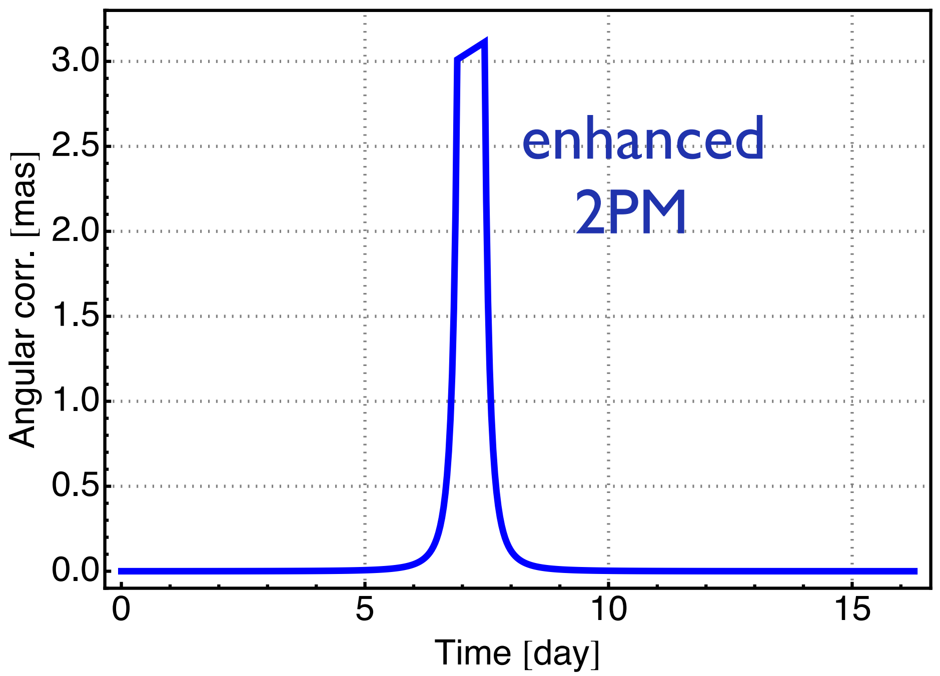
with $\kappa = 2 + 2\gamma - \beta + \frac{3}{4}\epsilon$

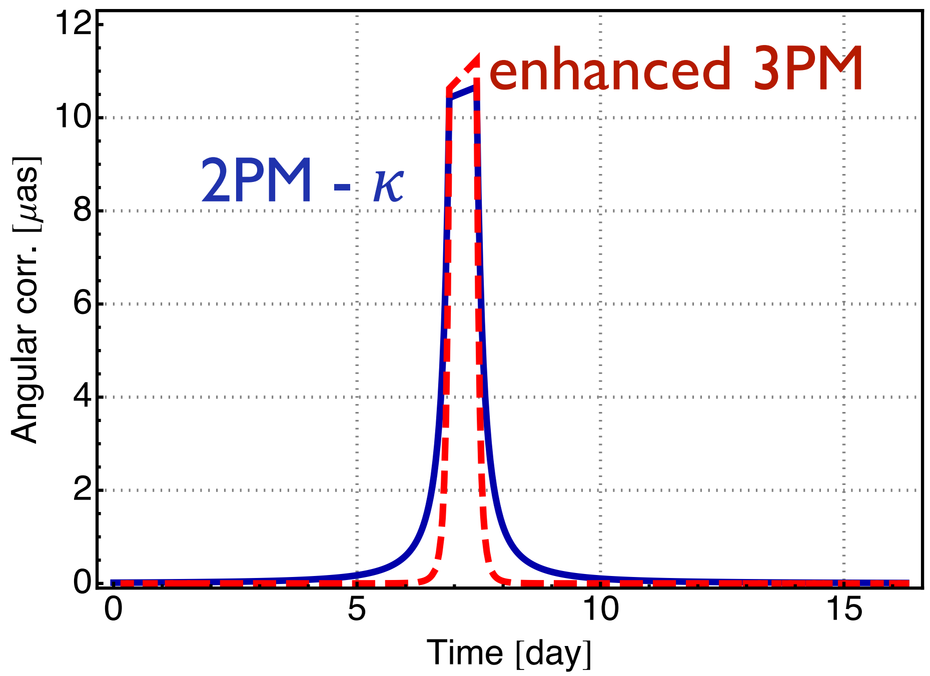
$\kappa_3 = 2\kappa - 2\beta(1+\gamma) + \frac{1}{4}(3\beta_3 + \gamma_3)$

and $\boldsymbol{n}_{A/B} = \frac{\boldsymbol{x}_{A/B}}{r_{A/B}}$

Is it necessary to go to the 3rd order?

