

ANNEX A: Adjustment (Theory of Operations)

1. Absolute data (read in the « config file »)

Loop **i** on all the lines: **ABSOLUT** station measure error

adjust.adata[] updated

```
adjust.adata[i].sta = station
adjust.adata[i].mes = measure
adjust.adata[i].err = error
adjust.adata[i].rei = 0
```

adjust.adjinfo.iabstot is the total absolute measures

2. Relative data (read in the « config file »)

Loop **i** on all the lines: **IRELFIL** file

adjust.adjinfo.cfg_in_rdata[i] updated

adjust.adjinfo.r_ftotal is the total relative measures

3. Relative data (reading of « c » or « r » files from CG6TOOL

For simplification, deletion of the term « adjust » in the rest of the document

3.1. Loop on all file of **adjinfo.cfg_in_rdata[]**

- Search for the current gravimeter in the **adjinfo.r_gid[i]** table
 - Not found = **NEW**

```
gc = adjinfo.r_gtotal // current gravimeter index
adjinfo.r_gid[gc] = gid // current gravimeter identifier
adjinfo.r_sbyg[gc] = 0 // number of stations for the current gravimeter
adjinfo.r_rbyg[gc] = 0 // number of files for the current gravimeter
s_total_curr = 0
s_uniq_curr = 0
```
 - Found

```
gc = i // current gravimeter index
s_total_curr = adjinfo.r_sbyg[gc] // number of stations
s_uniq_curr = adjinfo.r_sutot[gc] // number of unique stations
```
- Loop **nbsta** on all stations of the file and **rdata[]** updated

```
j = s_total_curr + nbsta
rdata[j][gc].id = gid
rdata[j][gc].tra = adjinfo.r_rbyg[gc] // profile number
rdata[j][gc].sta = current station
rdata[j][gc].mes = gi - g0
rdata[j][gc].err = current error
```

If new station :

 - **adjinfo.r_sunum[s_uniq_curr][gc]** = current station
 - **s_uniq_curr++**
- **rinfo[][]** update

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```
adjinfo.r_sbyg[gc] = adjinfo.r_sbyg[gc] + nbsta // stations counter
adjinfo.r_sbyr[adjinfo.r_rbyg[gc]][gc] = nbsta // stations/profile counter
adjinfo.r_sutot[gc] = s_uniq_curr // unique stations counter
adjinfo.r_rbyg[gc] = adjinfo.r_rbyg[gc] + 1 // profile counter
adjinfo.r_otal[gc] = adjinfo.r_otal[gc] + nbsta // observations counter
```

3.2. Unique stations counter

- Loop **ig** on all the gravimeters (**adjinfo.r_gtotal**)
 - Loop on all the stations of the current gravimeter **ns = adjinfo.r_sutot[ig]**
 - **adjinfo.r_usta[sta_total][i]** update
- **adjinfo.r_stotal = sta_total** // total number of stations

3.3. Count of repetition by station

- Loop **ig** on all the gravimeters (**adjinfo.r_gtotal**)
 - Loop on all the stations of the current gravimeter **nbs = adjinfo.r_sbyg[ig]**
 - **tc = rdata[nbs][ig].tra** // Current profile
 - **nbs_tc = adjinfo.r_sbyr[tc][ig]** // number of station of the current profile
 - Loop on all the stations of the current profile : **nbs → nbs + nbs_tc**
 - If repetition then **adjinfo.r_usta[j][1]++**

3.4. Examples

```
ftes1c16.061 : (profile 1 / Gravi 1) : 9 9 9 35 35 500 500 35 35 9 9
ftes1c16.062 : (profile 2 / Gravi 1) : 9 9 500 500 355 355 9 9
ftes2c16.061 : (profile 3 / Gravi 2) : 9 9 9 35 35 600 600 35 9 9
ftes2c16.062 : (profile 4 / Gravi 2) : 35 35 500 500 35 35
ftes1c16.063 : (profile 5 / Gravi 1) : 35 35 600 600 35 35
```

