The normal gravity determination

The normal gravity value γ on the surface of the ellipsoid is defined by the Somigliana formula :

$$\gamma = \gamma_e \frac{1 + k \sin^2\Phi}{\sqrt{(1 - e^2 \sin^2\Phi)}}, \text{ where } k = \frac{b\gamma_p - a\gamma_e}{a\gamma_e},$$

where γ_e gravity on the equator
γ_p gravity at the pole
a earth equatorial radius, semi-major axis of the ellipsoid
b polar radius, semi-minor axis of the ellipsoid
e first eccentricity
Φ geodetic (ellipsoidal) latitude

From this formula, we can derive a linear formula, called approximation of Chebychev, which doesn't require a square root extraction :

$$\gamma = \gamma_e (1 + a_2 \sin^2\Phi + a_4 \sin^4\Phi + a_6 \sin^6\Phi + ...)$$

which is more accurate than the conventional formula below :

$$\gamma = \gamma_e (1 + f^* \sin^2\Phi - \frac{1}{4} f_4 \sin^2 2\Phi) \quad \text{with} \quad f^* = \frac{\gamma_p - \gamma_e}{\gamma_e} \text{ (gravity flattening) and}$$

$$f_4 = -\frac{1}{2} f^2 + \frac{5}{2} f a \frac{\omega^2}{\gamma_e}$$ (f ellipsoidal flattening, ω angular velocity)

GRS 80 geodetic system :

γ_e	=	9.7803267715 $m s^{-2}$
γ_p	=	9.8321863685 $m s^{-2}$
a	=	6378137 m
b	=	6356752.3141 m
e^2	=	0.00669438002290
k	=	0.001931851353

The linear formula :

$$\gamma = \gamma_e (1 + 0.005\,279\,0414 \sin^2\Phi + 0.000\,023\,2718 \sin^4\Phi + 0.000\,000\,1262 \sin^6\Phi + 0.000\,000\,0007 \sin^8\Phi)$$

shows a relative error of 10^{-10} which corresponds to 10^{-3} mm.s^{-2} = 10^{-4} mgal.

The shorter but less accurate formula is frequently used :

$$\gamma = \gamma_e (1 + 0.005\,279\,0414 \sin^2\Phi + 0.000\,023\,2718 \sin^4\Phi)$$

The conventional series :

$$\gamma = 9.780\,327\,(1 + 0.005\,3024 \sin^2\Phi - 0.000\,0058 \sin^2 2\Phi)\; m s^{-2}$$

has an accuracy of less than 1 $mm.s^{-2}$ = 0.1 mgal. But it can be used to convert gravity anomalies from the GRS 1930 formula to the GRS 80 :

$$\gamma_{1980} - \gamma_{1930} = -16.3 + 13.7 \sin^2 \Phi \ \text{mgal} \quad (1 \text{ mgal} = 10^{-5} \text{ m.s}^{-2})$$

the difference coming mainly from the modification of the gravity value at Potsdam (-14 mgals)

To convert the gravity value issued from the 1967 formula to the 1980 formula, a more accurate development can be used :

$$\gamma_{1980} - \gamma_{1967} = (0.8316 + 0.0782 \sin^2 \Phi - 0.0007 \sin^2 2\Phi) \ \text{mgal}$$

GRS67 geodetic system :

$$\begin{aligned} \gamma_e &= 9.7803184558 \ ms^{-2} \\ \gamma_p &= 9.8321772792 \ ms^{-2} \\ a &= 6378137 \text{ m} \\ b &= 6356752.3141 \text{ m} \\ e^2 &= 0.00669460532856 \\ k &= 0.00193166338321 \end{aligned}$$

The linear formula :

$$\gamma = 9.780\,318\,4558 (1 + 0.005\,278\,895 \sin^2 \Phi + 0.000\,023\,462 \sin^4 \Phi) \text{ m.s}^{-2}$$

shows a maximum error of 0.004 mgal.

The conventional series :

$$\gamma = 9.780\,318\ (1 + 0.005\,3024 \sin^2 \Phi - 0.000\,0059 \sin^2 2\Phi) \text{ m.s}^{-2}$$

has an accuracy of only 1 $mm.s^{-2}$ = 0,1 mgal.

To convert gravity values issued from GRS 1930 to GRS 1967, one can use the following formula :

$$\gamma_{1967} - \gamma_{1930} = (-17.2 + 13.6 \sin^2 \Phi) \ \text{mgal}$$

1930 geodetic system :

$$\gamma = 9.780\,49 (1 + 0.005\,2884 \sin^2 \Phi - 0.000\,0059 \sin^2 2\Phi) \ \text{m.s}^{-2}$$

Advice :

Use the Somigliana formulas instead of the linear formulas, which are less accurate.
To convert anomalies from a geodetic system to another one :

- if the measured gravity is available, redetermine the normal gravity value using the Somigliana formula, then the gravity anomalies,

- if the measured gravity is not available, apply a correction to the gravity anomalies, adding the difference between the normal gravities determined for each of the two ellipsoids.