- In a conjunction geometry, at each order n, there are enhanced terms proportional to $(1+\gamma)^n$
- Ex. with light deflection for Sun grazing rays: AGP space mission (old GAME). Expected accuracy: μas
  $\Rightarrow$ 3pM term needed





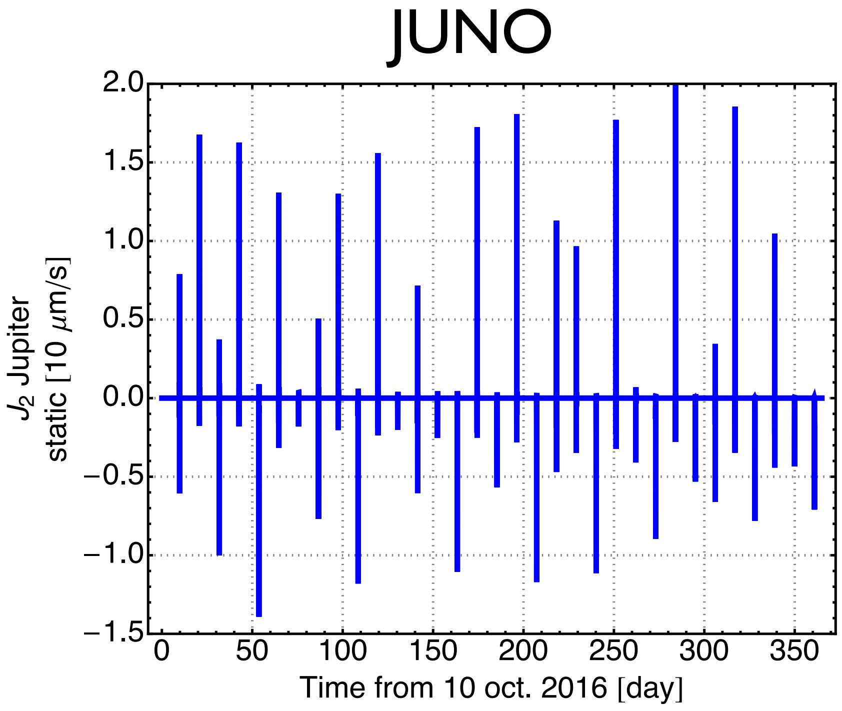
see A. Hees, S. Bertone, C. Le Poncin-Lafitte, PRD 89, 064045, 2014
P. Teyssandier, B. Linet, proceedings of JSR 2013, arXiv:1312.3510

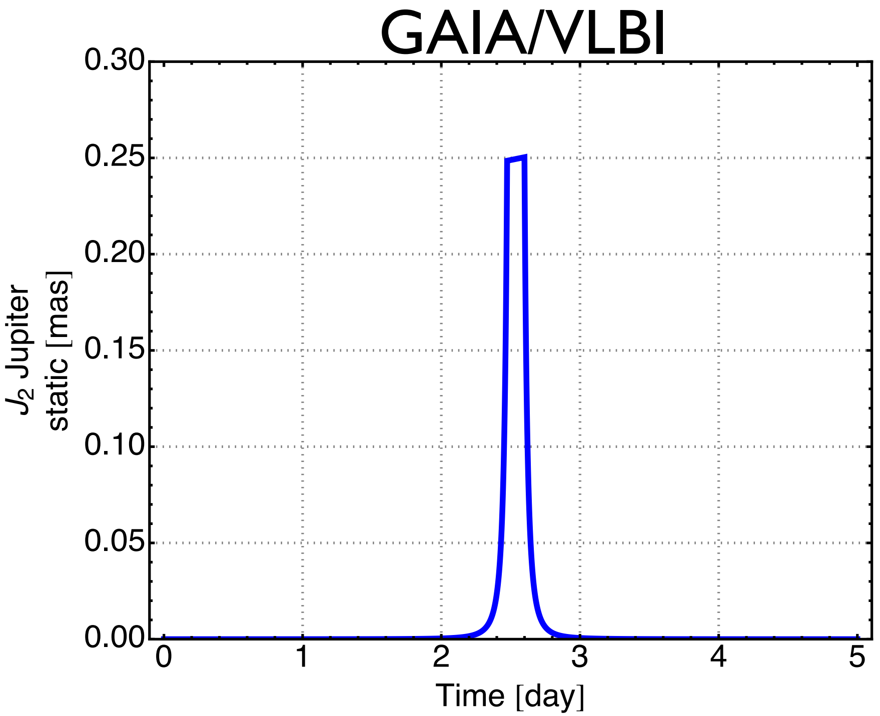
Analytical result around axisymmetric bodies