```
adjust.adjinfo.cfg_in_rdata[0] = « ftes1c16.061 »
adjust.adjinfo.cfg_in_rdata[1] = « ftes1c16.062 »
adjust.adjinfo.cfg_in_rdata[2] = « ftes2c16.061 »
adjust.adjinfo.cfg_in_rdata[3] = « ftes2c16.062 »
adjust.adjinfo.cfg_in_rdata[4] = « ftes1c16.063 »
```

```
adjust.adjinfo.r_ftotal = 5 // Total number of files (profiles)
```

- Reading the first file : profile 1 / Gravimeter 1
New gravimeter then **gc = adjinfo.r_gtotal = 0**

```
adjinfo.r_gid[0] = gid // current gravimeter identifier
adjinfo.r_sbyg[0] = 0 // number of stations for the current gravimeter initialized
adjinfo.r_rbyg[0] = 0 // number of files for the current gravimeter initialized
s_total_curr = 0 // total number of stations initialized
s_uniq_curr = 0 // total number of unique stations initialized
adjinfo.r_gtotal = 1 // total number of gravimeters updated
```

 - Reading line 1 of the file 1

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- Current station = 9
- First measure (**nbsta=0**) then I save **g₀**
- $i = s_total_curr + nbsta = 0$
- **rdata[0][0].id = 1**
rdata[0][0].tra = adjinfo.r_rbyg[0] = 0
rdata[0][0].sta = 9
rdata[0][0].mes = g₀ - g₀ = 0
rdata[0][0].err = $\sqrt{(e_0)^2 + (e_0)^2} = e_0 \sqrt{2}$
- New station : **adjinfo.r_sunum[0][0] = 9** et **s_uniq_curr = 1**
- **nbsta = 1**
- Reading line 2 of the file 1
 - Current station = 9
 - $i = s_total_curr + nbsta = 1$
 - **rdata[1][0].id = 1**
rdata[1][0].tra = adjinfo.r_rbyg[0] = 0
rdata[1][0].sta = 9
rdata[1][0].mes = g₁ - g₀
rdata[1][0].err = $\sqrt{(e_0)^2 + (e_1)^2}$
 - Old station
 - **nbsta = 2**
- Reading line 3 of the file 1
 - Current station = 9
 - $i = s_total_curr + nbsta = 2$
 - **rdata[2][0].id = 1**
rdata[2][0].tra = adjinfo.r_rbyg[0] = 0
rdata[2][0].sta = 9
rdata[2][0].mes = g₂ - g₀
rdata[2][0].err = $\sqrt{(e_0)^2 + (e_2)^2}$
 - Old station
 - **nbsta = 3**
- Reading line 4 of the file 1
 - Current station = 35
 - $i = s_total_curr + nbsta = 3$
 - **rdata[3][0].id = 1**
rdata[3][0].tra = adjinfo.r_rbyg[0] = 0
rdata[3][0].sta = 35
rdata[3][0].mes = g₃ - g₀
rdata[3][0].err = $\sqrt{(e_0)^2 + (e_3)^2}$
 - New station : **adjinfo.r_sunum[1][0] = 35** et **s_uniq_curr = 2**
 - **nbsta = 4**

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- Readin the last line (line 11) of the file 1
 - Current station = 9
 - $i = s_total_curr + nbsta = 10$
 - $rdata[10][0].id = 1$
 $rdata[10][0].tra = adjinfo.r_rbyg[0] = 0$
 $rdata[10][0].sta = 9$
 $rdata[10][0].mes = g_{10} - g_0$
 $rdata[10][0].err = \sqrt{(e_0)^2 + (e_{10})^2}$
 - Old station
 - $nbsta = 11$
- End of the file (profile) : **adjinfo** update
 - $adjinfo.r_sbyg[0] = adjinfo.r_sbyg[0] + nbsta = 11$ // stations/gravi
 - $adjinfo.r_sbyr[0][0] = nbsta = 11$ // station/profile
 - $adjinfo.r_sutot[0] = s_uniq_curr = 3$ // unique stations/gravi
 - $adjinfo.r_rbyg[0] = adjinfo.r_rbyg[0] + 1 = 1$ // profiles
 - $adjinfo.r_ototal = adjinfo.r_ototal + nbsta = 11$ // observations

	0	1	2	3	4	5	6	7	8	9	10
$rdata[][0].sta$	9	9	9	35	35	500	500	35	35	9	9
$rdata[][0].mes$	0	$g_1 - g_0$	$g_2 - g_0$	$g_3 - g_0$	$g_4 - g_0$	$g_5 - g_0$	$g_6 - g_0$	$g_7 - g_0$	$g_8 - g_0$	$g_9 - g_0$	$g_{10} - g_0$

$adjinfo.r_sunum[][0]$	9	35	500
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- Reading of the second file : profile 2 / Gravimeter 1

Old gravimeter with $gc = 0$

$s_total_curr = adjinfo.r_sbyg[0] = 11$ // Total stations initialized

$s_uniq_curr = adjinfo.r_sutot[0] = 3$ // Unique stations initialized

- End of file 2 (profile 2) : **adjinfo** update
 - $adjinfo.r_sbyg[0] = adjinfo.r_sbyg[0] + nbsta = 11 + 8 = 19$
 - $adjinfo.r_sbyr[1][0] = nbsta = 8$ // station/profile
 - $adjinfo.r_sutot[0] = s_uniq_curr = 4$ // unique stations /gravi
 - $adjinfo.r_rbyg[0] = adjinfo.r_rbyg[0] + 1 = 1 + 1 = 2$ // profiles
 - $adjinfo.r_ototal = adjinfo.r_ototal + nbsta = 19$ // observations

	0	10	11	12	13	14	15	16	17	18
$rdata[][0].sta$	9	9	9	9	500	500	355	355	9	9
$rdata[][0].mes$	0	$g_{10} - g_0$	0	$g_{12} - g_{11}$	$g_{13} - g_{11}$	$g_{14} - g_{11}$	$g_{15} - g_{11}$	$g_{16} - g_{11}$	$g_{17} - g_{11}$	$g_{18} - g_{11}$

$adjinfo.r_sunum[][0]$	9	35	500	355
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- Reading the third file : Profile 3 / Gravimeter 2

New gravimeter then $gc = \text{adjinfo.r_gtotal} = 1$

```

adjinfo.r_gid[1] = gid // current gravimeter identifier
adjinfo.r_sbyg[1] = 0 // number of stations for the current gravimeter initialized
adjinfo.r_rbyg[1] = 0 // number of files for the current gravimeter initialized
s_total_curr = 0 // total number of stations initialized
s_uniq_curr = 0 // total number of unique stations initialized
adjinfo.r_gtotal = 2 // total number of gravimeters updated
    
```

- End of the file 3 (profile 3) : adjinfo updated

- $\text{adjinfo.r_sbyg}[1] = \text{adjinfo.r_sbyg}[1] + \text{nbsta} = 0 + 10 = 10$
- $\text{adjinfo.r_sbyr}[0][1] = \text{nbsta} = 10$ // station/profile
- $\text{adjinfo.r_sutot}[1] = \text{s_uniq_curr} = 3$ // unique stations /gravi
- $\text{adjinfo.r_rbyg}[1] = \text{adjinfo.r_rbyg}[1] + 1 = 0 + 1 = 1$ // profiles
- $\text{adjinfo.r_ototal} = \text{adjinfo.r_ototal} + \text{nbsta} = 29$ // observations

	0	1	2	3	4	5	6	7	8	9
rdata[][1].sta	9	9	9	35	35	600	600	35	9	9
rdata[][1].mes	0	g1-g0	g2-g0	g3-g0	g4-g0	g5-g0	g6-g0	g7-g0	g8-g0	g9-g0

adjinfo.r_sunum[][1]	9	35	600
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- Reading of the last file : profile 5 / Gravimeter 1

Old gravimeter with $gc = 0$

```

s_total_curr = adjinfo.r_sbyg[0] = 18 // Total stations initialized
s_uniq_curr = adjinfo.r_sutot[0] = 4 // Unique stations initialized
    