- Influence of all the multipole moments J_n from the grav. potential

  see C. Le Poncin-Lafitte, P. Teyssandier, PRD 77, 044029, 2008 for a computation with the TTF
  or S. Kopeikin, J. of Math. Physics 38, 2587, 1997 for another approach

- Influence of Jupiter J_2 on the JUNO Doppler (1 μm/s accuracy) and for GAIA (10 μas acc.)





- terms important for the data analysis for these missions

see Hees, Bertone, Le Poncin-Lafitte, PRD 90, 084020, 2014

What happens if the body is moving ?

see Hees, Bertone, Le Poncin-Lafitte, PRD 90, 084020, 2014

- At first pM order, the TTF for uniformly moving bodies can easily be derived from the TTF generated by a static body

$$\Delta(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \gamma(1 - \boldsymbol{N}_{AB}.\boldsymbol{\beta})\tilde{\Delta}(\boldsymbol{R}_A + \gamma\boldsymbol{\beta}R_{AB}, \boldsymbol{R}_B)$$

TTF in the moving case

static TTF

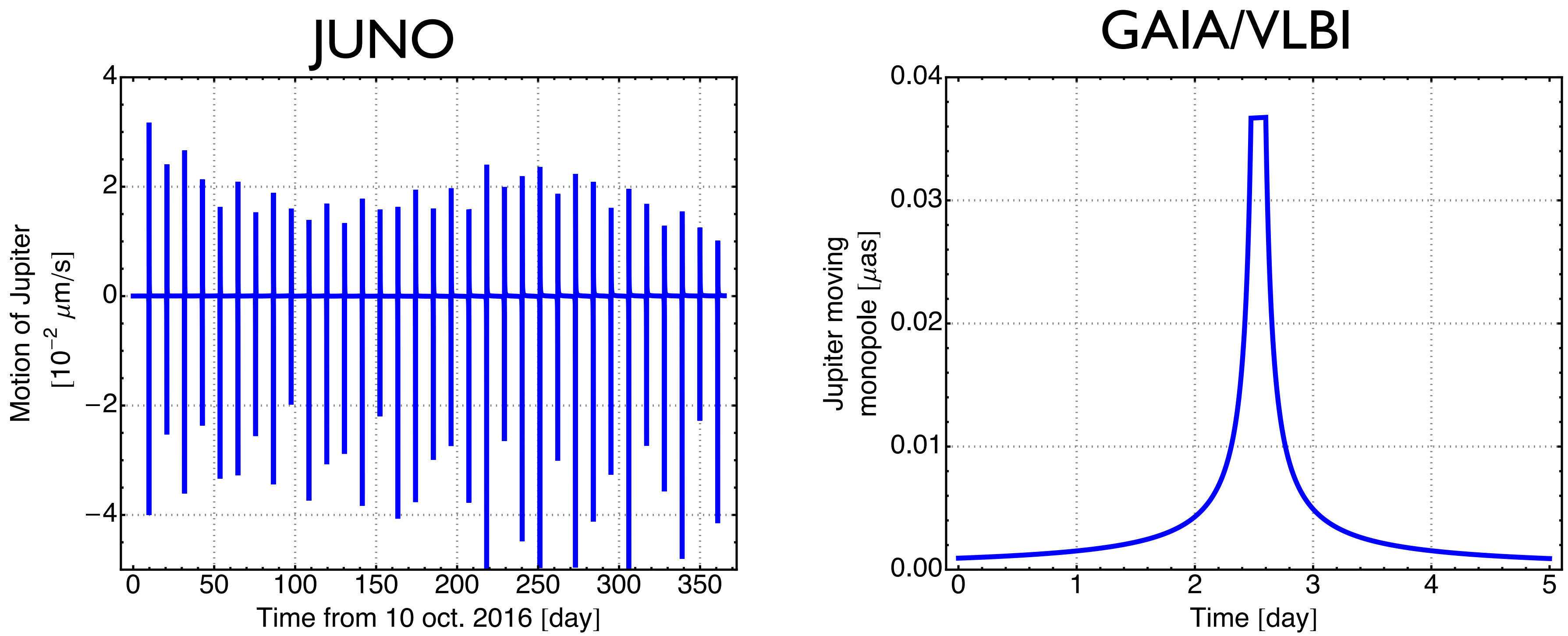
with $\boldsymbol{\beta} = \boldsymbol{v}/c, \quad \gamma = (1 - \beta^2)^{-1/2}$

and $\boldsymbol{R}_X$ depends on $\boldsymbol{x}_X, \boldsymbol{\beta}$

- All the analytical results computed for a static source can be extended in the case of a uniformly moving source

Ex.: motion of Jupiter

- Influence of Jupiter velocity on the JUNO Doppler (1μm/s accuracy) and for GAIA (10 μas acc.)



- depend highly on the orbit geometry: conjunction and $\beta.N_{AB}$
- In particular: should be reassessed for JUICE orbit

see Hees, Bertone, Le Poncin-Lafitte, PRD 90, 084020, 2014

Relativistic Celestial mechanics

- Objectives : to understand the motion of deflecting bodies == planets !
- Need to construct ephemerides fully consistent to GR

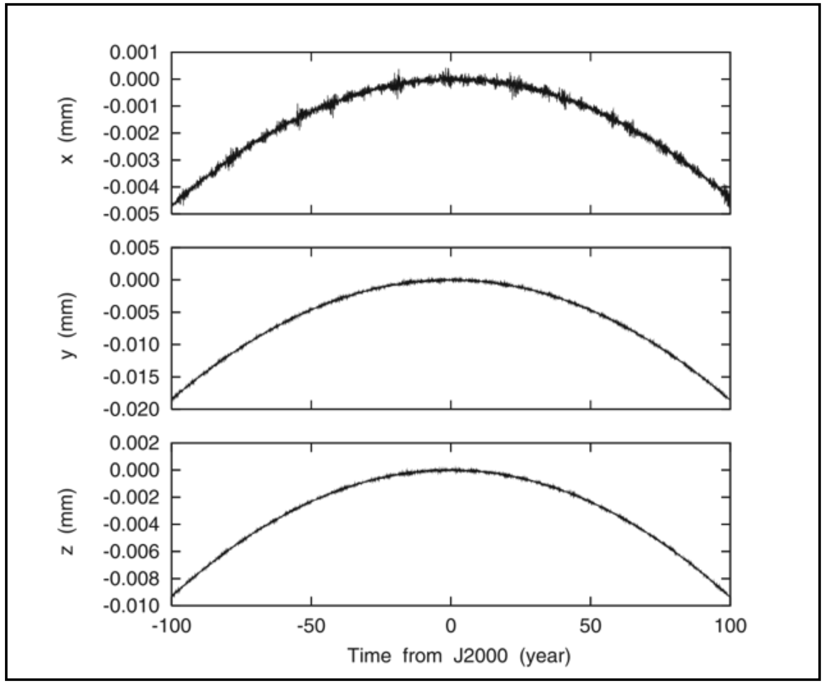
conservation laws

$$m^i_{EIH} = \sum_A M^A z^i_A \left[1 + \frac{1}{2c^2}\left(v_A^2 - \sum_{B\neq A}\frac{GM^B}{r_{AB}}\right)\right] + O\left(\frac{1}{c^4}\right),$$

$$p^i_{EIH} = \sum_A \left\{ M^A v^i_A \left[1 + \frac{1}{2c^2}\left(v_A^2 - \sum_{B\neq A}\frac{GM^B}{r_{AB}}\right)\right] - \frac{G}{2c^2}\sum_{B\neq A}\frac{GM^A M^B}{r_{AB}}(\mathbf{n}_{AB}.\mathbf{v}_B)\, n^i_{AB}\right\} + O\left(\frac{1}{c^4}\right).$$