```

- End of file 5 (profile 5) : adjinfo updated

- $\text{adjinfo.r_sbyg}[0] = \text{adjinfo.r_sbyg}[0] + \text{nbsta} = 18 + 6 = 25$
- $\text{adjinfo.r_sbyr}[2][0] = \text{nbsta} = 6$ // station/profile
- $\text{adjinfo.r_sutot}[0] = \text{s_uniq_curr} = 5$ // unique stations/gravi
- $\text{adjinfo.r_rbyg}[0] = \text{adjinfo.r_rbyg}[0] + 1 = 2 + 1 = 3$ // profiles
- $\text{adjinfo.r_ototal} = \text{adjinfo.r_ototal} + \text{nbsta} = 41$ // observations

	0	9	10	11	15	16	17	18	19	24
rdata[][0].sta	9	9	9	9	9	35	355	9	35	35
rdata[][1].sta	9	9	35	35	15					

adjinfo.r_sunum[][0]	9	35	500	355	600
adjinfo.r_sunum[][1]	9	35	600	500	

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- End of files reading : 5 Profiles / 2 Gravimeters

- **adjinfo.r_ototal** = 41 // Total Observations
- **adjinfo.r_gtotal** = 2 // Gravimeters used
- **adjinfo.r_stotal** = 5 // Total unique stations
- **adjinfo.r_rbyg[]** // Profiles (files) by gravimeter
 adjinfo.r_rbyg[0] = 3
 adjinfo.r_rbyg[1] = 2
- **adjinfo.r_sbyg[]** // Stations by gravimeter
 adjinfo.r_sbyg[0] = 25
 adjinfo.r_sbyg[1] = 16
- **adjinfo.r_sbyr[][]** // Number of stations by profile
 adjinfo.r_sbyr[][0]

11	8	6
10	6	

 adjinfo.r_sbyr[][1]
- **adjinfo.r_sutot[]** // Number of unique stations by profile
 adjinfo.r_sutot[0] = 5
 adjinfo.r_sutot[1] = 4
- **adjinfo.r_sunum[][]** // Unique stations number by gravimeter
 adjinfo.r_sunum[][0]

9	35	500	355	600
9	35	600	500	

 adjinfo.r_sunum[][1]
- **rdata[][]** // Observations (*sta, trav, gid, mes, err, rei, réo*)
 rdata[][0]

Profile 1 / Gravi 1	Profile 2 / Gravi 1	Profile 5 / Gravi 1
Profile 3 / Gravi 2	Profile 4 / Gravi 2	

 rdata[][1]
- **adjinfo.r_usta[][]** // Unique stations of the field campaign
 adjinfo.r_usta[][0]

9	35	500	355	600
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 Number

ANNEX A: Adjustment (Theory of Operations)

4. Determination of calibration coefficients (Scaling factors)

✓ Illustration of the calculation with the example above :

Reference Gravimeter **Gr**: $G_0 / 3$ profiles (*.r_rbyg[0]*) / 25 stations (*.r_sbyg[0]*)

Gravimeter to calibrate **Gc**: $G_1 / 2$ profiles (*.r_rbyg[1]*) / 16 stations (*.r_sbyg[1]*)

✓ Step 1 : Recuperation of all the profiles of the **Gc** gravimeter to calibrate

nbt is the number of profile for the gravimeter **g**, *nbsti* the number of station of the profile t_i with **g** and Δseg_{max} the final number of segment for **g**.

For each profile *i* realized with the gravimeter **g** : $nbsti = \text{adjinfo.r_sbyr}[t_i][g]$

$$\Delta seg_{max} = \sum_{i=0}^{nbt-1} (\sum_{j=1}^{nbsti-1} j)$$

For the gravimeter G_1 : $\Delta seg_{max} = \Sigma(10-1) + \Sigma(6-1) = 45+15 = 60$ segments

At the end we have an array with for each segment (**s1-s2**):

- the index (in *.r_usta* [[]]) of the stations s1 et s2,
- the difference ($g_{end}-g_{begin}$) and the associate error ($\sqrt{((e_{end})^2+(e_{begin})^2)}$).

adjinfo.r_usta [[0]	9	35	500	355	600	Number
	0	1	2	3	4	Index

rdata [[1.sta	9	9	9	35	35	600	600	35	9	9	35	35	500	500	35	35
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

	ΔS_i-S_0				ΔS_i-S_1				ΔS_9-S_8				ΔS_i-S_{10}				ΔS_i-S_{11}				$\Delta S_{15}-S_{14}$					
Gcal_s1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	1