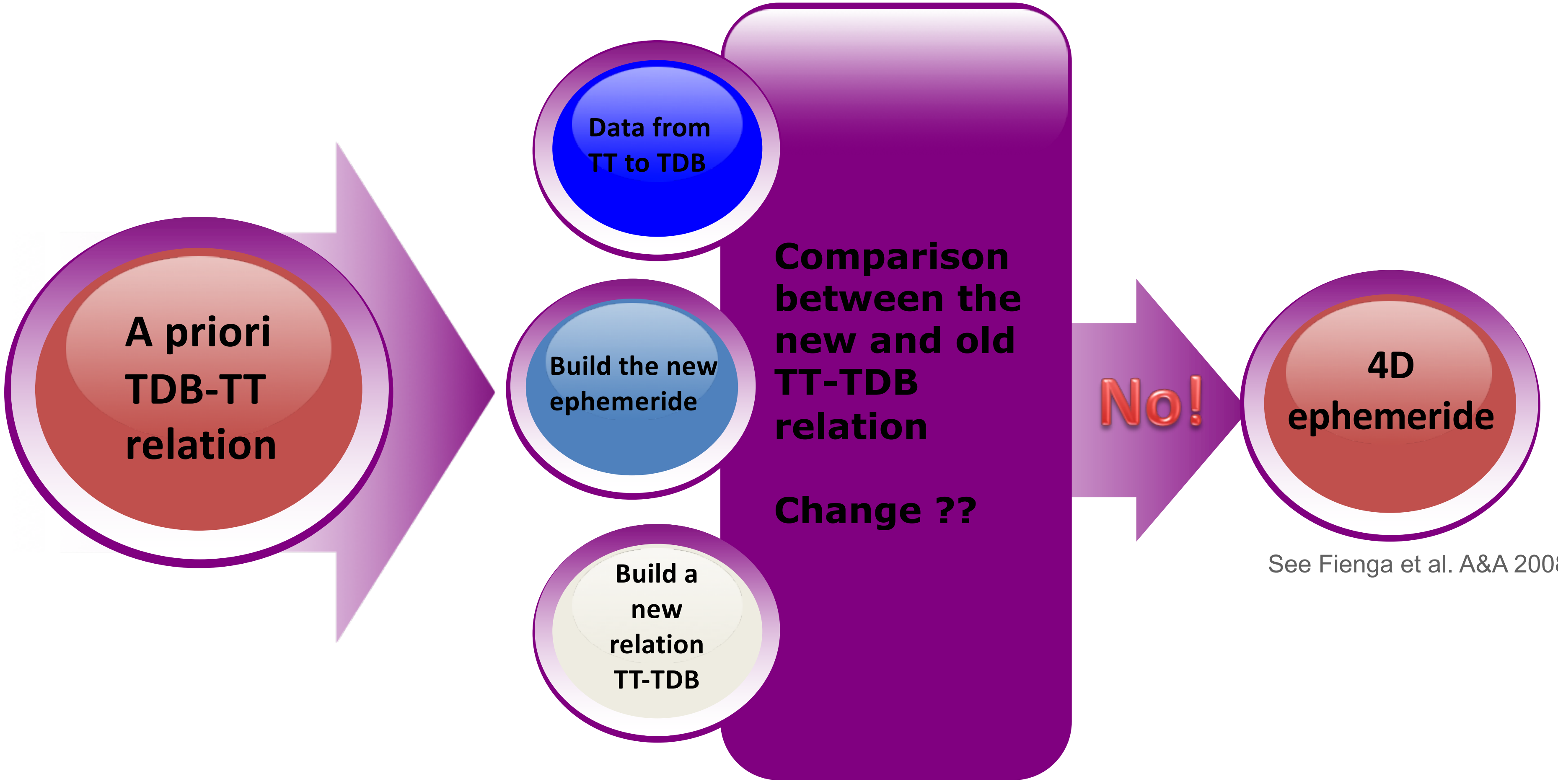
At least, EIH equations of motion

$$\begin{aligned}
\ddot{\boldsymbol{x}}_A = &-\sum_{B\neq A}\mu_B\frac{\boldsymbol{r}_{AB}}{|\boldsymbol{r}_{AB}|^3}\\
&+\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{\boldsymbol{r}_{AB}}{|\boldsymbol{r}_{AB}|^3}\left\{\sum_{C\neq B}\frac{\mu_C}{|\boldsymbol{r}_{BC}|} + 4\sum_{C\neq A}\frac{\mu_C}{|\boldsymbol{r}_{AC}|} + \frac{3}{2}\frac{(\boldsymbol{r}_{AB}\cdot\dot{\boldsymbol{x}}_B)^2}{|\boldsymbol{r}_{AB}|^2}\right.\\
&\qquad -\frac{1}{2}\sum_{C\neq A,B}\mu_C\frac{\boldsymbol{r}_{AB}\cdot\boldsymbol{r}_{BC}}{|\boldsymbol{r}_{BC}|^3}\\
&\qquad \left. -2\,\dot{\boldsymbol{x}}_B\cdot\dot{\boldsymbol{x}}_B - \dot{\boldsymbol{x}}_A\cdot\dot{\boldsymbol{x}}_A + 4\,\dot{\boldsymbol{x}}_A\cdot\dot{\boldsymbol{x}}_B\right\}\\
&+\frac{1}{c^2}\sum_{B\neq A}\mu_B\frac{\dot{\boldsymbol{x}}_A-\dot{\boldsymbol{x}}_B}{|\boldsymbol{r}_{AB}|^3}\left\{4\,\dot{\boldsymbol{x}}_A\cdot\boldsymbol{r}_{AB} - 3\,\dot{\boldsymbol{x}}_B\cdot\boldsymbol{r}_{AB}\right\}\\
&-\frac{1}{c^2}\frac{7}{2}\sum_{B\neq A}\frac{\mu_B}{|\boldsymbol{r}_{AB}|}\sum_{C\neq A,B}\mu_C\frac{\boldsymbol{r}_{BC}}{|\boldsymbol{r}_{BC}|^3} + O(c^{-4}),
\end{aligned}$$



Towards 4D ephemerides

The main problem is to distribute position/velocity but also Time scale transformation between TT and TDB



See Fienga et al. A&A 2008

The same problem with natural satellites ephemerides but in addition tidal effect as to be taken in to account.

PPN formalism and Sun J_2 : Gaia illustration

- highly correlated parameters: secular effect on orbital dynamics

$$\left\langle \frac{d\omega}{dt} \right\rangle = (2 + 2\gamma - \beta) n \frac{GM}{c^2 a(1-e^2)} + \frac{3}{2} n \frac{J_2 R^2}{a^2(1-e^2)^2}$$

- various asteroids orbital parameters help to decorrelate

- sensitivity:

	J_2	β
GAIA [5yr]	$\sigma_{J_2} \sim 5 \times 10^{-8}$	$\sigma_\beta \sim 4 \times 10^{-4}$
GAIA [10yr]	$\sigma_{J_2} \sim 1.5 \times 10^{-8}$	$\sigma_\beta \sim 10^{-4}$
INPOP	$(2.22 \pm 0.13) \times 10^{-7}$	$(0.0 \pm 6.9) \times 10^{-5}$

INPOP results from A. Fienga et al, Cel. Mech. Dyn. Astro. 2015

- correlation ~ 0.4

- complementary to planetary ephemerides: different analysis, not the same systematics BUT :

- Interesting: combined fit Gaia + planets

Lense-Thirring effect due to the Sun and PPN

Le Poncin-Lafitte, 2018, submitted PRD

- Relativistic frame dragging effect produced by the rotation of a body (due to the Spin S)
- impossible to estimate the Sun Lense-Thirring with planetary ephemerides: completely correlated with J_2 see W. Folkner et al, IPN, 2014
- Asteroids can decorrelate but Gaia not powerful enough

$$\frac{\sigma_S}{S} \sim 6.5 \quad [1.7 \text{ for } 10\text{yr}]$$

- Combination with radar observations to be considered
- But… not including the LT in the modeling leads to bias:

  - 10^{-8} on the J_2 (i.e. 10% of its value)

  - $5\text{x}10^{-5}$ on the β PPN

Must be included NOW in planetary ephemerides….

Conclusions : consequences on Conventions

- Last decade, huge activity on light propagation
  - taking into account Mass Multipole at 1PN approximation
  - considering motion of deflecting body at 2/3 PN approximation
  - All these funny things are not in IERS conventions

- Ephemerides 4D :
  - at 1PN approximation : Ok for the moment.
  - But testing GR : please, TAKE CARE and ASK to relativistic people before the disaster !

- Need to rewrite in a more compact, modern and easy-use the relativistic equations

- Local Relativistic Reference system for other planets may be needed, not crucial at present. But :
  - Relativistic Reference Systems and Alternative to GR. At 1PN, close to touch the limit. Need to consider seriously Mass/Spin multipole moments in Alternative Theory
  - Light at 2PN needed == Reference System at 2PN to be consistent.