Gcal_s2	0	0	1	1	4	4	1	0	0	1	1	4	4	1	0	0	0	0	1	1	1	2	2	1	1	1
Gcal_co	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4 5				12 13																					

✓ Step 2 : The duplicate segments are averaged for the gravimeter **Gc** to calibrate

Loop for all the segments in order to average the duplicates.

Gcal_s1	0	0	0	1	1	1	4	4	1	2
Gcal_s2	0	1	4	1	4	0	1	0	2	1
Stations	9:9	9:35	9:600	35:35	35:600	35:9	600:35	600:9	35:500	500:35

For each segment *j* identical and *n* the number of duplicate segments

<ul style="list-style-type: none"> • $n > 1$: $\overline{\Delta Gc_j} = \frac{\sum_1^N w_i \Delta Gc_i}{\sum_1^N w_i}$ ($w_i = \frac{1}{\Delta Ec_i^2}$) et $\overline{\Delta Ec_j} = \sqrt{\frac{\sum_1^N w_i \Delta Gc_i^2}{\sum_1^N w_i} - \overline{\Delta Gc_j}^2}$ • $n = 1$: $\overline{\Delta Gc_j} = \Delta Gc_j$ et $\overline{\Delta Ec_j} = \Delta Ec_j$
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✓ Step 3 and 4 : same processing with the reference gravimeter **Gr**

ANNEX A: Adjustment (Theory of Operations)

✓ Step 5 : Linear relation between the two gravimeters

The purpose of this step is to calculate k such that $k \cdot G_c = G_r$ using a Least-square method. We search a straight line that minimize the sum of the squares of the residual values $(G_r - k \cdot G_c - b)$. Each element of the sum is weighted by the weight $w_i = 1 / ((\Delta E_{c_i})^2 + (\Delta E_{r_i})^2)$

- Matrix generation and resolution of the obtained system (**PASS 1**) :

$$\begin{array}{c}
 \left[\mathbf{W} \right] \times \begin{array}{|c|} \hline \Delta G_{c1} \\ \hline \Delta G_{c2} \\ \hline \cdot \\ \hline \cdot \\ \hline \cdot \\ \hline \Delta G_{cn} \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline \cdot \\ \hline \cdot \\ \hline \cdot \\ \hline 1 \\ \hline \end{array} \\
 \times \begin{array}{|c|} \hline \mathbf{k}_1 \\ \hline \mathbf{b}_1 \\ \hline \end{array} \\
 = \left[\mathbf{W} \right] \times \begin{array}{|c|} \hline \Delta G_{r1} \\ \hline \Delta G_{r2} \\ \hline \cdot \\ \hline \cdot \\ \hline \cdot \\ \hline \Delta G_{rn} \\ \hline \end{array}
 \end{array}$$

At the end of **PASS 1** we obtain the calibration coefficients (k_1, b_1) as well as the **standard deviation** σ_1 for the residuals.

- Removal of isolated points ($r_{\text{resi}} > 3\sigma_{i-1}$) : (**PASS 2 – PASS 20**)

20 iterations with deletion of data whose residue is greater than three times the standard deviation calculated during the previous iteration.

The information is displayed on the screen.

- Determination of the final calibration coefficient k

Use of the remaining data to calculate the parameter k of the line passing through the origin ($b=0$).

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5. Adjustment of all data (Adjustment)

- ✓ Step 1 : Recovery of “user” k_g coefficients of each gravimeter
- ✓ Step 2 : Value estimation for the first station (base) of each profile (file)
 - Loop of all absolute stations read in the *config_file* and stored in **adata[]**. The stations used in the adjustment (stored in **r_usta[][0]**) allow to build the C, B and W matrices :

adata[].sta	Sa_0	Sa_1	Sa_i	Sa_{n-1}	n absolute sites
adata[].mes	Ga_0	Ga_1	Ga_i	Ga_{n-1}	
adata[].err	Ea_0	Ea_1	Ea_i	Ea_{n-1}	

r_usta[][0]	S_0	S_1	S_i	S_{m-1}	m adjusted stations
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If all the absolute values are used in the adjustment then the value read in **adata[i]** (station in **r_usta[j]**) will cause the update of the following cells (**i-th** column of A array) :

$$C[i][j] = 1 ; B[i] = Ga_i ; W[i][i] = 1/(Ea_i)^2$$

- Loop of all relative stations stored in **rdata[]**.
 - The segment [**S1(i):S1(j)**] (repetition/reoccupation) will be use only if **S1** is an absolute site (**adata[ii]**) with index **jj** in **r_usta[]** :

ind = current index
Kc = current calibration coefficient

$m1 = rdata[i].mes ; e1 = rdata[i].err$
 $m2 = rdata[j].mes ; e2 = rdata[j].err$

C[ind][jj] = 1 ;

B[ind] = $Ga_{ii} + (m2 - m1) * Kc$

W[ind][ind] = $1 / ((e2)^2 + (e1)^2)$

- Segment [**S1(i):S2(j)**]. The station **S1** (**S2**) have an index **ii** (**jj** respectively) in **r_usta[]**

ind = current index
Kc = current calibration coefficient

$m1 = rdata[i].mes ; e1 = rdata[i].err$
 $m2 = rdata[j].mes ; e2 = rdata[j].err$

C[ind][ii] = -1 ; C[ind][jj] = 1 ;

B[ind] = $(m2 - m1) * Kc$

W[ind][ind] = $1 / ((e2)^2 + (e1)^2)$

- Determination of the linear system : **[W] [C] [X] = [W] [B]**

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- ✓ Step 3 : Determination of a value for all the stations in the network
 - If the first station of a profile is unknown (because it's not an absolute site) then stations are undetermined and another calculation is automatically realized.
 - Segment [S1:S1] with S1 absolute site : same above
 - Segment [S1(i):S1(j)] (S1 adjusted site in step 2) :

$$\begin{aligned} ind &= \text{current index} \\ Kc &= \text{current calibration coefficient} \\ m1 &= rdata[i].mes ; e1 = rdata[i].err \\ m2 &= rdata[j].mes ; e2 = rdata[j].err \\ G_{adj} &= \text{Value for S1 obtained in step 2} \\ e_{adj} &= \text{Error for S1 obtained in step 2} \\ C[ind][jj] &= 1 ; \\ B[ind] &= G_{adj} + (m2 - m1) * Kc \\ W[ind][ind] &= 1 / ((e2)^2 + (e1)^2 + (e_{adj})^2) \end{aligned}$$

- Segment [S1(i):S2(j)] : same above

- ✓ Step 4 : Data filtering (Option)

Another iteration is realized if the keyword **ADJNSIG** is activate in the *config_file*.

All the observations with residues biggest than **ADJNSIG * σ** are eliminated.

6. Summary

In each site **S** of a profile we have measured with a gravimeter **m** the gravity **g** (with an error **e**). (**g₀**, **e₀**) is the first measure for the base **S₀** (first station). (**g_i**, **e_i**) is the measure for the station **S_i**.

- ✓ Calculation of all the segments **S₀-S_i** : $\Delta g_i = g_i - g_0$, $E_i \sqrt{((e_0)^2 + (e_i)^2)}$
- ✓ Duplicate segments (**n**) are averaged : $\overline{\Delta g_i}$, $\overline{\Delta E_i}$
- ✓ Calculation of the calibration coefficient **K_m** | $G_{ref} = K_m * G_m$
- ✓ Adjustment of all the network stations (absolute and relative data)
 - The absolute measures (**a_i**, **e_i**) are weighted by $1/(e_i)^2$
 - The segments **S_i[g_i,e_i]:S_j[g_j,e_j]** with **i≠j** are weighted by $1/((e_i)^2 + (e_j)^2)$
 - The segments **S_n[g_i,e_i]:S_n[g_j,e_j]** (with **S_n** no absolute) are weighted by $1/((e_i)^2 + (e_j)^2 + (\sigma_n)^2)$ with **σ_n** (standard deviation) obtained for **S_n** in step 2.